M. Haque *et al.*, Phys. Rev. 152, 1148 (1966); W. Hoogland *et al.*, Nucl. Phys. B21, 381 (1970); J. S. Lindsey *et al.*, Phys. Rev. 147, 913 (1966); G. W. London *et al.*, Phys. Rev. 143, 1034 (1966); J. Mott *et al.*, Phys. Rev. 177, 1966 (1969); D. G. Scotter *et al.*, Nuovo Cimento 62A, 1057 (1969).

[6]S. Okubo, Phys. Lett. 5, 165 (1963); G. Alexander *et al.*, Phys. Rev. Lett. 17, 412 (1966).

[7]G. S. Abrams *et al.*, Phys. Rev. Lett. 23, 673 (1969); J. C. Anderson *et al.*, Phys. Lett. 45B, 165 (1973); N. Armenise *et al.*, Nuovo Cimento 65A, 637 (1970); M. S. Farber *et al.*, Nucl. Phys. B29, 237 (1971); L. E. Holloway *et al.*, Phys. Rev. D 8, 2814 (1973); J. A. J. Matthews *et al.*, Phys. Rev. Lett. 26, 400 (1971).

[8]L. E. Holloway *et al.*, Phys. Rev. Lett. 27, 1671 (1971).

[9]See for example C. W. Akerlof *et al.*, Phys. Rev. Lett. 27, 539 (1971), and unpublished.

[10]S. Frautschi, Nuovo Cimento 12A, 133 (1972), and private communication.

Permanently Bound Quarks–New Solutions to Old Field Equations

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A class of relativistic-field-theoretic models of permanently bound quarks are shown to exhibit new and unconventional properties. They display crossing symmetry, but their Green's functions have an infinite array of singularities in momentum space and are singular everywhere in coordinate space. Nevertheless, these models are consistent quantum field theories that describe a system of interacting bound states which can decay into one another but not into quarks.

A conventional perturbative solution of a quantum field theory has asymptotically free quanta of the fundamental fields. However, since quarks are not observed, a conventional solution is inappropriate in theories with quark fields. Johnson[1] has suggested a new mechanism, which is a relativistic generalization of an r^2 potential binding quarks, which could account for the permanent binding of quark fields and thus the nonexistence of asymptotically separating quarks. In this paper we develop a new self-consistent field-theoretic perturbation method to describe strongly interacting particles which in lowest order incorporates this mechanism.

We will generalize some of our remarks to fermion fields in future papers,[2] but here we confine our discussion to a boson field φ with a φ^4 interaction. Triality is thus replaced by field parity $\varphi \to -\varphi$. A "quark" state has odd field parity and a "meson" state has even field parity.

We begin by assuming that there exist states of n mesons and states of n mesons and one quark, and we denote the corresponding projection operators onto these states by P_n and $P_{q,n}$. Projecting with $P_{q,n}$ and isolating the one-quark, n-meson contribution to φ^3, the quark field equation becomes

$$(-\Box^2 - m_0^2)P_{q,n}\varphi(x) = \lambda P_{q,n}\varphi^2 P_{q,n}\varphi(x) + \epsilon\lambda P_{q,n}\varphi^2(x)(1-P_{q,n})\varphi(x). \tag{1}$$

ϵ, and not λ, is the perturbation parameter for this theory. Matrix elements are taken to be power series in ϵ. Each calculation is concluded by setting $\epsilon = 1$.

The basic assumption of this theory is that the one-quark to one-quark matrix element of the quark current $I(x) \equiv \lambda\varphi^2(x)$ is extremely singular at zero momentum transfer; this is how the theory incorporates an effective long-range potential between quarks. We further assume that to zeroth order in ϵ, the one-quark, n-meson matrix element of the quark current is dominated by this singularity:

$$\langle q, p_1, \ldots, p_n | I(0) | q', p_1', \ldots, p_n'\rangle^0 = \langle p_1, \ldots, p_n | p_1', \ldots, p_n'\rangle^0 [\gamma(\partial/\partial q^\mu)(q^\mu q^\nu - g^{\mu\nu}q^2)(\partial/\partial q^\nu) + \sigma] \times (2\pi)^3 2q'^0 \delta^3(q-q'). \tag{2}$$

Here γ and σ depend on λ. Equation (2) is a manifestly covariant generalization in differential form of the mechanism proposed by Johnson.[1] The consistency of this assumption must be verified once the theory is solved to zeroth order in ϵ. This has been accomplished for the corresponding model in two-dimensional space-time.[2]

