as a three-level system. Calculation of the refractive index must consider optical pumping of inhomogeneous lines in the presence of strong hole burning. Such a theory is difficult and has not previously been worked out. Also the nonlinearity is not strictly local since the time constant for optical pumping is comparable to the excited-state lifetime and thermal atoms move tens of micrometers during this time. For tuning many Doppler widths off resonance the calculation is simplified since the sodium vapor may then be treated as an inhomogeneously broadened two-level system. For this case we have shown that the refractive index is of the form $n(\nu)-1=-\delta n \times \mathrm{Im}[w(\Delta+iy)]$,[14] where w is the error function for complex arguments,[15] Δ is a normalized detuning, and y is the normalized width of the power-broadened hole burned in the line. For sodium, $\delta n = 2.7\times10^{-17}N$, which is more than enough to account for the observed focal-spot diameters.[8] The above result can be used to estimate the maximum detuning for self-focusing to occur at a given intensity. For $I/I_s \approx 10^4$ this estimate agrees with our observation of strong self-focusing as far as 3 GHz above the $F=1$ transition. The allowed detuning increases for larger N and increased input power.

Detailed comparison of accurate cw experiments like ours with a complete theory would be worthwhile. Understanding of self-action effects in simple atomic vapors may be applicable to similar effects in more complicated media such as Kerr liquids. Our demonstration of self-trapping should stimulate further work to elucidate the general conditions under which self-trapping can occur.

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Atomic Capture of Negative Mesons*

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The atomic capture of negative mesons is examined in detail, using the model of Fermi and Teller. We calculate the initial distribution in angular momentum of the mesons upon capture and that after de-exciting through the electron cloud. The mesonic x-ray intensities are computed from a quantum-mechanical cascade and compared with experiment for K^- mesons. Our results agree fairly well with the average (over Z) of the experimental data.

The physics involved in the precise calculation of the *energy shifts* and *widths* of mesonic x-ray lines has received considerable attention.[1] In contrast, the physics that determines the *intensities* of these lines has been comparatively neglected.[2] The usual procedure for calculating the line intensities involves starting a quantum-mechanical cascade from a large negative energy E_n with an *ad hoc* "initial" distribution in angular momentum $P(l)$ of the form

$$P(l) \propto (2l+1)\exp(al), \tag{1}$$

the parameter a being adjusted to fit experimental data. In this Letter we report on our calculations of the atomic capture and de-excitation, which include *computing* the "initial" distribution $P(l)$ (with *no* adjustable parameters) and using it to predict x-ray intensities. In our view, there have been no even moderately convincing calculations of $P(l)$ heretofore.[3] The major goal of the present work is to determine how well, on the basis of the Fermi-Teller model,[4] one can describe the initial capture and early de-excitation of the meson, and hence calculate $P(l)$.

A secondary goal is to provide a framework for discussion of the recent K^- absolute-yield data of Wiegand[5] and Wiegand and Godfrey.[6] This fits in very well with our major goal since the K^- absolute yields are sensitive to the shape of the initial distribution. While one might think that dealing with hadronic atoms would be more complicated than dealing with muonic atoms, in fact the nuclear absorption brings about some helpful technical simplifications[7] and, for K^-, can easily be calculated with the required accuracy. Thus our numerical calculations have been carried out for K^- capture; however, most of our results, those which have to do only with atomic physics, can easily be scaled to apply to other "mesons."

In their pioneering work on negative-meson capture, Fermi and Teller[4] treated the meson as a classical particle undergoing continuous energy loss. Subsequently, there have been a few attempts to improve upon this description by making quantum-mechanical calculations of the initial atomic-capture process.[8] There are two major difficulties with these quantum-mechanical calculations: (i) The large mass of the meson (compared to the electron mass) means very rapidly varying meson wave functions (very large quantum numbers); (ii) the use of Born approximation is inappropriate since the meson is moving slowly enough to allow the electron wave functions to accomodate to its presence to a large extent (adiabatic limit). Of course, (i) is just the reason that the classical description of the meson's motion is a good one; and, as for (ii), the Fermi and Teller formulation should be valid near the adiabatic limit. Thus we conclude that the classical approach is the more promising one, and we are led to explore the Fermi-Teller model in some detail. We neglect any effects of electron depletion, so our results are most applicable to metals.

