

ity)<sup>10</sup>

$$I(T) = \frac{1}{y_c^2} \left( \frac{x(T)}{1-x(T)} \right)^{\epsilon} \left( 1 - \frac{[1-x(T)]^{\epsilon/2} y_c}{e^{\epsilon x(T)/2}} \right)^2. \quad (26)$$

For small  $y_c$ ,  $I(T)$  is a rapidly varying function of  $x(T)$ ; for  $y_c$  near 1,  $I(T)$  varies very slowly. For  $x(T)$  monotonically increasing,  $I(T)$  is also monotonic in  $T$ , cutting each renormalization-group trajectory exactly once. Similar behavior holds for general  $n$ .

If  $x(T) \rightarrow 1$  as  $T \rightarrow \infty$ , the temperature trajectories all pass through the infinite Gaussian point at  $x=1$ ,  $y=0$ . This requires that  $r(T) \rightarrow \infty$  for  $T \rightarrow \infty$ . For realistic Hamiltonians,  $r(T)$  has a finite limit at infinite temperature,<sup>11</sup> and the formal cross-over properties of the renormalization-group equations are not completely realized. Moreover, even before the limiting values of  $x$  and  $y$  are approached (whether these limits are at the infinite Gaussian point or not) the correlation length and other thermodynamic functions will be dominated by their high-temperature behavior, rather than by the limiting behavior of an expression such as Eq. (23).

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<sup>1</sup>F. J. Wegner and A. Houghton, Phys. Rev. A **8**, 401 (1972).

<sup>2</sup>K. G. Wilson and J. Kogut, to be published.

<sup>3</sup>E. K. Riedel and F. J. Wegner, Phys. Rev. B **9**, 294 (1974).

<sup>4</sup>The range of validity within which mean-field theory is applicable has been discussed in several papers; see, e.g., V. L. Ginsburg, Fiz. Tverd. Tela. **2**, 2031 (1969) [Sov. Phys. Solid State **2**, 1824 (1960)]; L. P. Kadanoff *et al.*, Rev. Mod. Phys. **39**, 395 (1967), and references contained therein.

<sup>5</sup>Our result should provide a logical step towards the understanding of the matching condition between the mean-field and critical region. See Y. Imry, G. Deutscher, D. Bergman, and S. Alexander, Phys. Rev. A **7**, 744 (1973); M. K. Grover, Phys. Rev. A **8**, 2754 (1973).

<sup>6</sup>F. J. Wegner, Phys. Rev. B **5**, 4529 (1972).

<sup>7</sup>K. G. Wilson and M. E. Fisher, Phys. Rev. Lett. **28**, 278 (1972).

<sup>8</sup>The ordering field is completely uncoupled from the remainder of the renormalization group for Wilson's Hamiltonian as first pointed out by J. Hubbard, Phys. Lett. **40A**, 111 (1972).

<sup>9</sup>L. P. Kadanoff, Physics **2**, 263 (1966). See also Ref. 4.

<sup>10</sup>This case, of course, exhibits no changes in the effective values of exponent  $s$ .

<sup>11</sup>M. E. Fisher and P. Pfeuty, Phys. Rev. B **6**, 1889 (1972).

## 4843-keV, $1^+$ Level of $^{208}\text{Pb}^\dagger$

C. P. Swann

*Bartol Research Foundation of the Franklin Institute, Swarthmore, Pennsylvania 19081*

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From measurements of resonance-fluorescence cross section, angular distribution, and polarization, the 4843-keV level of  $^{208}\text{Pb}$  has been shown to have a  $1^+$  character and a width of  $5.1 \pm 0.8$  eV with all of the decays to the ground state. As the probable lower member of the giant  $M1$  excitation, this state is at a significantly lower energy and has a decay strength which is an order of magnitude larger than the predictions of simple shell-model calculations.

Recently I reported on a number of states in nuclei in the lead region which were observed using the resonance-fluorescence technique.<sup>1</sup> Among these was a spin-1 state in  $^{208}\text{Pb}$  at 4843 keV with a width of 5 eV. The level was also observed by Earle *et al.*<sup>2</sup> through the  $(d, p\gamma)$  reaction. They also gave a spin-1 assignment but were unable to determine the parity. Using a two-slab Ge(Li) polarimeter,<sup>3</sup> I have now measured the linear

polarization of the resonantly scattered radiation from this state, and the results show that the parity must be positive. The corresponding ground-state  $M1$  radiative strength is 2.3 Weisskopf units, a surprisingly strong  $M1$  transition for this low an energy.