Zeroth-order solution, meson spectrum.—We take the matrix element of the field equation (1) between states of n mesons and one quark and an arbitrary state $|x\rangle$, retain terms to zeroth order in ϵ, and substitute Eq. (2). We then replace m_0, the bare-quark mass, and the parameter σ by the single parameter m, the physical-quark mass, by letting $n=0$ and $|x\rangle$ be the vacuum state. The resulting wave equation is

$$[(p_x - p_{\text{mesons}} - q)^2 - m^2 - \gamma(\partial/\partial q^\mu)(q^\mu q^\nu - g^{\mu\nu}q^2)(\partial/\partial q^\nu)]\langle p_1, \ldots, p_n, q|\varphi(0)|x\rangle = 0. \tag{3}$$

Letting $|x\rangle$ be a meson of mass M, momentum P, and spin J, we write

$$\langle q|\varphi(0)|p\rangle^0|_{\vec{p}=0} = g(z)Y_J^{\,J}(\Omega_{\vec{q}}), \tag{4}$$

where $Mmz = \vec{p}\cdot\vec{q}$. Combining Eqs. (3) and (4) gives an equation for $g(z)$:

$$[(z^2-1)d^2/dz^2 + 3z\,d/dz - J(J+1)/(z^2-1) + (\mu^2 - 2\mu z)/\gamma]g(z) = 0, \tag{5}$$

where $\mu = M/m$. This equation has regular singular points at $z=\pm 1$ and an irregular singular point at $z=\infty$. But $z=\mu/2$, which corresponds to the momentum transferred to the quark field being the square of the quark mass, $(p-q)^2 = m^2$, is a regular point. Thus $g(\mu/2)$ is finite, verifying that a meson does not decay into two quarks.

Evaluating the form factor $\langle p|J(0)|p\rangle$ by inserting a complete set of states and requiring it to be finite places boundary conditions on Eq. (5): $g(1)=g(\infty)=0$. This leads to a discrete mass spectrum for the mesons which in a WKB-like approximation is given by

$$4\mu_n\sqrt{\gamma}[\ln(4\mu_n) - 2] = \pi(4n + 2J + 3), \quad n = 0, 1, \ldots, n > J. \tag{6}$$

We have also solved Eq. (5) numerically for the eigenvalues μ and have displayed the results on the Chew-Frautschi plot in Fig. 1. The meson masses lie on almost linearly rising trajectories.

To complete the lowest-order solution one must consider states $|x\rangle$ beyond the one-meson state. Equation (3) implies that the most general state $|x\rangle$ consists of n noninteracting mesons whose mass spectrum is already given in Eq. (6). That the mesons are noninteracting is illustrated by the following property of the solution:

$$\langle p_1, \ldots, p_n, q|\varphi(0)|p_1', \ldots, p_{n+1}'\rangle^0 = \langle p_1, \ldots, p_n|p_1', \ldots, p_n'\rangle^0\langle q|\varphi(0)|p_{n+1}'\rangle^0 + \text{permutations}.$$

All other matrix elements of the field are zero. Thus, in this lowest approximation the model has only states with n free mesons or n free mesons and one quark.

The details of proof of the consistency of the initial assumption in Eq. (2) are too complex to be presented here. However, we emphasize that the only way to reproduce such a singularity by summing over intermediate states is for there to be an infinite number of mesons; but, the very existence of these mesons depends on the presence of a long-range force and hence the singularity at $q=q'$. We also remark that the consistency condition is nonlinear and thus it fixes the overall normalizations of the matrix elements $\langle q|\varphi(0)|p\rangle$, which are needed to calculate the me-

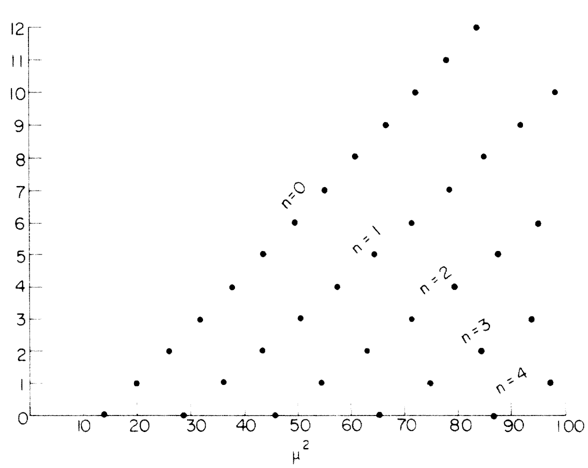

FIG. 1. Plot of J versus $\mu^2 = M^2/m^2$ for $\gamma = 1$.

son decay amplitudes.

