Bhabha's modified cross section. The total cross section given by Racah[6] is in close agreement with it. But the theory of Murota, Weda, and Tanaka[6] gives a slightly higher cross section than the modified Bhabha's theory for a given primary energy in the same region of transferred energy. In all these theories the total electron-pair-production cross section depends on the ratio γ ($=E/m$) and not on E and m separately or on the type of incoming particle. It is very interesting that although the γ factor in both our experiments (p and μ) are of the same order of magnitude, the cross section values vary by a factor of more than 2.5 for the small regions of E_0 and Q. We may point out that all these theories have been computed using perturbation theory and have neglected both nuclear recoil and the finite size and structure of the particles, which might be essential to include at such high energies. Nuclear emulsion has a large detection efficiency for low-energy particles and here we have been able to detect electrons with a kinetic energy of 0.07 MeV. We feel that the present experimental observations will be useful to theoreticians and that these discrepancies should be investigated very seriously.

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φ-Meson Production in π^-p and K^-p Interactions from 3 to 6 GeV/c*

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Cross sections and density-matrix elements for $\pi^-p \to \varphi n$ have been measured for $-t \lesssim 1.5$ GeV^2 at 3, 4, 5, and 6 GeV/c, using the Argonne effective-mass spectrometer to observe the decay $\varphi(1019) \to K^+K^-$. This is the first observation of the reaction in this energy range. The remarkably flat differential cross section at 4 GeV/c and the strong energy dependence suggest a production mechanism not normally seen at these energies. Data on $K^-p \to \varphi\Lambda$ and $K^-p \to \varphi\Sigma^0$ from the same experiment are also presented.

In an experiment to study $\pi^-p \to K^+K^-n$ with the Argonne effective-mass spectrometer, we have observed simultaneously the reactions

$$\pi^-p \to \varphi n, \tag{1}$$

$$K^-p \to \varphi\Lambda, \tag{2}$$

$$K^-p \to \varphi\Sigma^0. \tag{3}$$

Data were taken at 4, 5, and 6 GeV/c for all reactions, and also at 3 GeV/c for reaction (1). This Letter will focus on Reaction (1) which, because of its small cross section and the large S-wave background, had not previously been observed between 2.2 and 11 GeV/c.[1,2] Our sample of 300 events above background for this reaction is more than the total number of events collected in all previous experiments at all energies.

The spectrometer, which has been described elsewhere,[3] consisted of a large magnet and magnetostrictive-readout spark chambers. It was used to measure the forward-going K^+K^- from the φ decay; the recoil particle was not detected. The trigger required an incident π^- or K^-, two or more particles through the spectrometer, and no count in a large threshold Cherenkov counter which vetoed reactions with fast pions. Veto counters around the magnet aperture and hydrogen target were used in the incident pion trigger to suppress events with extra particles.

Reaction (1) was identified by assuming the events to be of the form $\pi^-p \to A^+A^-n$ and requiring the calculated mass of A to be close to the kaon mass. The $M_A{}^2$ distributions in Fig. 1(a) show a clean separation of $\pi^-p \to K^+K^-n$ from $\pi^-p \to \pi^+\pi^-n$ at the lower momenta, and very little non-K^+K^-n background at 4, 5, and 6 GeV/c (where the Cherenkov counter was used). The effective-mass spectra of Fig. 1(b) show the φ on top of a smooth background of K^+K^- events from the S-wave threshold effect. Reactions (2) and (3) were identified by cutting on the missing mass in $K^-p \to K^+K^-X$; $M_x{}^2$ distributions for the three incident momenta are given in Fig. 1(c). The effective-mass spectra shown in Figs. 1(d) and 1(e) have very little background. The mass and width of the φ were obtained by fitting the K^+K^- mass spectra from all three reactions to a simple Breit-Wigner form plus background and correcting for the ±1-MeV resolution of the spectrometer: $M_\varphi = 1019.5 \pm 0.6$ MeV and $\Gamma_\varphi = 4.5 \pm 0.7$ MeV, in good agreement with previous data.[4]

