Slowing Down of Ions by Ultrahigh-Density Electron Plasma

A. Dar, J. Grunzweig-Genossar, A. Peres, M. Revzen, and A. Ron
Department of Physics, Technion-Israel Institute of Technology, Haifa, Israel
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The slowing down of ions by a degenerate electron plasma is calculated by means of the random-phase approximation. It is shown that high compressions greatly reduce the ability of electrons to slow down ions. These results indicate that nuclear burning of a DT plasma can proceed as a *chain reaction*, even at low temperature, at densities about 10^5 times the solid density.

It was shown long ago by Fermi and Teller[1] that a degenerate electron gas loses its efficiency for stopping heavy charged particles when the velocity of the latter falls below the velocity of the electrons at the Fermi surface. Some years later, Gryziński[2] pointed out that if the Fermi surface were raised high enough when compressing a DT plasma, then energetic ions resulting from nuclear reactions would give their energy mostly to other ions by nuclear collisions, rather than to electrons as usual. They would thus produce showers of fast ions, some of which could undergo further fusion reactions. In this fashion, DT fusion could proceed as a fast *chain reaction*, rather than as a thermonuclear reaction. Gryziński's calculations, however, were fraught with many errors and attracted only scant attention.[3]

There has recently been a renewal of interest in fast charged-particle reactions in a plasma,[4,5] in view of claims that laser-driven implosions might yield densities 10^4 times the normal solid density.[6,7] We have investigated the critical conditions to reach a fusion chain reaction (detailed calculations will be published in a separate paper) and have found that the slowing down due to electrons appears in the transport equations via the expression

$$\tau(E_i) = (1/E_i)\int(E_i - E_f)d\sigma(E_i, E_f), \tag{1}$$

where $d\sigma(E_i, E_f)$ is the effective cross section for scattering an ion with initial energy E_i into the final energy range E_f to $E_f + dE_f$. Note that τ has the dimensions of an area. Its physical meaning is given by the energy-range relationship $dE/dx = -En_e\tau$.

In order to compute τ, we consider a high-density electron gas at zero temperature, with a compensating static positive background due to the plasma ions. The electron density is assumed to be much larger than a_0^{-3}, where $a_0 = \hbar^2/me^2$ is the Bohr radius for an electron of mass m and charge $-e$. We now consider an ion of mass M, charge Ze, and initial energy $E_i = Mv_i^2/2$ being scattered by the electron plasma. In the Born approximation, the differential cross section for the ion scattering is given by

$$\frac{d^2\sigma}{d\Delta\, dq} = \frac{2Z^2e^4M}{\hbar^3E_iq^3} S_{-\vec{q}}\left(\frac{\Delta}{\hbar}\right), \tag{2}$$

where $\Delta = E_i - E_f$ is the energy transfer, $\hbar\vec{q}$ is the momentum transfer, and

$$S_{\vec{k}}(\omega) = \int_0^\infty dt\, e^{i\omega t}\langle \rho_{\vec{k}}(t)\rho_{-\vec{k}}(0)\rangle \tag{3}$$

is the electron plasma form factor. Here, $\rho_{\vec{k}}$ denotes the Fourier transform of the electron particle density $\rho(\vec{r})$ and the angular brackets denote an ensemble average. The energy loss of the ion is then given by

$$\frac{dE}{dx} = \frac{1}{V}\int d\Delta\, \Delta \int dq \frac{d^2\sigma}{d\Delta\, dq}, \tag{4}$$

where V is the volume of the plasma and the limits of integration are determined by the ion-electron collision kinematics.

Since our plasma is very dense, namely, much denser than a metallic electron plasma, the random-phase approximation (RPA) should yield a very accurate result for the form factor $S_{\vec{k}}(\omega)$. It is convenient to express $S_{\vec{k}}(\omega)$ in terms of the dielectric function $\epsilon_{\vec{k}}(\omega)$:

$$S_{\vec{k}}(\omega) = (2\hbar V/v_k)\mathrm{Im}[1/\epsilon_{\vec{k}}(\omega)], \tag{5}$$

where $v_k = 4\pi e^2/k^2$ is the Fourier transform of the Coulomb interaction of the electrons. The RPA dielectric function is given by[8]

$$\epsilon_{\vec{k}}(\omega) = 1 - \frac{2v_k}{V}\sum_{\vec{p}} \frac{f_{\vec{p}+\vec{k}} - f_{\vec{p}}}{E_{\vec{p}+\vec{k}} - E_{\vec{p}} + \hbar\omega}, \tag{6}$$

where $E_p = \hbar^2p^2/2m$ is the electron energy, and f_p is the Fermi distribution. Using dimensionless variables $y = q/2p_0$ and $x = \Delta/4E_0$, where $p_0 = (3\pi^2n_e)^{1/3}$ is the Fermi wave number and E_0

