The multicomponent Bose gas, which has been discussed recently by a number of authors,[16] differs in certain respects from the helium case, and will be considered in a separate paper.

---

*Junior Fellow, Society of Fellows, Harvard University. Supported in part by the National Science Foundation under Grant No. GH-32774.

[1] K. G. Wilson and J. Kogut, to be published.

[2] B. I. Halperin, P. C. Hohenberg, and S. Ma, Phys. Rev. B (to be published), and Phys. Rev. Lett. 29, 1548 (1972).

[3] K. Kawasaki, Phys. Rev. 148, 375 (1966); L. P. Kadanoff and J. Swift, Phys. Rev. 165, 310 (1968).

[4] K. Kawasaki, Ann. Phys. (New York) 61, 1 (1970), and references therein.

[5] L. P. Kadanoff and J. Swift, Phys. Rev. 166, 89 (1968), and Ann. Phys. (New York) 50, 312 (1968); J. Swift, Phys. Rev. 173, 257 (1968).

[6] Details will be given elsewhere.

[7] P. C. Martin, E. D. Siggia, and H. A. Rose, Phys. Rev. A 8, 423 (1973).

[8] J. V. Sengers, in *Transport Phenomena,* AIP Conference Proceedings No. 11, edited by J. Kestin (American Institute of Physics, New York, 1973), p. 229.

[9] See Eqs. (1) and (10) of K. Kawasaki and S. M. Lo, Phys. Rev. Lett. 29, 48 (1972).

[10] See B. I. Halperin and P. C. Hohenberg, Phys. Rev. 188, 898 (1969).

[11] B. I. Halperin and P. C. Hohenberg, Phys. Rev. 177, 952 (1969).

[12] P. C. Hohenberg, M. DeLeener, and P. Résibois, Physica (Utrecht) 65, 505 (1973).

[13] R. Ferrell *et al.*, Ann. Phys. (New York) 47, 565 (1968).

[14] A. M. Polyakov, Zh. Eksp. Teor. Fiz. 57, 2144 (1969) [Sov. Phys. JETP 30, 1164 (1970)]; Y. Yamashita and T. Tsuneto, to be published.

[15] M. Suzuki and G. Igarashi, to be published.

[16] R. Abe and S. Hikami, to be published; I. Kondor and P. Szépfalusy, to be published; M. Suzuki and F. Tanaka, to be published.

Observation of the Critical Velocity Peak in Superfluid Films*

K. Telschow and I. Rudnick
Department of Physics, University of California, Los Angeles, California 90024

and

T. G. Wang
Jet Propulsion Laboratory, Pasadena, California 91103

(Received 20 March 1974)

Critical flow velocities measured by Doppler-shifted third sound in unsaturated helium II films are shown to exhibit a well-defined maximum as a function of film thickness and at this maximum to be considerably higher than those usually found in film flow. In addition the critical velocity is strongly temperature dependent below the maximum and relatively temperature independent above.

It is a well-known observation that the critical flow velocity in superfluid helium increases as the channel width or film thickness, d, decreases.[1,2] Many relations such as $V_{sc}=d^{-1/4}$ cgs (the Leiden critical velocity)[2] and $V_{sc}=\hbar/md$ (the Feynman critical velocity)[3] have been suggested in the literature. On the other hand, when d becomes so small that it approaches the onset thickness for superflow, V_{sc} must vanish.[4] These observations imply that at some intermediate thickness V_{sc} has a maximum value, a result which, while recognized, seems never to have been explicitly stated in the literature and certainly never directly observed. Using Doppler-shifted third sound in unsaturated films we present here the first direct observations of this maximum and find it can be considerably higher than values usually associated with film flow.

The experimental apparatus is shown schematically in Fig. 1 and described briefly below.[5] An unsaturated film is condensed onto the tubular glass substrate which has outside diameters of 0.395 and 1.625 in. at the top and bottom sections, respectively. The surface of the entire glass substrate was flamed smooth and a nonsealing soap fillet provides a smooth transition from the glass surface to the copper housing. A heater at the top drives the film up from the larger bottom section. Evaporation from the heater, recondensation on the copper walls, and flow back to the bottom section complete the cycle. A supply of Al_2O_3 powder (500-Å grain size) acts as a film

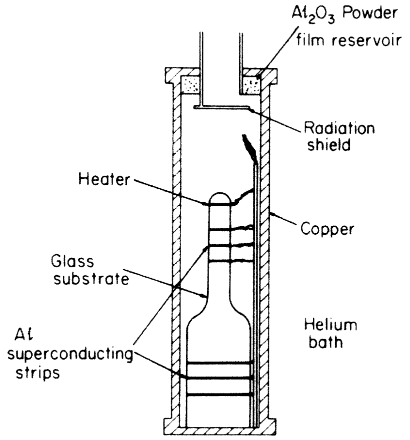

FIG. 1. Schematic diagram of the experimental chamber. The glass substrate is about 6 in. long and the figure is drawn to scale. The He II film is drawn up the substrate by the heater. Its velocity is obtained by measuring the Doppler shift of third sound using the aluminum superconducting strips.