The resonance-fluorescence technique has been adequately described in the literature.<sup>4,5</sup> The 4843-keV level of  $^{208}\text{Pb}$  was excited by brems-

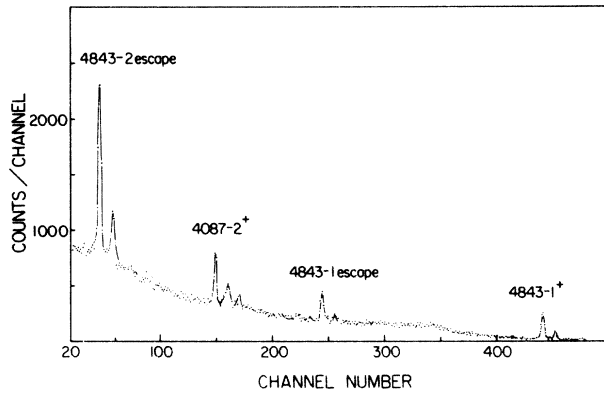


FIG. 1. Spectrum obtained for a bremsstrahlung endpoint energy of 4.95 MeV, a scattering angle of  $98^\circ$ , and with the plane of the Ge(Li) slabs perpendicular to the scattering plane. Energies are given in keV. The unlabeled peaks are lines from  $^{206}\text{Pb}$  and  $^{207}\text{Pb}$  (see Ref. 1).

strahlung produced by passing analyzed electrons from the Bartol accelerator through a thin gold foil. For the spin determination measurements were made for scattering angles of  $98$  and  $127^\circ$  which are close to the minimum and the zero of the Legendre polynomial of order 2. A spin-2 state would result in a counting-rate ratio,  $N_{98^\circ}/N_{127^\circ}$ , of 2.05, whereas a spin-1 level would give a ratio of 0.74. The experimental result of  $0.63 \pm 0.14$  is clearly consistent only with a spin-1 assignment, being 10 standard deviations away from the spin-2 possibility. The width of the state was calculated from the scattering cross section assuming a 100% ground-state decay. Such an assumption is entirely reasonable; the partial widths for possible decays to the  $2^+$  or the  $3^-$  states would be very small compared to the width for decay to the ground state. Of course the intensity of the exciting radiation must also be known, and this was determined by extrapolating the "standards" curve obtained previously.<sup>4</sup>

The width I obtain is  $5.1 \pm 0.8$  eV, where the error reflects principally the uncertainty in the intensity of the exciting radiation. This error has been increased over that given previously.<sup>1</sup>

The technique for measuring the linear polarization using a slab-type Ge(Li) detector has been described by Litherland, Ewan, and Lam.<sup>6</sup> In the present case there are two rectangular slabs measuring  $5.8 \times 3.8 \times 0.8$  cm<sup>3</sup> which are treated electronically as separate detectors. Measurements were made with the slabs in the plane and perpendicular to the plane of the scattered radiation. Figure 1 presents the spectrum of the data obtained using a normal Pb scatterer with the plane of the slabs perpendicular to the scattering plane. In the spectrum one also observes the two-escape peak resulting from pair production. One does not expect any significant polarization effect<sup>6</sup> for these photons, and indeed within the errors no effect was observed. From the measurements one obtains  $(N_{\parallel} - N_{\perp})/(N_{\parallel} + N_{\perp})$ , where  $N_{\parallel}$  and  $N_{\perp}$  are the respective counting rates for the two orientations. The sign of the effect is the determining factor as to whether the parity of the state involved is positive or negative; the magnitude of the effect is only important to the extent that its error must be small enough to allow for a sign selection. From experience one expects the magnitude at 4.8 MeV to be no less than 4%.

For the 4843-keV radiation from  $^{208}\text{Pb}$  I obtain a value for  $(N_{\parallel} - N_{\perp})/(N_{\parallel} + N_{\perp})$  of  $(-5.41 \pm 2.86)\%$ . I conclude, therefore, that the sign is negative to a probability of better than 99.9%. The negative sign corresponds to a positive parity. The  $B(M1, 1^+ \rightarrow 0^+)$  value for this level becomes  $3.8(e\hbar/2Mc)^2$  and is listed in line 3 of Table I.

Recently, Bowman *et al.*<sup>7</sup> reported on a large concentration of M1 strength to the ground state of  $^{208}\text{Pb}$  just above the  $(\gamma, n)$  threshold. Toohey and Jackson<sup>10</sup> question many of the  $1^+$  assign-

TABLE I. Comparison of the experimental and theoretical results for the "giant" M1 excitation in  $^{208}\text{Pb}$ .

Experimental		Theoretical <sup>a</sup>	
$E_{\gamma}$ (MeV)	$B(M1, 1^+ \rightarrow 0^+)$ $[(e\hbar/2Mc)^2]$	$E_{\gamma}$ (MeV)	$B(M1, 1^+ \rightarrow 0^+)$ $[(e\hbar/2Mc)^2]$
7.41 to 8.24 <sup>b</sup> (7 levels)	(11.6)	7.52	16.0
7.28 <sup>c</sup>	0.2	...	...
4.84 <sup>d</sup>	3.8	5.45	0.4

<sup>a</sup>See Ref. 9.

<sup>b</sup>See Ref. 7.

<sup>c</sup>See Ref. 8.