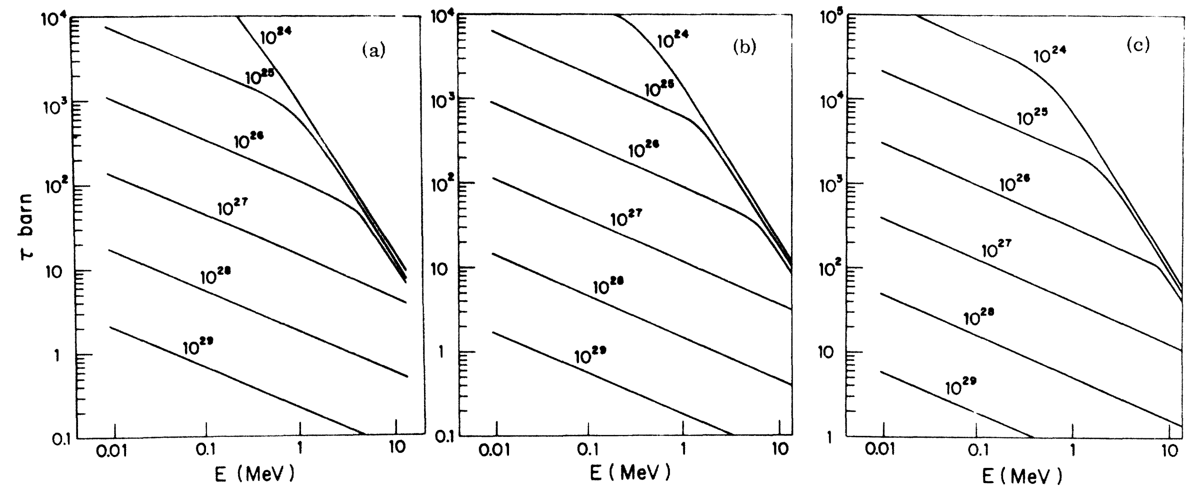

FIG. 1. Slowing-down cross section τ as a function of ion energy E, for $n_e = 10^{24}$ to 10^{29} cm^{-3}, for (a) deuterons, (b) tritons, and (c) α particles.

$=\hbar^2 p_0^2/2m$ is the Fermi energy, we obtain

$$\epsilon_{-\vec{q}}(\Delta/\hbar) = 1 - (y_0^2/y^3)[R(z_-) + R(z_+)] - i(y_0^2/y^2) I(y, x), \qquad (7)$$

with $z_\pm = y \pm x/y$, and $y_0^2 = \kappa^2/4p_0^2 = 1/\pi a_0 p_0$. Note that y_0 is the inverse Thomas-Fermi screening length κ in units of $2p_0$. In Eq. (7) we used

$$R(z) = \tfrac{1}{2}[(1 - z^2)\ln|(1 - z)/(1 + z)| - 2z] \qquad (8)$$

and

$$\begin{aligned} I(y, x) &= -\pi x/2y \text{ for } z_+^2 \leq 1, \\ &= -\pi(1 - z_-^2)/8y \text{ for } z_-^2 \leq 1 \\ &\qquad \text{and } z_+^2 > 1, \\ &= 0 \text{ otherwise.} \end{aligned} \qquad (9)$$

Note that in the long-wavelength limit, the static dielectric function ϵ tends to $1 + y_0^2/y^2$ and the dynamic one to $1 - x_0^2/x^2$, where $x_0 = h\omega_p/4E_0$ is the plasma frequency ω_p, in our units. However, since the integrals of Eq. (4) cover the whole range of q and Δ, we have to use the complete expression for ϵ in Eq. (7) while integrating the cross section to find the energy loss.

Because of the large ion-to-electron mass ratio, the kinematic limitations on the energy and momentum exchange, namely

$$x \geq \beta y - (m/M) y^2, \qquad (10)$$

give essentially $x \geq \beta y$, where $\beta = m v_i/\hbar p_0$ is the ion-to-electron velocity ratio. Thus the integrations in the xy plane are limited by Eqs. (9) and (10), and the region where the cross section does not vanish is severely restricted by the electron degeneracy. This makes the plasma *more transparent* to ions than classical calculations would indicate.[1]

