Magnetic Form Factor of ^{3}He†

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(Received 18 April 1974)

The ^{3}He magnetic form factor is calculated in the impulse approximation using a wave function obtained previously from a complete solution of the Faddeev equations with the Reid soft-core potential. The *S-D* matrix elements contribute significantly. The convection current contribution is found to be small. The form factor has a minimum at $Q^2 = 7.1$ fm^{-2}.

Calculations of bound-state properties of the three-nucleon system are a useful means for studying the nucleon-nucleon interaction. Of particular interest are electromagnetic form factors, which for large values of the momentum transfer should depend sensitively on the short-range nuclear potential. We have previously reported[1] on a complete solution of the Faddeev equations[2] for ^{3}He using the entire Reid soft-core potential.[3] In this paper we use the resulting wave function to calculate the ^{3}He magnetic form factor, $F_{\mathrm{MAG}}{}^{3\mathrm{He}}$. To our knowledge, this is the first complete Faddeev calculation of $F_{\mathrm{MAG}}{}^{3\mathrm{He}}$ which involves realistic nuclear forces, off-diagonal *S-D* contributions, and the nucleon convection current.

In calculating $F_{\mathrm{MAG}}{}^{3\mathrm{He}}$, we use the usual impulse approximation and nonrelativistic reduction of the nucleon-current matrix elements.[4,5] Contributions from meson-exchange currents are neglected. A convenient expression for $F_{\mathrm{MAG}}{}^{3\mathrm{He}}$ is obtained if we (a) use the fact (following from charge-current conservation) that only the component of the nucleon current perpendicular to the electron momentum transfer $\vec{Q}$ contributes, (b) orient our coordinate system so that $\vec{Q}$ lies in the x direction, and (c) choose the z axis as the quantization axis for magnetic quantum numbers. We then find that

$$F_{\mathrm{MAG}}{}^{3\mathrm{He}}(Q^2) = (3/Q)\int d^3p_1 \int d^3q_1\, \psi^*(\vec{p}_1, \vec{q}_1 - \vec{Q}/\sqrt{3}) J_y{}^{(1)} \psi(\vec{p}_1, \vec{q}_1), \qquad (1)$$

where $\psi(\vec{p}_1, \vec{q}_1)$ is the ^{3}He center-of-mass momentum-space wave function with $\mathcal{J} = \mathcal{J}_z = \frac{1}{2}$. (Spin and isospin variables are not indicated.) The momenta, $\vec{p}_1$ and $\vec{q}_1$, are given in terms of the nucleon momenta $\vec{k}_i$ by

$$\vec{p}_1 = \tfrac{1}{2}(\vec{k}_2 - \vec{k}_3), \quad \vec{q}_1 = (\vec{k}_2 + \vec{k}_3 - 2\vec{k}_1)/2\sqrt{3}. \qquad (2)$$

$J_y{}^{(1)}$ is the y component of $\vec{J}^{(1)}$, the momentum matrix element of the current operator for nucleon 1, consisting of convection and magnetic-spin-current contributions[4,5]:

$$\vec{J}^{(1)} = \vec{J}_C{}^{(1)} + \vec{J}_M{}^{(1)} = -[(4/\sqrt{3})\vec{q}_1 + \vec{Q}]e_1 + i(\vec{\sigma}_1 \times \vec{Q})\mu_1. \qquad (3)$$

$\vec{\sigma}_1$ is the Pauli spin operator for nucleon 1 and

$$e_1 = f_{\mathrm{CH}}{}^p(Q^2)\tfrac{1}{2}(1+\tau_z{}^{(1)}) + f_{\mathrm{CH}}{}^n(Q^2)\tfrac{1}{2}(1-\tau_z{}^{(1)}), \qquad (4)$$

$$\mu_1 = [+2.793 f_{\mathrm{MAG}}{}^p(Q^2)]\tfrac{1}{2}(1+\tau_z{}^{(1)}) + [-1.913 f_{\mathrm{MAG}}{}^n(Q^2)]\tfrac{1}{2}(1-\tau_z{}^{(1)}). \qquad (5)$$

