higher-order terms will involve insertions of φ^4, $(\nabla\varphi)^2$, etc. in the SDE and thus will depend on new anomalous dimensions in addition to those of φ and φ^2, which sufficed hitherto.[18]

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Dispersion Curves for Surface Electromagnetic Waves with Damping*

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The recent observation of backbending in the dispersion curves of surface plasmons on silver can be explained by use of Fresnel's equations. In the presence of damping, the results of attenuated–total-reflection measurements can be displayed as dispersion curves either with or without backbending. The measurements for silver with backbending are expected for experiments in which the frequency is fixed and the propagation constant (or angle of incidence) is swept.

Recently Arakawa *et al.*[1] have reported the observation of unusual behavior in the dispersion curve of surface plasmons on silver. Instead of a monotonic increase of wave vector with energy toward an asymptote of infinite wave vector at the surface-plasmon energy, they observed a decrease in wave vector with increasing energy at energies close to the surface-plasmon energy. We show that this backbending is to be expected from calculated reflectances using Fresnel's equations for the attenuated–total-reflection (ATR) geometry (see Fig. 1, inset). We also prove that the details of the experiment and method used to display the results dictate whether or not the dispersion curve exhibits backbending.

We have calculated the ATR reflectance as a

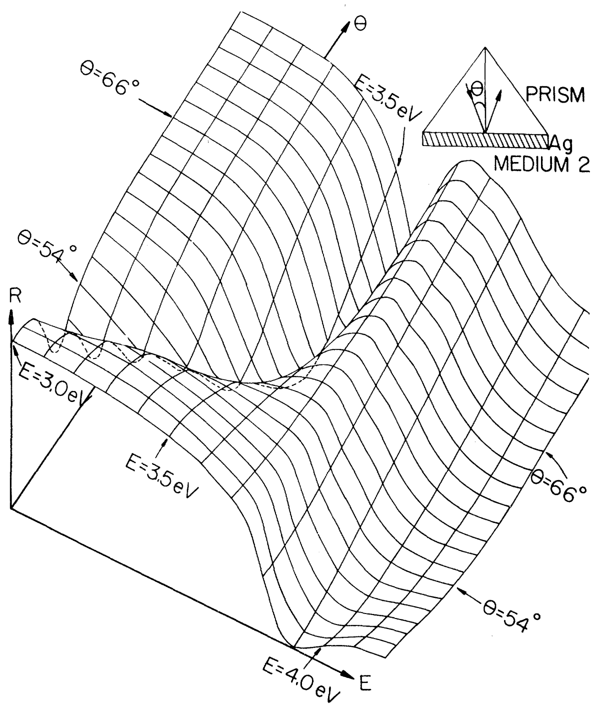

FIG. 1. The ATR reflectance of a 340-Å-thick Ag film on a CaF_2 prism plotted as a function of incident photon energy $h\nu$ and angle of incidence θ. The indicated cross sections of this ATR reflectance surface are plotted in Fig. 2. Inset, ATR geometry of a prism or semicylinder (index of refraction n_p) with a silver film, and a second interface with another medium.

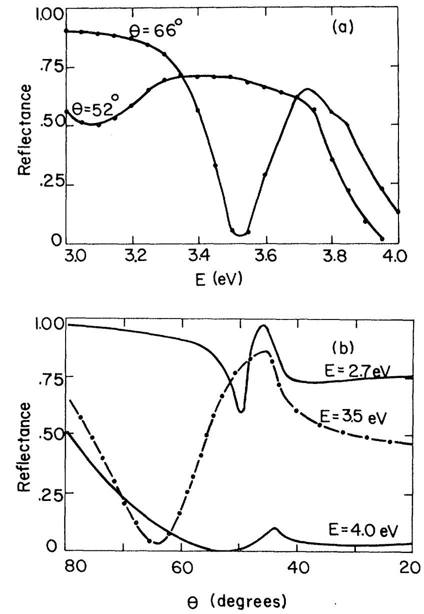

FIG. 2. (a) Cross sections of the ATR reflectance surface of Fig. 1 obtained by holding the angle of incidence θ fixed and varying the incident photon energy. (b) Cross sections of the ATR reflectance of Fig. 1 obtained by holding incident photon energy fixed and varying θ.

function of *both* the angle of incidence θ and the incident photon energy. The multilayer reflectance equations of Wolter[2] were used, and calculations were done for a set of energies from 3.0 to 4.25 eV and incident angles from 45 to 75° in the prism (CaF_2, $n = 1.434$). The film thickness was 340 Å. The complex dielectric function for Ag was obtained from Johnson and Christy.[3]

The results are displayed in Fig. 1 as a reflectance surface, where the vertical axis is the reflectance, the lower axis is the photon energy, and the axis into the page is the incident angle. If the experimenter fixes the incident angle and scans energy, the results will follow lines going from left to right. If the experimenter fixes energy and scans incident angle, the results will follow lines going from bottom to top.