<sup>d</sup>Present study.

ments made by Bowman *et al.* but nevertheless conclude that a "giant"  $M1$  is probable. A combined width of 50.8 eV was obtained by Bowman *et al.* for the seven states they assign as  $1^+$ . Allowing for additional observed but unassigned levels they get a width as large as 65 eV. This corresponds to the  $B(\lambda=1, 1^+ \rightarrow 0^+)$  value of  $11.6 \times (e\hbar/2Mc)^2$  as given in the first line of Table I; I have parenthesized this number because of the uncertainty as to the true value. It is also possible that a number of additional  $1^+$  states exist below the  $(\gamma, n)$  threshold. One such state with a 100% ground-state decay and a width of 0.78 eV was observed by Wolf *et al.*<sup>8</sup> at 7.28 MeV and the  $B(M1)$  value is given on line 2 of Table I. A number of other levels with strong ground-state transitions have been observed in  $(\gamma, \gamma)$  studies,<sup>11,12</sup> but are assumed to have  $J^\pi = 1^-$  since they were also observed through the decay of analog resonances formed by the proton bombardment of  $^{208}\text{Pb}$ .<sup>13</sup> The one possible exception may be one of the doublets at 7.1 MeV.<sup>11</sup>

The simplest shell model predicts two  $1^+$  states in  $^{208}\text{Pb}$  as mixtures of 1p-1h (one-particle, one-hole) excitations from the  $i_{13/2}$  neutron and  $h_{11/2}$  proton shells, and a total  $B(M1)$  sum-rule strength of about  $16.5(e\hbar/2Mc)^2$ .<sup>14</sup> Several theoretical calculations assuming 1p-1h transitions but using different residual interactions have been made.<sup>9,15,16</sup> The results of these studies give different energies and relative  $B(M1)$  strengths for the two excitations. The energies and the strengths, however, are related. Within the framework of this simple model a reduction in the energy of the lower state requires more mixing of the neutron and proton excitations, which because of incoherence reduces the  $B(M1)$  strength of the lower state. Of course, the mixing will be coherent in the upper state and increased mixing will result in a somewhat higher energy and greater  $B(M1)$  strength.

In Table I we give the results obtained by Vergados,<sup>9</sup> the most recent of the calculations. As can be seen, the experimental  $B(M1)$  strength of the lower excitation is an order of magnitude

greater than the theoretical prediction even though the energy is significantly lower. It is quite clear that the simple 1p-1h model cannot reproduce this lower state, assuming, of course, that this state is indeed related to the giant  $M1$  excitation.

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<sup>1</sup>C. P. Swann, in *Proceedings of the International Conference on Photoneuclear Reactions and Applications, Pacific Grove, California, 1973*, edited by B. L. Berman (Lawrence Livermore Laboratory, Livermore, Calif., 1973), Vol. 1, p. 317.

<sup>2</sup>E. D. Earle, A. J. Ferguson, G. Van Middelkoop, G. A. Bartholomew, and I. Bergqvist, *Phys. Lett.* **32B**, 471 (1970).

<sup>3</sup>F. R. Metzger and V. K. Rasmussen, *Phys. Rev. C* **8**, 1099 (1973).

<sup>4</sup>C. P. Swann, *Nucl. Phys.* **A172**, 569 (1971).

<sup>5</sup>F. R. Metzger, *Ann. Phys. (New York)* **66**, 697 (1971).

<sup>6</sup>A. E. Litherland, G. T. Ewan, and S. T. Lam, *Can. J. Phys.* **48**, 2320 (1970).

<sup>7</sup>C. D. Bowman, R. J. Baglan, B. L. Berman, and T. W. Phillips, *Phys. Rev. Lett.* **25**, 1302 (1970).

<sup>8</sup>A. Wolf, R. Moreh, A. Nof, O. Shahal, and J. Tenenbaum, *Phys. Rev. C* **6**, 2276 (1972).

<sup>9</sup>J. D. Vergados, *Phys. Lett.* **36B**, 12 (1971).

<sup>10</sup>R. E. Toohy and H. E. Jackson, *Phys. Rev. C* **6**, 1440 (1972).

<sup>11</sup>C. P. Swann, *Nucl. Phys.* **A201**, 534 (1973).

<sup>12</sup>J. W. Knowles and A. M. Kahn, *Bull. Amer. Phys. Soc.* **12**, 538 (1967), and as referred to in Ref. 2.

<sup>13</sup>J. G. Creamer, P. von Brentano, G. W. Phillips, H. Ejiri, S. M. Ferguson, and W. J. Braithwaite, *Phys. Rev. Lett.* **21**, 297 (1968).

<sup>14</sup>Various other particle-hole excitations might be considered but all of these would be expected to reduce the total  $B(M1)$  strength. For example, Vergados (Ref. 9) obtained a 20% reduction upon including the  $M1$  core polarization in the ground state of  $^{208}\text{Pb}$ .

<sup>15</sup>V. Gillet, A. M. Green, and E. A. Sanderson, *Nucl. Phys.* **88**, 321 (1966).

<sup>16</sup>R. A. Broglia, A. Molinari, and B. Sørensen, *Nucl. Phys.* **A109**, 353 (1968).