$f_{\mathrm{CH}}(Q^2)$ and $f_{\mathrm{MAG}}(Q^2)$ are, respectively, the charge and magnetic form factors for the appropriate nucleon, which we take to be the analytic forms of Janssens *et al.*[6]

We have calculated $F_{\mathrm{MAG}}{}^{3\mathrm{He}}(Q^2)$ numerically using the 27 components of $\psi(\vec{p}, \vec{q})$ as given in Ref. 1. We find that the *P*-wave components contribute negligibly and can be ignored.

The results of using only the spin term of $\vec{J}^{(1)}$ are shown in Fig. 1 in order to make comparisons with earlier works.[7-9] The dot-dashed line, with a minimum around 14.7 fm^{-2}, is the form factor given by the *S*-wave components of the ^{3}He wave function alone. There is good agreement between this curve and the results of Ref. 8. (Strict comparisons should not be made since more *S*-wave compo-

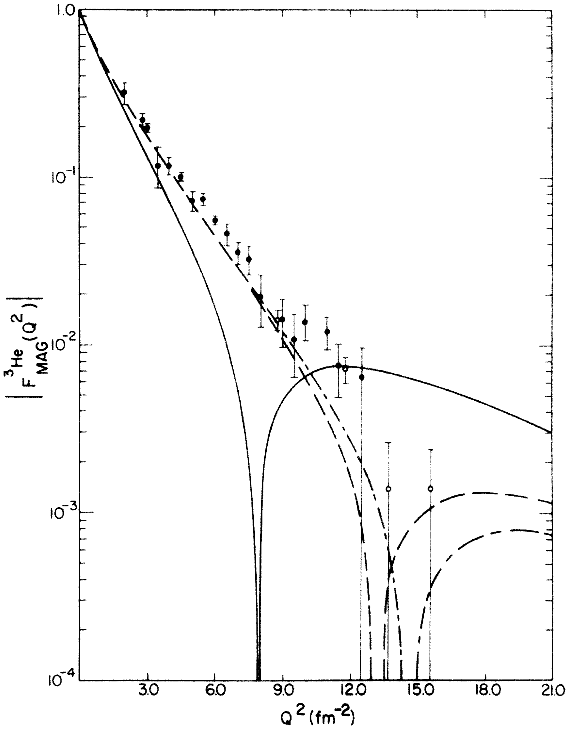

FIG. 1. ^{3}He magnetic form factor with only the $i\vec{\sigma}\times\vec{Q}$ term in $\vec{J}^{(1)}$. The solid-circle data points are from Ref. 10 and the open-circle points from Ref. 11. The dot-dashed curve includes only S-wave components of the wave function. The solid and dashed curves represent all components, with and without off-diagonal matrix elements, respectively.