One can see two extended minima regions or "valleys." The curved valley at lower photon energy corresponds to the presence of surface plasmons. As can be seen, the valley exists at large angles for relatively constant energy. This increasing angle corresponds to larger wave-vector values for the surface plasmon [$k = (2\pi\nu/c)n_p \times \sin\theta$].

Fig. 2(a) shows cross sections made across the reflectance surface following the lines of fixed angle. If the observed minima coordinates are converted to wave vectors and these plotted versus photon energy, the dash-dotted line in Fig. 3 is obtained. This dispersion curve shows the usual asymptotic behavior at the surface-plasmon energy (for which $\mathrm{Re}\,\epsilon \equiv \epsilon_1 = -1$). This also corresponds to the experimental methods used by Otto,[4] Reshina, Gerbstein, and Merlin,[5] and Marschall and Fischer.[6]

Figure 2(b) shows cross sections made across the reflectance surface following the lines of fixed photon energy varying the angle of incidence θ. Plotting the wave vector corresponding to the value of θ for the reflectance minimum in each cross section as a function of the energy yields the solid line of Fig. 3. Thus fixing the photon energy and sweeping θ produces the backbending. This corresponds to the experimental procedure of Arakawa *et al.*,[1] and their data points are indicated as crosses on the graph. The slight discrepancy between experiment and theory is due to small differences between the sample of Johnson and Christy and the sample of Arakawa *et al.*

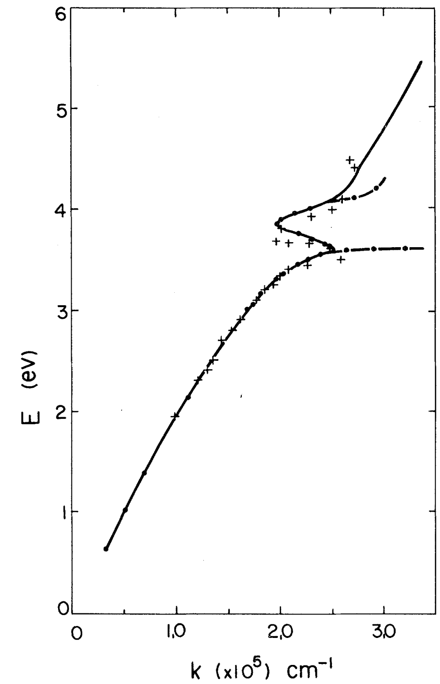

FIG. 3. Dispersion curves for a 340-Å silver film on a CaF_2 prism. Solid curve, obtained from cross sections of the ATR reflectance surface of Fig. 1 for fixed energies of the type shown in Fig. 2(b). This curve (lower branch) also results from solving Eq. (1) using $\epsilon_2(\nu) \neq 0$. Dash-dotted line, from cross sections for fixed angle of the type shown in Fig. 2(a). This curve also (for the lower branch) results from solving Eq. (1) with $\epsilon_2(\nu) = 0$. Crosses, experimental points of Arakawa *et al.*

In this region, $\epsilon_1 \simeq -1$, the real part of the denominator of Eq. (1) becomes small, and the reflectivity minima are very sensitive to the value used for ϵ_1 and ϵ_2. Note that ϵ_2 is relatively small in this region. Also, the reflectance minima are very broad in this region.

For energies near the surface-plasmon energy, the cross sections of Fig. 1 for fixed energies do not go cleanly across the valley but instead go along the sides of the valley or along the ridge. There are still minima in these reflectance cross sections which result in the backbending in the dispersion curve. However, there is now no direct simple relation between the location of the reflectance minima and the surface or bulk plasmon excitation energies.