The electrons, of course, must be treated quantum mechanically; in the Fermi-Teller model they are assumed to form a degenerate Fermi gas, with Fermi momentum $p_{\rm F}$ corresponding to the depth of the atomic potential at the instantaneous position of the meson. The electrons feel the (shielded) Coulomb potential from the meson and, since the meson is moving, can pick up energy in a collision with it. Real collisions occur when an electron is scattered out of the Fermi sphere (Auger ejection), and integrating over all possible initial and final electron momenta yields the Fermi-Teller expression for the *average* rate of energy loss. If we also include the contribution of radiation, which is rather small, the expression is (in atomic units)

$$-\frac{dW}{dt}=\frac{4}{3\pi}(\epsilon V)^2\ln\sqrt{p_{\rm F}}+\frac{2}{3c^3}(\epsilon A)^2, \tag{2}$$

where V is the meson speed and A its acceleration, and where ϵ, the Fried-Martin factor,[9] expresses the effect of nuclear motion. The angular-momentum (l) loss is given in terms of dW/dt by

$$-dl/dt=-(dW/dt)l/MV^2, \tag{3}$$

M being the meson reduced mass. As an atomic potential (which determines both $p_{\rm F}$ and the meson motion) we use the (spherically symmetric) neutral Thomas-Fermi potential, modified to vanish smoothly at the effective atomic radius.

For a given incident energy and impact parameter, we integrate (numerically) the meson's radial equation of motion to calculate the energy and angular-momentum loss, and hence determine whether the meson will be captured. This information determines the capture cross section and, assuming the incident mesons to be distributed uniformly in energy, the initial distributions. These are shown in Fig. 1. The distribution in initial l, $f(l_i)$, is distinct from the distribution in final l at zero energy, $f'(l_f)$, because the meson loses angular momentum during its capture and descent to zero energy.[10]

The next question is how this distribution changes as the mesons de-excite through the electron cloud. Following individual orbits would be extremely laborious; however, what counts is the relation between energy loss and angular-

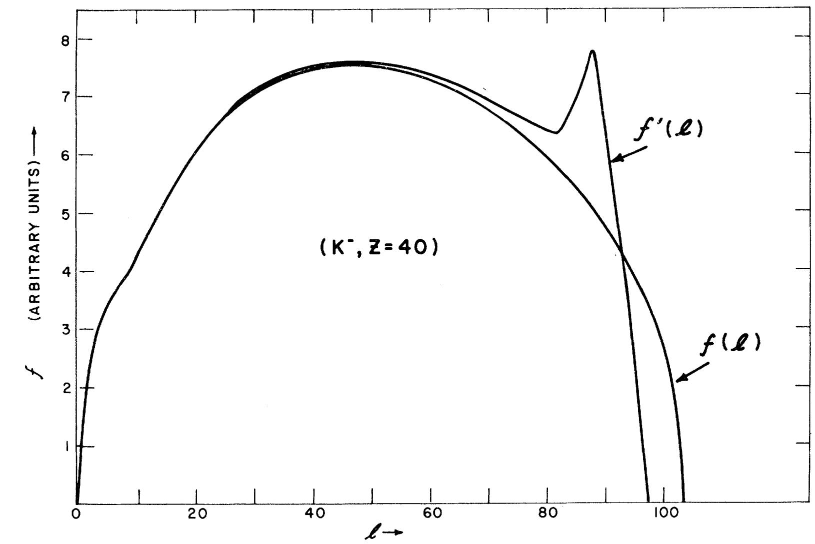

FIG. 1. Distribution of mesons in incident l, $f(l)$, and in l at $E=0$, $f'(l)$.

momentum loss, and the ratio $\delta W/\delta l$ is found by integrating Eqs. (2) and (3) over one period. The problem can then be efficiently handled by putting this $\delta W/\delta l$ into the equation of continuity for the distribution function $F(E, l)$ and integrating this numerically, with the appropriate boundary condition $[F(0, l) \equiv f'(l)]$. Remarkably, we find that the *shape* of the distribution remains *quantitatively unchanged* as we go from zero energy down to the electronic K-shell energy E_n.[11] We consider the computation of $f'(l)$ and $P(l)$ $[\equiv F(E_n, l)]$ to be the major results of this work.

