In the experiment under discussion,[4] the counting rate associated with the reactor was, before various corrections, determined to be 16 ± 3 sec^{-1}. Since these corrections reduce the rate which could be attributed to (1) by a factor ~ 10, a conservative upper limit on the decay rate is obtained by using the value 10/sec.

If the lifetime of the $\bar{\nu}_e$ against the decay for (1) is τ, then the limiting lifetime against decay τ_e is given by

$$dN/dt = -N/\tau_e, \tag{2}$$

where $dN/dt = -10/\text{sec}$, $f = 1.3 \times 10^{13}\ \bar{\nu}_e/\text{cm}^2\ \text{sec}$, $V = 1.4 \times 10^6\ \text{cm}^3$, and $c = 3 \times 10^{10}$ cm/sec so that $N = fV/c = 6 \times 10^8\ \bar{\nu}_e$. As a result, $\tau_e = 6 \times 10^8/10 = 6 \times 10^7$ sec and $\tau > 6 \times 10^7$ sec.

The decay length L for E_γ in the range 0.1–0.5 MeV[6] from $\bar{\nu}_e$ from fission[7] (≥ 1 MeV) is therefore

$$L = c\tau > 1.8 \times 10^{18}\ \text{cm or}\ L > 10^5\ \text{astronomical units.}$$

Accordingly, this particular mode of decay is ruled out as an explanation of the Davis result.[8]

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[7]The limits are reasonably ascribed to ν_e decay as well.

[8]We note that the γ detection efficiency of the large detector employed is $\sim 100\%$. If the product γ is assumed instead to be an unknown particle which has a lower detection efficiency, the decay length would be correspondingly decreased. Further, as pointed out by H. A. Bethe (private communication), if the masses of $\bar{\nu}_e$ and ν' are sufficiently close, the γ will receive a small fraction of the $\bar{\nu}_e$ energy, i.e.,

$$E_\gamma/E_{\bar{\nu}_e} = (m_{\bar{\nu}_e} - m_{\nu'})/m_{\bar{\nu}_e}.$$

This kinematic effect would reduce the sensitivity of the experiment.

Strengths of Strong and Other Interactions*

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The requirement of compatibility of vector-meson dominance and partial conservation of axial-vector current in $\pi^0 \to 2\gamma$ decay [along with $SU(6)_W$ symmetry] determines the absolute value of the $\rho\pi\pi$ coupling constant. The strength of strong interactions results inversely proportional to the number of postulated quarks. In dynamical models of spontaneous symmetry breaking the electromagnetic, weak, and gravitational interactions are found also to have strengths inversely proportional to the corresponding numbers of fundamental fermions.

The $\pi^0 \to 2\gamma$ decay rate can be calculated either from vector-meson dominance[1] (VMD) or on the basis of partial conservation of axial-vector current (PCAC) (including the anomaly term) and the quark model.[2] Both calculations agree with experiment, yet their equivalence is by no means obvious. We shall explore here the dynamical basis of this equivalence. This leads us to a relation that, in its strongest form, forces the strong $\rho\pi\pi$ coupling constant to equal a certain

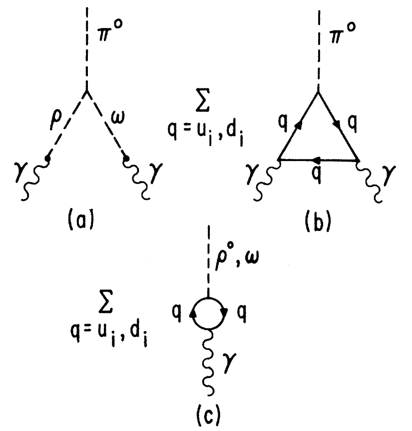

FIG. 1. Feynman diagrams relevant to the calculation of the $\pi^0 \to 2\gamma$ amplitude: (a) the VMD diagram, (b) the anomalous PCAC diagram, and (c) the quark-model diagram for f_ρ and f_ω.

real number:

$$g_{\rho\pi\pi}^2/4\pi = \tfrac{2}{3}\sqrt{2}\,\pi. \qquad (1)$$

More generally, a picture emerges in which the strength of strong interactions turns out to be inversely proportional to the number of postulated quarks. We then show that in dynamical models of spontaneous symmetry breaking, the strengths of the electromagnetic, weak, and gravitational interactions exhibit a similar dependence on the number of fundamental fermions.