First-order solution, meson decays.— Taking matrix elements of Eq. (2) and keeping terms to first order in ϵ gives

$$[(q-p_1-p_2)^2-m^2]\langle q|\varphi(0)|p_1,p_2\rangle^1=\sum_{q'}\langle q|I(0)|q'\rangle^0\langle q'|\varphi(0)|p_1,p_2\rangle^1 + \sum_{\text{all mesons},\, p',q'}\langle q|I(0)|q',p'\rangle^0\langle q',p'|\varphi(0)|p_1,p_2\rangle^0, \tag{7}$$

where we have set $\epsilon=1$ and used $\langle q|\varphi(0)|p_1,p_2\rangle^0=0$.

Substituting the zeroth-order matrix elements into Eq. (7) gives

$$[(q-p_1-p_2)^2-m^2-\gamma(\partial/\partial q^\mu)(q^\mu q^\nu-g^{\mu\nu}q^2)(\partial/\partial q^\nu)]\langle q|\varphi(0)|p_1,p_2\rangle^1 = \lambda\langle q|\varphi(0)|p_1\rangle^0\int[d^3q'/(2\pi)^3 2q'^0]\langle q'|\varphi(0)|p_2\rangle+(1\longleftrightarrow 2). \tag{8}$$

This is a first-order version of Eq. (3).

The solution of this equation is singular whenever $(p_1+p_2)^2$ approaches the square of the mass of a meson, p^2. The singularity has the form

$$\langle q|\varphi(0)|p_1,p_2\rangle^1\sim c\langle q|\varphi(0)|p\rangle^0[(p_1+p_2)^2-p^2]^{-1},$$

so c may be identified as the meson decay amplitude $\langle p|p_1,p_2\rangle^1$. Manipulation of Eq. (8) yields the decay formula

$$\langle p|p_1,p_2\rangle^1=\frac{\int[d^3q/(2\pi)^3 2q^0]\langle q|\varphi(0)|p\rangle^0\langle q|\varphi(0)|p_1\rangle^0\int[d^3q'/(2\pi)^3 2q'^0]\langle q'|\varphi(0)|p_2\rangle^0}{\int[d^3q/(2\pi)^3 2q^0](1-\vec{p}\cdot\vec{q}/p^2)|\langle q|\varphi(0)|p\rangle^0|^2}+(1\longleftrightarrow 2). \tag{9}$$

We have evaluated this expression in two-dimensional space-time for two distinct decay processes. We find that the cascade mode $(M\to M_1+M_2,\ M_1\lesssim M,\ M_2\ll M)$ is strongly preferred over the symmetric mode $(M\to M_1+M_2,\ M_1\simeq M_2\ll M)$. The respective amplitudes A_1 and A_2 for these decays are

$$A_1=\frac{m^2(\mu_2\gamma)^{1/2}\cos[\gamma(\mu-\mu_1)]\sin[\gamma(\mu-\mu_1)(4\ln\mu-1)]}{\sqrt{\lambda}\,\gamma^2\ln(4\mu_1)\ln(4\mu_1)(\mu-\mu_1)},\quad A_2=\frac{m^2\Gamma(\frac{1}{3})(\mu_1\gamma/3)^{1/6}\mu_1^{3/2}}{2\sqrt{\lambda}\,\gamma\ln^2(4\mu_1)}. \tag{10}$$

This result suggests that the selection rules for decay processes may already be included in the dynamical assumptions of the model.

Further calculations to first order in ϵ show that $\langle q|\varphi(0)|p\rangle^1$ and $\langle q|\varphi(0)|p_1,p_2\rangle^1$ have no quark poles. Thus mesonic states continue to be stable against decay into quarks. We also find that $\langle q,p|\varphi(0)|0\rangle^1$ is the analytic continuation of $\langle q|\varphi(0)|-p\rangle^0$ (plus an additional term when the spin of the meson is 0). Thus the theory is crossing symmetric to first order in ϵ. Recall that crossing symmetry is the momentum-space statement of locality. We speculate that a crossed matrix element in $(n+1)$th order is the analytic continuation of the direct matrix element in nth order, plus possible corrections from higher orders.