To predict line intensities we switch from the Fermi-Teller model to a quantum-mechanical cascade calculation, using our $P(l)$ as initial distribution (starting at $n=30$ for K^-'s). This part of our work is novel only in that, instead of a perturbation calculation for the Auger rates, we use Ferrell's formula[12]: This relates each Auger rate $[\Gamma_A(\omega)]$ to the corresponding radiation rate $[\Gamma_{rad}(\omega)]$ and the photoelectric cross section $\sigma_{\gamma e}{}^{Z-1}(\omega)$ for a $Z-1$ atom, for which experimental data are available.[13] Ferrell's formula is

$$\Gamma_A(\omega) = \Gamma_{rad}(\omega)\sigma_{\gamma e}{}^{Z-1}(\omega)/(Z-1)^2\sigma_T, \quad (4)$$

where σ_T is the Thomson cross section. Nuclear absorption for K^- is found by integrating the Klein-Gordon equation with a phenomological optical potential that reproduces existing kaonic x-ray energy-shift and -width data for light nuclei.[14]

Figure 2 compares our results for the most intense lines with the *absolute-yield* data of Wiegand.[5] These data (and the more recent data of

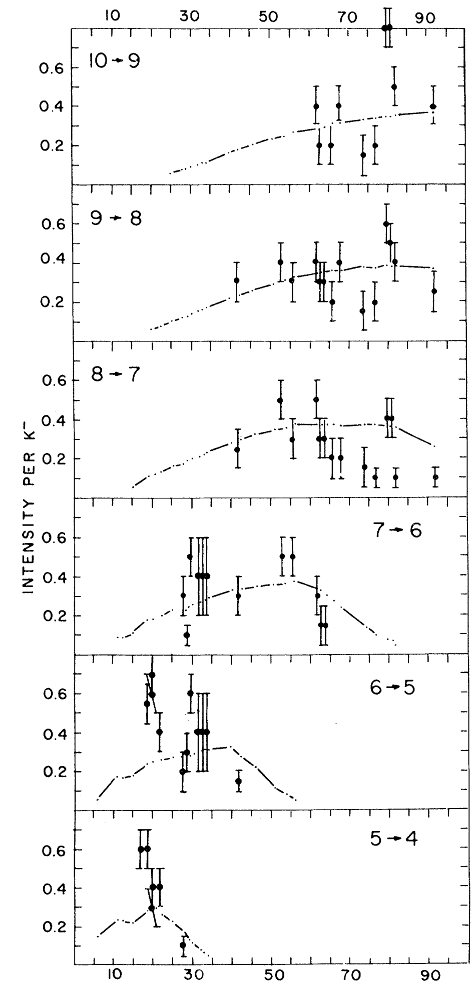

FIG. 2. Calculated absolute kaonic x-ray yields compared with the data of Wiegand (Ref. 5). The experimental data are shown with error bars.

Wiegand and Godfrey[6]) exhibit sharp intensity variations with Z which must be coming from variations in atomic structure. Since in the Fermi-Teller model individual atomic structure is abandoned in favor of the statistical model, it is only sensible to compare our results with the experimental intensities *smoothed over* Z. While this considerably weakens the test, we think that the agreement is fairly good. Our maximum yields are in the range (20–40)%; in contrast, Eisenberg and Kessler[2] and Martin,[8] because they used a "statistical" $P(l)$ [i.e., Eq. (1) with $a = 0$], found maximum yields of about 70%.

Hence we conclude that, to the extent that one is willing to ignore effects due to individual atomic structure, the Fermi-Teller model provides an adequate framework for calculating initial capture and de-excitation. A detailed account of this work will appear elsewhere.

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Temporal Electrostatic Instabilities in Inhomogeneous Plasmas*

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The parametric process where the electromagnetic wave drives two Langmuir oscillations near $2\omega_p$ can be shown to be temporally unstable even in inhomogeneous plasmas. The instability arises because of wave trapping which prevents the convection of wave energy out of the unstable region.

Recently there has been much interest in parametric instabilities in inhomogeneous plasmas which have applications both in controlled fusion experiments and in high-intensity laser-pellet interactions. Rosenbluth[1] presented a theory of parametric three-wave interactions within the WKB approximation. His results show that, for the case $k = k'(0)x$, instabilities can be saturated by spatial convection of wave energy out of the unstable region. Subsequent investigation by Drake and Lee[2] indicates the existence of temporal instabilities when the WKB approximation is no longer valid. In this paper we study the parametric decay instabilities of Langmuir oscillations at $\omega_0 \simeq 2\omega_p$. We find that both Lang-