The VMD calculation corresponds to the Feynman diagram of Fig. 1(a). It yields the matrix element

$$M_{\pi^0\to 2\gamma}^{\mathrm{VMD}} = 8\pi\alpha g_{\omega\rho\pi}(f_\rho/m_\rho^2)(f_\omega/m_\omega^2). \qquad (2a)$$

The PCAC calculation starts from the triangle diagram of Fig. 1(b) and leads to

$$M_{\pi^0\to 2\gamma}^{\mathrm{PCAC}} = -4\pi\alpha S/(\pi^2 f_\pi). \qquad (2b)$$

In Eqs. (2a) and (2b), f_π is the charged-pion decay amplitude, f_ρ (f_ω) the ρ^0-photon (ω^0-photon) transition amplitude, $g_{\omega\rho\pi}$ the $\omega\rho\pi$ coupling constant, α the fine structure constant, m_ρ the ρ-meson mass, and

$$S = \sum_{i=1}^{N}(Q_i - \tfrac{1}{2}),$$

with Q_i the charge of the u (i.e., "protonlike") quark of the ith triplet and N the number of fundamental quark triplets. For later use we also write down here the Goldberger-Treiman relation

$$f_\pi g_{\pi\bar{N}N} = 2m_N G_A, \qquad (3)$$

and the basic VMD relation

$$f_\rho g_{\rho\pi\pi}/m_\rho^2 = 1. \qquad (4)$$

Here $g_{\pi\bar{N}N}$ and $g_{\rho\pi\pi}$ are the πN and the $\rho\pi\pi$ coupling constants ($g_{\pi\bar{N}N}^2/4\pi \simeq 14$; $g_{\rho\pi\pi}^2/4\pi \simeq 2.8$ corresponding to $\Gamma_\rho = 146$ MeV).

One is faced here with the not unusual situation of two apparently uncorrelated dynamical explanations of the same number $\Gamma_{\pi\to\gamma\gamma}$. The natural idea is that they should be equivalent. If that were indeed the case then interesting new relations among the fundamental constants entering Eqs. (2) could be derived. While that is our main goal, let us first show that the equivalence of the two Eqs. (2) is theoretically not unexpected.

The expression (2b) depends directly on the quark charges via S. No such dependence appears in (2a). How can this be? Of the quantities entering in Eq. (2a) $g_{\omega\rho\pi}$, m_ρ^2, and m_ω^2 are strong-interaction parameters and as such *cannot* depend on the quark charges. These could only enter via the electromagnetic transition amplitudes f_ρ and f_ω. In the quark model these ρ-γ and ω-γ transitions can be pictured as in Fig. 1(c). It is readily seen that

$$f_\rho = \tfrac{1}{2}CN, \quad f_\omega = CS, \qquad (5)$$

where C is a dynamical parameter—for our purposes irrelevant—and as before N is the number of quark triplets and

$$S = \sum_{i=1}^{N}(Q_i - \tfrac{1}{2}).$$

The ratio f_ω/f_ρ is, of course,

$$f_\omega/f_\rho = \tfrac{1}{3}, \qquad (6)$$

as long as the "color singlet" part of the electromagnetic current is a pure SU(3) octet. Inserting Eq. (5) into (2a) we see that the same dependence on quark charges (i.e., $\propto S$) as in Eq. (2b) appears. We then introduce into (2a) also a dependence on the SU(3)-invariant quantity N. This however can be balanced by an N dependence of the strong couplings. We shall return to this point below. For the time being let us conclude that the dependence on the quark charges is the *same* in Eqs. (2a) and (2b).