Finally, we calculate the quark propagator to second order in ϵ. In general

$$\langle 0|T[\varphi(x)\varphi(0)]|0\rangle=\int d\mu^2\rho(\mu^2)\Delta_{\mu^2}^{(F)}(x),$$

where

$$\rho(k^2)=\delta(k^2-m^2)+(2\pi)^{-3}\sum_{\text{mesons}}\int(d^3p\,d^3q/2p^0 2q^0)|\langle 0|\varphi(0)|p,q\rangle^1|^2\delta^4(p+q-k)+O(\epsilon^3).$$

Thus the propagator has a quark pole even though quarks are not produced in a scattering process. However, the matrix element $\langle 0|\varphi(0)|p,q\rangle^1$ is singular near the threshold $(p+q)^2=(M_n+m)^2=S_n$, which corresponds to the $z=-1$ singular point of Eq. (5). Near this singularity $\rho(k^2)\sim\theta(k^2-S_n)(k^2-S_n)^{-J-1/2}$, where J is the spin of the meson. Except for $J=0$, these contributions are nonintegrable and we must conclude that the two-field Green's function, and probably all Green's functions, do not exist (are infinite everywhere) in coordinate space. This theory exists only as a momentum-space theory.

This remarkable result is crucial because it frees the theory from any conflict with the cluster-decomposition theorem. This theorem states that at widely separated bunches of space-time points, Green's functions in a local quantum field theory factor. This theorem would contradict the assumption of permanently bound fundamental quark fields, but the nonexistence of coordinate-space Green's functions obviates the proof of the theorem. Nevertheless this model probably retains the physical consequences of the cluster-decomposition theorem for mesons, as is indicated in the lowest approximation where the mesons are free particles.

We have thus shown that it is possible to formulate, in a mathematically consistent manner, local relativistic quantum field theories which have permanently bound quarks. The general properties of such theories are obviously unconventional, but we see no reason to criticize these theories for exhibiting them, especially as they follow from very reasonable physical assumptions.

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[1]K. Johnson, Phys. Rev. D 6, 1101 (1972).

[2]C. M. Bender, G. S. Guralnik, and J. E. Mandula, to be published.

Breaking the Hadron String–Quarks at the National Accelerator Laboratory?*

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I consider the average displacement from its near neighbors of a parton in a hadron string after absorbing a highly virtual photon. It is found that for $Q^2 \gtrsim 60$ GeV^2, the struck parton would find itself separated from its near neighbors by a distance greater than the average internucleon spacing in a nuclear matter. I discuss the consequences of this result *vis à vis* deep inelastic scattering of leptons from complex nuclei.

The deep inelastic lepton-hadron scattering experiments at the Stanford Linear Accelerator Center,[1] CERN,[2] and the National Accelerator Laboratory[3] seem to establish Bjorken scaling[4] over a spectacular range of Q^2 and ν. The simplest realization of Bjorken scaling is provided by the parton model,[5] in which the currents scatter incoherently from elementary constituents within the nucleon. The proximity of the ratios $\sigma_{\bar{\nu}}/\sigma_\nu$ and $-\int xF_3(x)dx/\int F_2(x)dx$ to the value $\frac{1}{3}$ and 1, respectively,[2] lend support to the characterization of partons as quarks. The validity of the impulse approximation requires that the forces responsible for binding the partons be fairly soft,[5] and hence that a parton can travel freely for a considerable distance after being struck.[6] The crisis is then clear: Why do we not see quarks in nature? Various models of containment have been proposed,[7] but the nature of their conclusions (i.e., *absolute* containment) makes an experimental test of these models extremely problematic.

A different approach to the problem is taken in the present work. We first emphasize the lack of any basis for deciding how far a parton with nonzero triality may wander from its neighbors before being pulled back—it may be much farther than 1 fm. We then focus our attention on the scattering of leptons from *complex nuclei*, and examine the behavior of a parton struck by a virtual photon (or $W^\pm$). Within the dynamics of the phenomenological dual string model,[8] we find that for $Q^2 > 60$ GeV^2, $\omega \simeq 3$, the average displacement from its near neighbors of the struck parton exceeds the average *internucleon* distance in a complex nucleus. It is then proposed that the normal string might fail to retrieve the struck parton because of screening by the intervening nuclei (the existence of bilocal operators implies that quarks of nonzero triality do scatter absorptively from nucleons). We then discuss some possible experimental consequences of this result.

For simplicity, we work in an infinite-momentum frame, and formulate the string model in the