A number of workers have calculated the properties of surface plasmons using several approaches. Chiu and Quinn[7] used a macroscopic approach, Fuchs and Kliewer[8] a microscopic approach, and Ritchie[9] a hydrodynamic treatment. Ritchie[10] provides an extensive bibliography which can be consulted.

Most derivations of the dispersion curve have assumed no damping, i.e., $\epsilon_2(\nu) = 0$ in $\epsilon(\nu) = \epsilon_1(\nu) + i\epsilon_2(\nu)$. However, Bell *et al.*[11] have shown that the form of the dispersion curve is unchanged for $\epsilon_2(\nu) \neq 0$ and

$$k = \frac{2\pi\nu}{c}\left[\frac{\epsilon(\nu)\eta(\nu)}{\epsilon(\nu)+\eta(\nu)}\right]^{1/2}, \tag{1}$$

where $\eta(\nu) = \eta_1(\nu) + i\eta_2(\nu)$ is the dielectric function of the second material at the interface. In our case, this medium is air ($\eta_1 = 1$ and $\eta_2 = 0$). If the real part of k obtained from Eq. (1) is plotted as a function of energy as in Fig. 3, the solid curve is obtained in the region where $\epsilon_1 < -1$.[11] Note that this is exactly the same curve as obtained by plotting the reflectance minima for fixed energy as discussed above. If we neglect damping and set $\epsilon_2 \equiv 0$, the plot of the real part of k versus energy exhibits asymptotic behavior and follows the lower dash-dotted line in Fig. 3. Note that this is the same curve obtained above by plotting reflectance minima for fixed angle. The upper dash-dotted line is obtained by plotting reflectance minima for photon energies above the surface-plasmon energy.

A very similar behavior has been seen by us in the ATR reflectance of n-type InSb. Here the reflectance surface is more complicated because of the presence of LO phonons which are coupled to the plasmon modes. This produces two surface modes, and the reflectance surface has two distinct surface mode valleys. The same dispersion curve behavior (including backbending) can be demonstrated by examination of this surface.[11,12] ATR measurements on this material have been made by Reshina, Gerbstein, and Mirlin.[5]

Once $\epsilon_2(\nu) \neq 0$, the use of a dispersion curve to represent the response of a system to an applied force becomes ambiguous. A three-dimensional plot or surface as in Fig. 1 is required to adequately describe the response. Barker[13] has shown how the usual asymptotic dispersion curve is obtained from a three-dimensional plot of a response function (which is related to the reflectance) but did not show how to obtain dispersion curves with backbending. This same problem caused much discussion in the literature with respect to bulk excitations. Barker[14] points out some of the problems in detail and gives references.

In summary, we have shown that the backbending in the surface-plasmon dispersion curve observed by Arakawa *et al.* is obtained from Fresnel's equations. Furthermore, this result is *not* inconsistent with the asymptotic behavior previously observed. A correct interpretation of the ATR reflectivity minima in terms of dispersion curves requires a plot of reflectance versus both incident photon energy and incident angle and a consideration of the experimental method. Backbending will occur when the energy is fixed and the incident angle varied. Asymptotic behavior will occur when the incident angle is fixed and the energy varied. Finally, the minima observed for photon energies above the surface-plasmon frequency (i.e., for photon energies where $\epsilon_1 > -1$) are consistent with Fresnel's equations, and are due to ϵ_1 going through zero at the bulk-plasmon frequency.

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Dislocations as a Model for One-Dimensional Systems: Magnetoabsorption in Strained Te

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Tellurium crystallizes in parallel-packed helixlike chains of covalent-bound atoms. The bonds between the chains are much weaker than the ones within the chains; thus the dislocations in Te arrange themselves parallel to the crystallographic c axis to avoid cutting covalent bonds. Electrons bound to the dislocation lines therefore form quasi-one-dimensional states. We used magneto-optical absorption to study the nature of these bound states. They split up into highly anisotropic sub-bands, depending on strength and orientation of the magnetic field.

The absorption spectra at the band edge of Te show a pronounced anisotropy[1] originated by the allowed and forbidden direct transitions for the two polarization directions.[2] A sharp absorption line in the spectra for $\vec{E} \parallel \vec{c}$ was previously interpreted as an exciton.[3-5] By using highly polarized light it can, however, be shown that the shape of the line discussed there results from an