We shall now equate these two expressions, representing two alternative but *equivalent* pictures of the $\pi^0 \to 2\gamma$ decay[3]:

$$M_{\pi^0\to 2\gamma}^{\mathrm{VMD}} = M_{\pi^0\to 2\gamma}^{\mathrm{PCAC}}. \qquad (7)$$

To explore the implications of Eq. (7) we use

$SU(6)_W$ symmetry to relate $g_{\omega\rho\pi}$ to $g_{\rho\pi\pi}$:

$$g_{\omega\rho\pi} = -(2/m_\rho)g_{\rho\pi\pi}. \quad (8)$$

Collecting Eqs. (2a), (2b), (4), and (6)–(8) and using $m_\rho \simeq m_\omega$, we find

$$(f_\pi/m_\rho)(f_\rho/m_\rho^2) = 3S/4\pi^2. \quad (9)$$

Experimentally $f_\pi \simeq 0.198$ GeV, $f_\rho \simeq 0.105$ GeV^2, and $m_\rho = 0.77$ GeV so that the left-hand side of this equation is 0.045 with a typical error of ± 0.09 while its right-hand side is $0.071S$, which suggests $S = +\frac{1}{2}$ [$|S| = \frac{1}{2}$ already follows from Eq. (2a) and the experimental $\pi^0 \to 2\gamma$ rate] as in the fractionally or integrally charged three-triplet models.

By noting that the already used (and experimentally well obeyed) assumptions of $SU(6)_W$ invariance of the strong couplings of $L = 0$ multiplets and of VMD also give

$$g_{\pi\bar{N}N} = (10M_n/3m_\rho)g_{\rho\bar{p}p}, \quad g_{\rho\bar{p}p} = \tfrac{1}{2}g_{\rho\pi\pi}, \quad (10)$$

we can rewrite Eq. (9) [using Eqs. (3), (4), and (10)] in the form

$$g_{\rho\pi\pi}^2/4\pi = (\sqrt{2}\,\pi/3S)(G_A/\tfrac{5}{3}\sqrt{2}\,). \quad (11)$$

This relation shows that as long as G_A and S are of order unity, so is $g_{\rho\pi\pi}^2/4\pi$. If instead, for instance, S was large (as would be the case for many triplets) the hadronic coupling would be weaker. One can further restrict the argument by accepting the "elusive" Kawarabayashi-Suzuki-Fayazuddin-Riazuddin (KSFR) relation[4]

$$f_\pi = \sqrt{2}\,f_\rho/m_\rho, \quad (12)$$

which when combined with Eqs. (9) and (4) gives

$$g_{\rho\pi\pi}^2/4\pi = \sqrt{2}\,\pi/3S. \quad (13)$$

This equation reduces for the three-triplet model ($S = \frac{1}{2}$) to the Eq. (1) mentioned in the introduction. In this case it gives $g_{\rho\pi\pi}^2/4\pi = 2.96$ in agreement with experiment. Equation (13) can be obtained directly from Eq. (11) since[5] under the stated assumptions $G_A = \frac{5}{3}\sqrt{2}$. Both Eqs. (11) and (13) are remarkable because they express a typical strong-coupling constant $g_{\rho\pi\pi}$ as either a pure number or a number times G_A. These equations do *not* give ratios of coupling constants but *absolute* values. In short they tell us why strong interactions are strong, if one is willing to accept either a G_A of order unity or the semiempirical KSFR relation.

We emphasize that the assumptions we made —VMD, PCAC, $SU(6)_W$ symmetry, and the KSFR relation—entail errors of typically ~10–20%. One should therefore trust the relations (11), (13), and (1) only to within this typical margin of error. We also note here that VMD and the $SU(6)_W$ symmetry relation (8) give a satisfactory account of the $\pi^0 \to 2\gamma$, $\omega \to 3\pi$, and $\omega \to \pi\gamma$ decay widths. With $g_{\rho\pi\pi}^2/4\pi = 2.8$, as determined from $\Gamma_{\rho\to 2\pi}^{\text{exp}} = 146$ MeV, the observed masses of the ω, ρ, and π mesons, and $g_{\omega\rho\pi}$ given by Eq. (8), we find $\Gamma_{\pi^0\to 2\gamma}^{\text{VMD}} = 9.9$ eV, $\Gamma_{\omega\to 3\pi}^{\text{VMD}} = 9.7$ MeV, and $\Gamma_{\omega\to\pi\gamma}^{\text{VMD}}/\Gamma_{\omega\to 3\pi} = 10\%$, to be compared with the experimental values[6] of 7.8 ± 0.9 eV, 9.8 ± 0.5 MeV, and 8.9%, respectively. We record these results, as there have been claims[7] in the literature that VMD for these decays disagrees with experiment. These claims were based on preliminary, and by now revised, results of colliding-beam experiments.