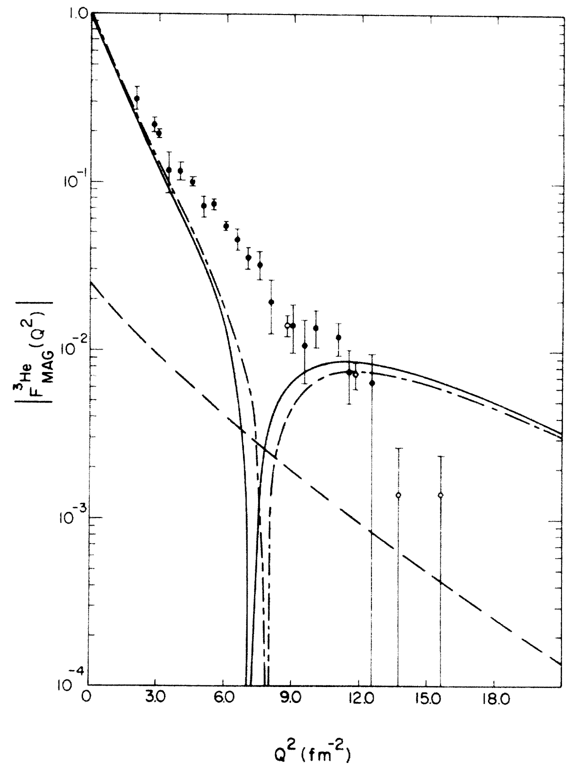

FIG. 2. ^{3}He magnetic form factor. The data points are the same as in Fig. 1. The solid curve gives the total magnetic form factor, while the dashed and dot-dashed curves represent the contributions from the convection and $i\vec{\sigma}\times\vec{Q}$ terms in $\vec{J}^{(1)}$, respectively.

nents are included in the present calculation.) The dashed curve is calculated using all wave-function components but neglecting contributions from off-diagonal matrix elements. The addition of these contributions results in a shift of the minimum from about 13 to 7.8 fm^{-2}. This downward shift in the minimum, due mainly to S-D matrix elements, is in qualitative agreement with the results of Hadjimichael and Barroso.[12]

In Fig. 2, the solid curve gives $|F_{MAG}^{^3He}(Q^2)|$ calculated with the total $\vec{J}^{(1)}$. The separate contributions from the spin and convection-current terms are plotted as dot-dashed and dashed curves, respectively. All three curves are renormalized so that $|F_{MAG}^{^3He}(Q^2=0)|=1$. The effect of the convection current is to shift the minimum from 7.8 to 7.1 fm^{-2}.

The calculated (unrenormalized) form factor, extrapolated to $Q^2=0$, is in good agreement with the single-particle contribution to the ^{3}He magnetic moment ($=1.74\mu_N$) calculated using the wave function probabilities of Ref. 1. If we neglect $P(P)$, the small P-state probability of the ^{3}He wave function, the single-particle contribution is given by

$$F_{MAG}^{^3He}(Q^2=0)=\mu^{(1)}(^3\text{He})=\mu_s^{(1)}-\mu_v^{(1)},$$

where

$$\mu_s^{(1)}=\tfrac{1}{2}(\mu_p+\mu_n)[P(S)+P(S')-P(D)]+\tfrac{1}{2}P(D),\quad \mu_v^{(1)}=\tfrac{1}{2}(\mu_p-\mu_n)[P(S)-\tfrac{1}{3}P(S')+\tfrac{1}{3}P(D)]-\tfrac{1}{6}P(D),$$
$$\mu_p=2.793\mu_N,\quad \mu_n=-1.913\mu_N. \tag{6}$$

In Harper *et al.*[13] it is shown that most of the discrepancy between the single-particle contribution and the experimental magnetic moment ($2.128\mu_N$) can be accounted for by two-body meson-exchange currents calculated in the one-pion-exchange approximation. The same contributions to the magnetic form factor at nonzero Q^2 values may account for the lack of a minimum in the experimental data.[10,11] Meson-exchange corrections as calculated by Hadjimichael and Barroso[14] indeed indicate that this may be the case. However, Hadjimichael and Barroso use an approximate method for obtaining the ^{3}He D-state component which may not be very accurate.[15] This may, in part, account for the fact that their exchange correction extrapolated to $Q^2=0$ from their Fig. 2, of about $0.28\mu_N$, is considerably less than the amount ($\approx 0.38\mu_N$) required to fit the experimental magnetic moment. In any case, there is a need for calculating meson-exchange-current corrections more accurately. Also, the accurate measurement of $|F_{MAG}{}^{^3He}(Q^2)|$ for $Q^2 > 12$ fm^{-2} would be extremely valuable.

We wish to thank Dr. B. F. Gibson for several helpful conversations.

†Research supported in part by the U. S. National Science Foundation and by the U. S. Atomic Energy Commission.
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