Cross sections for φ production were calculated by rotating each observed event about the beam direction to obtain a geometrical acceptance weight, and subtracting weighted events in background regions from events in the ±6-MeV φ region. This subtraction eliminates any non-φ background, whether from nonresonant K^+K^- or pion contamination. The result was corrected for the 24% loss of φ's not in the central region and for the 6% of the φ's included in the background regions (±12 to 24 MeV). Event weights included geometric apertures, kaon-decay probabilities,

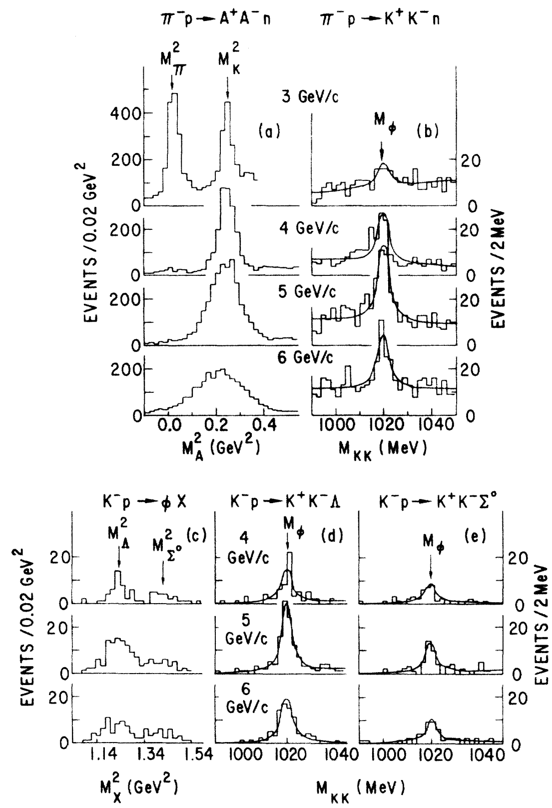

FIG. 1. (a) $M_A{}^2$ distributions for π^-p events with $M_{KK} < 1050$ MeV. The 3-GeV/c data, which show a prominent peak at $M_\pi{}^2$, were taken without the aid of the large Cherenkov counter used to veto the $\pi^-p \to \pi^+\pi^-n$ events at higher momenta. (b) Effective-mass distributions for events in the $M_K{}^2$ peak with $0.15 \leq -t' \leq 0.90$. The t' cut considerably enhances the φ signal relative to the s-wave K^+K^- background. (c) $M_x{}^2$ distributions for K^-p events with K^+K^- effective mass within ±6 MeV of the φ mass. (d), (e) Effective-mass distributions for events in the $M_\Lambda{}^2$ and $M_{\Sigma^0}{}^2$ peaks. The curves in (b), (d), and (e) are from fits to a Briet-Wigner plus linear background.

corrections for veto-counter losses, trigger inefficiencies, and decays inside the Cherenkov counter. The apparatus had finite acceptance for all φ decay configurations out to the maximum momentum transfers t_{max}' given in Table I, where $t' = t - t_{min}$. The cross sections were corrected for rate effects, absorption in the spectrometer, spark chamber and software inefficiencies, and the $\varphi \to K^+K^-$ branching ratio of 0.468.[4] Reactions (2) and (3) were also corrected for cross contamination. Uncertainty in the various corrections leads to an overall ±14% systematic uncertainty in our results for all reactions. Total cross sections, integrated from t_{min} to t_{max}', are given in Table I. Figure 2 shows our results

TABLE I. The total cross sections σ and $d\sigma/dt'$ slopes b for $|t'| \leq |t_{max}'|$. The errors shown include both statistical errors and the ±14% systematic uncertainty.