We now return to another interesting consequence of our dynamical assumption (7). As was stated before, by combining Eqs. (2), (5), and (7) one can learn about the dependence of strong couplings on the number of quarks N that underlie the quark model. In detail, combining the quark model relation (5) for f_ρ with the VMD formula (4) we find

$$\tfrac{1}{2}g_{\rho\pi\pi}CN/m_\rho^2 = 1. \quad (14)$$

We notice [see Fig. 1(c)] that C is proportional to the ρ-quark coupling. If, as is natural, we assume ρ universality to hold at the quark level, i.e., $g_{\rho\bar{q}q} = \frac{1}{2}g_{\rho\pi\pi}$, then C is of the form

$$C \simeq (\pi D)^{-1}(\tfrac{1}{2}g_{\rho\pi\pi}), \quad (15)$$

where D is a new constant *independent* of the strength of strong interactions and of the number N of quark triplets. The last two equations give

$$(1/m_\rho^2)g_{\rho\pi\pi}^2/4\pi = D/N. \quad (16)$$

Thus, either m_ρ^2 or $g_{\rho\pi\pi}$, or both, must depend on the number of quarks. Particularly interesting is the case in which m_ρ is independent of N. Then the typical strong-coupling constant $g_{\rho\pi\pi}$ must decrease like $1/\sqrt{N}$. Strong interactions are thus found to be strong because there are only nine quarks ($N = 3$). Were there to be more quarks, strong interactions would then *not* be so strong.

At this point it is tempting to speculate that strong interactions are not the only ones whose strength depends on the number of fundamental objects ("quarks") that participate in them. Could it be that electromagnetic and weak interactions are weaker because more fermions (quarks and leptons) can participate in them? More precisely, could it be that also the fine structure con-

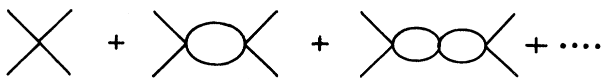

FIG. 2. Diagrams generating the photon in a theory with spontaneous breaking of Lorentz invariance (see Ref. 9).

stant α can be expressed in the form

$$\alpha = D'/N' , \tag{17}$$

where D' is a new constant and N' the number of electrically charged fundamental fermions (quarks, leptons)? In fact, a familiar dynamical picture yields precisely Eq. (17). The photon can be viewed as the Nambu-Goldstone boson corresponding to the spontaneous breakdown of Lorentz invariance.[8] A model known to yield such a photon is that in which the bubble diagrams of Fig. 2 are assumed.[9] It leads to

$$\alpha = C_\alpha \left(\sum_{i=1}^{N'} \ln \frac{\Lambda}{m_i} \right)^{-1}, \tag{18}$$

where C_α is a constant, Λ a cutoff, and m_i the masses of the fermions in the bubbles. If $\Lambda \gg m_i$ then

$$\alpha \simeq \frac{C_\alpha}{\ln(\Lambda/m_{\rm av})} \frac{1}{N'}, \tag{19}$$

which is of the form (17).[10] In the spirit of unified gauge theories of weak and electromagnetic interactions a similar argument can be made for a $1/N'$ dependence of weak couplings. A similar model for the graviton (the divergence being quadratic) leads to[11]

$$G \simeq (C_G/\Lambda^2)(1/N_G), \tag{20}$$

where G is Newton's gravitational constant, C_G a constant comparable to C_α, and N_G the number of gravitating "basic" fermions. Equations (19) and (20) give the well-known Landau-type[12] relation $\alpha \sim \ln(Gm_{\rm av}^2)$.

It is remarkable that the interaction strengths (g^2) of the massless bosons implied by dynamical spontaneous symmetry breaking are inversely proportional to the number of their fermionic constituents. We conjecture the strength of all interactions to have this property.

To sum up, by requiring that VMD and PCAC both hold for $\pi^0 \to 2\gamma$ we were able [after use of further VMD and PCAC relations and of the $SU(6)_W$ symmetry of strong interactions] to determine correctly the strength of strong interactions provided either G_A is of order unity or the KSFR relation holds. The same arguments have shown that this strength is inversely proportional to the the number of postulated fundamental fermions (quarks). A similar dependence was then derived for electromagnetic, weak, and gravitational interactions in dynamical models of symmetry breaking.

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