We have calculated numerically the energy loss for deuterons, tritons, and α particles, using Eq. (4). The results are shown in Fig. 1. The two extreme limits, namely, slow ($\beta \ll 1$) and fast ($\beta \gg 1$) ions, can be handled analytically to yield the following expressions[9]: (a) For slow ions the energy loss is linear in β,

$$\frac{dE}{dx} = -\frac{4Z^2 E_0 y_0^2}{3a_0}\beta\left(\ln(1 + y_0^{-2}) - \frac{1}{1 + y_0^2}\right), \qquad (11)$$

and at very high density ($y_0 \ll 1$) behaves as $\ln n_e$ rather than as n_e, for nondegenerate plasmas. (b) For fast ions we obtain the classical expression

$$dE/dx = -(2\pi M Z^2 e^4 n_e/m E_i)\ln\beta^2. \qquad (12)$$

Figure 2 depicts the energy loss as a function of β for three types of calculation: Thomas-Fermi cutoff, static dielectric function, and dynamical screening (RPA). The latter is seen to deviate most markedly from static calculations when the ion velocity becomes larger than that of electrons. Apparently, the electron cloud is then unable to follow and screen the ion.

Let us now draw some conclusions from Fig. 1: As nuclear cross sections are of the order of

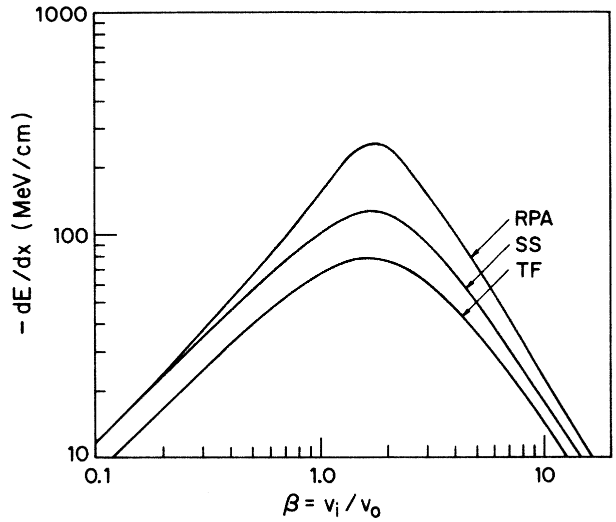

FIG. 2. Proton energy loss for $n_e = 10^{23}$ cm^{-3} as a function of velocity ratio β for three methods of calculation: dynamical screening (RPA), static self-consistent screening (SS), and Thomas-Fermi static screening (TF).

barns, it is clear that electrons dominate the slowing-down process for densities much below 10^{27} cm^{-3}, but become negligible for densities much above 10^{28} cm^{-3}. Somewhere between these two values lies the critical density for a fusion chain reaction. Detailed numerical calculations indeed confirm this estimate (the exact value depends on the fraction of the neutron energy deposited in the DT pellet, i.e., on the volume of the latter). The density required then is about 1 order of magnitude higher than densities hitherto contemplated in laser fusion. On the other hand, a high temperature is not required. Further calculations and experiments are clearly needed to decide which approach (high temperatures or high densities) is more promising.[10]

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[10]It is in fact quite possible that the best solution will involve some combination of both effects, because high temperatures too make an electron plasma more transparent: For slow ions, the energy loss in a nondegenerate plasma was given by S. Gasiorowicz, M. Neumann, and R. J. Riddell, Jr. [Phys. Rev. 101, (1956)] as

$$\frac{dE}{dx} = -\frac{4}{3}\left(\frac{2\pi m}{(kT)^3}\right)^{1/2} Z^2 e^4 n_e v_i \ln\left(\frac{3(kT)^{3/2}}{2Z^2 e^3 (4\pi n_e)^{1/2}}\right).$$

It appears that a fusion chain reaction may proceed in a nondegenerate DT plasma at temperatures of a few keV, at densities like those currently contemplated in laser fusion. However, larger DT pellets would be necessary to have an appreciable fraction of the 14.1-MeV neutron energy deposited in the plasma.

Infrared Conductivity of Tetrathiofulvalene Tetracyanoquinodimethane (TTF - TCNQ) Films*

D. B. Tanner, C. S. Jacobsen,† A. F. Garito, and A. J. Heeger
Department of Physics and Laboratory for Research on the Structure of Matter, University of Pennsylvania, Philadelphia, Pennsylvania 19174
(Received 4 March 1974)

Infrared studies of thin films of TTF-TCNQ are analyzed to obtain the frequency-dependent conductivity. In the metallic state the results indicate an energy gap (0.14 eV) with a collective mode at zero frequency. Below 58 K the conductivity peak moves away from zero frequency into the far infrared.

In earlier studies, it was proposed that above 58 K TTF-TCNQ is a one-dimensional (1D) metal exhibiting strong electron correlations such that the conductivity greatly exceeds the limitations of single-particle scattering.[1-4] This Letter describes infrared measurements on thin films of TTF-TCNQ which span the frequency range between the previous experiments.