	p_{lab} (GeV/c)	$-t_{max}'$ (GeV^2)	σ (μb)	b (GeV^{-2})
$\pi^- p \to \varphi n$	3	0.9	4.9±2.1	0.4±1.8
	4	1.2	1.66±0.32	0.1±0.5
	5	1.7	0.58±0.15	1.2±0.4
	6	2.3	0.47±0.14	1.6±0.6
$K^- p \to \varphi\Lambda$	4	0.9	55±13	1.2±0.9
	5	1.5	38±8	1.6±0.4
	6	1.5	27±6	2.8±0.7
$K^- p \to \varphi\Sigma^0$	4	0.9	37±14	0.7±1.3
	5	1.5	14±5	0.4±0.7
	6	1.5	8±4	2.1±1.1

compared to those of previous experiments.

Differential cross sections are shown in Fig. 3, along with straight-line fits to $d\sigma/dt' = Ae^{bt'}$; the fitted slopes are given in Table I. While the slopes for Reactions (2) and (3) agree with previous measurements,[5] the results for Reaction (1) are new, and show a dramatic change in behavior over our energy range. Production at 3 and 4 GeV/c is isotropic out to t_{max}', agreeing with the results of Dahl *et al.* at lower energies.[1] The cross sections integrated to $t' = -0.9$ GeV^2 shown in Fig. 2(a) for 3 and 4 GeV therefore represent only a fraction of the total cross section for φ production (if production at 4 GeV/c were truly isotropic, the $t' = -0.9$ GeV^2 cutoff would include only 16% of the cross section. We have parametrized the energy dependence of Reaction (1) from 3 to 6 GeV/c and $-t < 0.9$ as a function of t by fitting with the form $d\sigma/dt = As^{2\alpha(t)-2}$. The resulting α's give a good fit to $\alpha(t) = (-0.3 \pm 0.6) + (2.4 \pm 1.3)t$.

K^+K^- angular distributions for Reactions (1) and (2) were fitted using a maximum likelihood method to obtain the quantities $\rho_{ij}\, d\sigma/dt$ as functions of K^+K^- mass, where the ρ_{ij}'s are helicity-frame density-matrix elements. The φ density-matrix elements shown in Table II were calculated by a background subtraction much like that used previously for cross sections. This procedure again eliminates the effect of background from other processes. S and P waves were used in 5-MeV mass bins to fit Reaction (1), while pure P wave and 12-MeV mass bins were taken for Reaction (2). Density-matrix elements for different energy and momentum-transfer ranges were averaged when they were mutually consis-

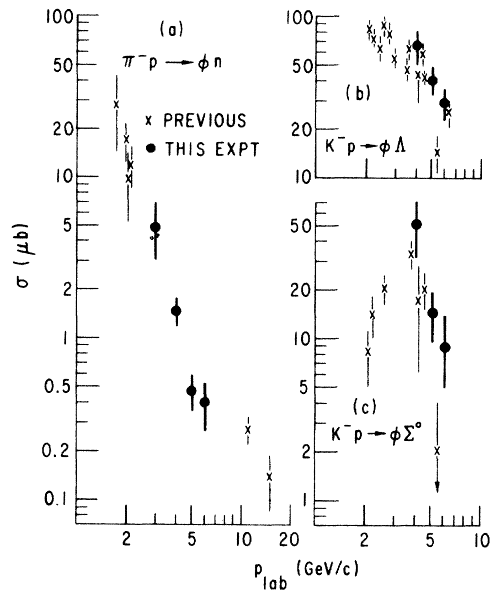

FIG. 2. Cross sections from this experiment compared with previous data. In (a) the differential cross sections were integrated out to $-t' = 0.9$ GeV^2; published cross sections below 3 GeV/c (Ref. 1) have been reduced to correspond to this range. In (b) and (c) our results have been scaled up using bubble-chamber data (Ref. 5) to correct for φ's produced at t values beyond our acceptance cutoff. Error bars include both statistical and systematic uncertainties.

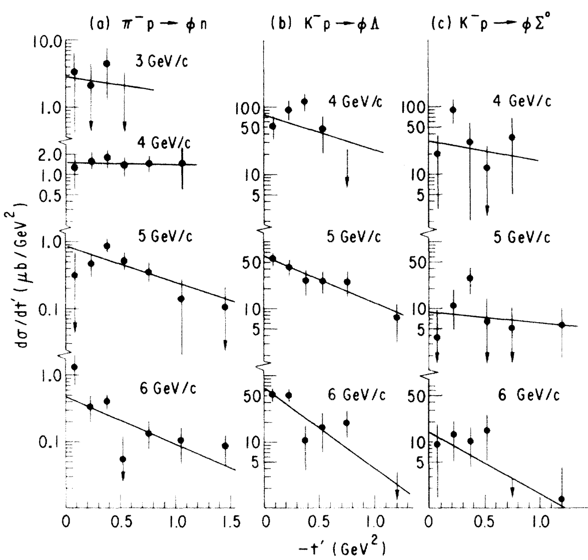

FIG. 3. Differential cross sections for the three reactions. The straight lines result from fitting $d\sigma/dt'$ to $Ae^{bt'}$ with the slopes given in Table I. Only statistical errors are shown.

TABLE II. Density-matrix elements in the helicity frame. The 5- and 6-GeV/c $\pi^- p \to \varphi n$ results have been averaged, as have those for $K^- p \to \varphi\Lambda$ at 4, 5, and 6 GeV/c. All s-p interference terms are consistent with zero, except $\mathrm{Re}\rho_{1s} = 0.12 \pm 0.04$ for $\pi^- p \to \varphi n$ at 5.5 GeV/c.

Reaction	p_{lab}	$-t'$ range	$\rho_{11}+\rho_{1-1}$	ρ_{00}	$\rho_{11}-\rho_{1-1}$	$\mathrm{Re}\rho_{10}$
$\pi^- p \to \varphi n$	4	0.15 − 0.90	0.12 ± 0.14	0.26 ± 0.17	0.61 ± 0.17	0.14 ± 0.10
	5.5	0.15 − 0.90	0.38 ± 0.08	0.20 ± 0.09	0.43 ± 0.09	0.06 ± 0.05
$K^- p \to \varphi\Lambda$	5	0 − 0.15	0.30 ± 0.13	0.44 ± 0.14	0.26 ± 0.13	0.07 ± 0.08
	5	0.15 − 0.45	0.62 ± 0.09	0.12 ± 0.07	0.25 ± 0.08	0.11 ± 0.04
	5	0.45 − 0.90	0.52 ± 0.16	0.27 ± 0.14	0.20 ± 0.17	0.00 ± 0.09

tent. Results are given in terms of the natural- ($\rho_{11}+\rho_{1-1}$) and unnatural-parity exchange (ρ_{00}, $\rho_{11}-\rho_{1-1}$) contributions, and show both to be important in Reactions (1) and (2). In addition, Reaction (1) shows evidence for interference between helicity-1 φ production and the S-wave background. The density-matrix elements for Reaction (1) are consistent with the 3.9 and 4.6 GeV/c data of Aguilar-Benitez *et al.*,[5] and show no energy dependence between 4 and 6 GeV/c.

The cross-section ratio $\sigma(K^- p \to \varphi\Lambda)/\sigma(\pi^- p + \varphi n) \approx 60$ is one measure of the suppression of φ production in $\pi^- p$, and can be explained qualitatively by the quark model, in which the φ is composed mainly of strange-quark pairs.[6] In this model, the reaction $\pi^- p \to \varphi n$ proceeds only through the very small admixture of nonstrange quarks in the φ, as given for example by the ω-φ mixing angle, while the reactions $K^- p \to \varphi\Lambda$, $\varphi\Sigma^0$ couple strongly to the large strange-quark component of the φ.

Aside from the strong suppression, the model predicts the processes $\pi^- p \to \varphi n$ and $\pi^- p \to \omega n$ to be rather similar. Although our data indicate increasing similarity at higher energies, significant differences remain: In the 5- to 6-GeV/c region the ω cross section[7] has a steeper falloff, $e^{(3 \text{ or } 4)t}$ from $-t = 0.2$ to 1 GeV^2, than does φ production; the 5.5-GeV/c density-matrix elements for $-t < 1$ GeV^2 show the ratio of natural- to unnatural-parity exchange to be 1.35 ± 0.25 for ω production[8] and only 0.6 ± 0.1 for φ production. In the quark model the cross-section ratio $\sigma(\pi^- p \to \varphi n)/\sigma(\pi^- p \to \omega n)$ is equal to the fraction of nonstrange quarks in the φ; our 5- and 6-GeV/c data give 0.0035 ± 0.0010 for this ratio.

The remarkably flat t distribution at 4 GeV/c and the very steep energy dependence below 5 GeV/c bear even less resemblance to ω production. These characteristics are, however, quite reminiscent of exotic or double-particle exchange reactions.[9] Alternatively, they may indicate the importance of s-channel effects[10] (i.e., $\pi^- p \to N^* \to \varphi n$), but this would require N^* resonances with unexpectedly large couplings to φn.

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Permanently Bound Quarks–New Solutions to Old Field Equations

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A class of relativistic-field-theoretic models of permanently bound quarks are shown to exhibit new and unconventional properties. They display crossing symmetry, but their Green's functions have an infinite array of singularities in momentum space and are singular everywhere in coordinate space. Nevertheless, these models are consistent quantum field theories that describe a system of interacting bound states which can decay into one another but not into quarks.

A conventional perturbative solution of a quantum field theory has asymptotically free quanta of the fundamental fields. However, since quarks are not observed, a conventional solution is inappropriate in theories with quark fields. Johnson[1] has suggested a new mechanism, which is a relativistic generalization of an r^2 potential binding quarks, which could account for the permanent binding of quark fields and thus the nonexistence of asymptotically separating quarks. In this paper we develop a new self-consistent field-theoretic perturbation method to describe strongly interacting particles which in lowest order incorporates this mechanism.

We will generalize some of our remarks to fermion fields in future papers,[2] but here we confine our discussion to a boson field φ with a φ^4 interaction. Triality is thus replaced by field parity $\varphi \to -\varphi$. A "quark" state has odd field parity and a "meson" state has even field parity.

We begin by assuming that there exist states of n mesons and states of n mesons and one quark, and we denote the corresponding projection operators onto these states by P_n and $P_{q,n}$. Projecting with $P_{q,n}$ and isolating the one-quark, n-meson contribution to φ^3, the quark field equation becomes

$$(-\Box^2 - m_0^2)P_{q,n}\varphi(x) = \lambda P_{q,n}\varphi^2 P_{q,n}\varphi(x) + \epsilon\lambda P_{q,n}\varphi^2(x)(1-P_{q,n})\varphi(x). \qquad (1)$$

ϵ, and not λ, is the perturbation parameter for this theory. Matrix elements are taken to be power series in ϵ. Each calculation is concluded by setting $\epsilon = 1$.

The basic assumption of this theory is that the one-quark to one-quark matrix element of the quark current $I(x) \equiv \lambda\varphi^2(x)$ is extremely singular at zero momentum transfer; this is how the theory incorporates an effective long-range potential between quarks. We further assume that to zeroth order in ϵ, the one-quark, n-meson matrix element of the quark current is dominated by this singularity:

$$\langle q, p_1, \ldots, p_n | I(0) | q', p_1', \ldots, p_n' \rangle^0 = \langle p_1, \ldots, p_n | p_1', \ldots, p_n' \rangle^0 [\gamma(\partial/\partial q^\mu)(q^\mu q^\nu - g^{\mu\nu}q^2)(\partial/\partial q^\nu) + \sigma] \times (2\pi)^3 2q'^0 \delta^3(q-q'). \qquad (2)$$

Here γ and σ depend on λ. Equation (2) is a manifestly covariant generalization in differential form of