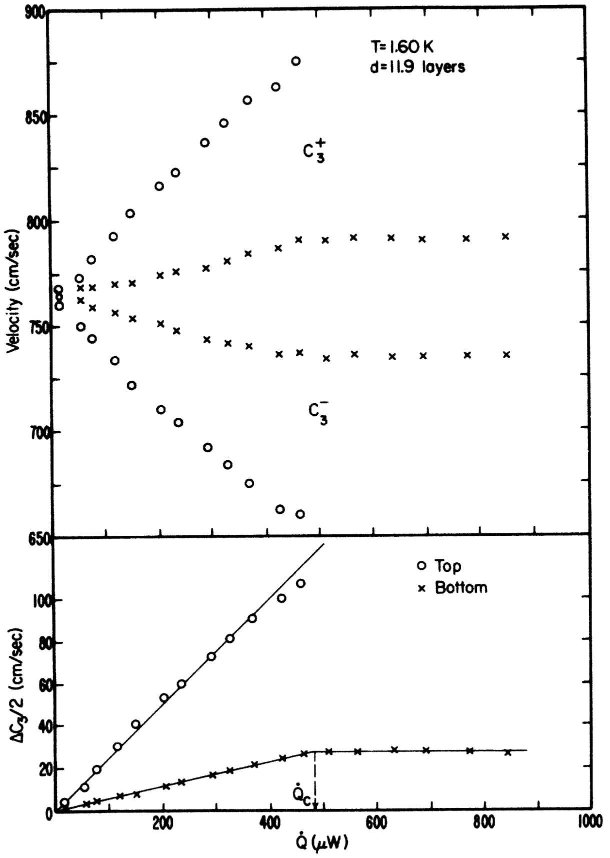

FIG. 2. Typical measured Doppler-shifted velocities upstream (C_3^-) and downstream (C_3^+) on both the top (circles) and bottom (crosses) sections. The lower portion shows the net Doppler shift $\Delta C_3/2 = (C_3^+ - C_3^-)/2$ and the dashed line gives the critical heat current, $\dot{Q}_c$.

reservoir. Thin aluminum superconducting strips deposited around the glass cylinders and placed 1 cm apart act as transmitters and receivers of the third sound pulses. With a small bias current and an external magnetic field the transition temperature of these strips can be adjusted to occur at the desired operating temperature. With the control of these parameters only a small amount of heat (0.5–20 μW depending on film thickness) is injected into the film during the detection process.

The film flow velocity is directly determined by measuring the upstream and downstream velocities, C_3^- and C_3^+, respectively, of a third-sound pulse. Their difference, twice the Doppler shift, is given by

$$2(\langle\rho_s\rangle/\rho)V_s = C_3^+ - C_3^-, \tag{1}$$

where V_s is the flow velocity of the superfluid (the normal component is locked) and $\langle\rho_s\rangle$ the average superfluid density in the film. In other experiments $\langle\rho_s\rangle$ is found to be given by[4,6,7]

$$\langle\rho_s\rangle d = \rho_{sb}(d - D), \tag{2}$$

where ρ_{sb} is the bulk superfluid density and D is a temperature-dependent nonsuperfluid thickness. We have independently verified Eq. (2) and find values of D in substantial agreement with those given in Ref. 7. The film thickness is obtained from the known saturated vapor pressure P_0 at the temperature of the experiment T and the measured vapor pressure P in equilibrium with the adsorbed film through

$$\alpha/d^3 = (kT/m)\ln(P_0/P), \tag{3}$$

where α is the Van der Waals constant for the glass substrate,[8] m is the mass of a helium atom, and k is Boltzmann's constant.

Figure 2 shows typical results for C_3^+, C_3^-, and their difference [which gives V_s through Eqs. (1) and (2)] for both the large bottom and the small top sections. The flow velocities on these two sections should be in the inverse ratio of the diameters which they are to within a discrepancy of a few percent which can be traced to the effect of partial recondensation of vapor between the two measuring regions. When the critical velocity is reached on the top section no additional increase in heating rate can cause the Doppler-shifted velocities on the bottom section to increase as is evident from Fig. 2. This gives an unambiguous indication of when the critical velocity is reached on the top section and this critical velocity itself is obtained from the values of $C_3^+ - C_3^-$ for either the top or bottom section at this point. An interesting phenomenon occurs on the top section as the critical heating current, $\dot{Q}_c$, is approached. The upstream signal decreases in amplitude and disappears when $\dot{Q}_c$ is reached; the downstream signal increases in amplitude and never disappears. This happens because superfluid flows toward the third-sound source which is just a heater. The propagating third-sound pulse is thus a dimple in the surface of the film. Although the dimple propagates away from the source the helium particle velocity is directed toward the source. In upstream propagation the convective flow velocity and the particle velocity add and in downstream propagation they subtract. If their sum cannot exceed the critical velocity then one expects to see no upstream signal when the flow velocity itself has this value. Therefore the vanishing of this upstream signal is clear evidence that the critical velocity is being reached on the top section.

For subcritical flows the heater converts the superfluid to vapor according to $\dot{Q} = \langle\rho_s\rangle V_s d 2\pi r T S_v$, where $2\pi r$ is the perimeter of the top section and S_v is the entropy of the vapor which is assumed to be an ideal gas. This expression gives an additional means of determining V_s and was used to substantiate Eq. (1) for the Doppler shift.[9]

The measured critical velocities at $T = 1.50$, 1.60, and 1.80°K are shown in Fig. 3. Also shown are the onset thicknesses d_0 for the temperatures as measured in this experiment. It is seen that $V_{sc} \to 0$ when $d \to d_0$ as originally discovered in persistent-current experiments.[4] V_{sc} is seen to be temperature dependent below the maximum and to approach a temperature-independent behavior for the thicker films. The experimental observation that $\dot{Q}_c \propto \ln(d/d_0)$ leads us to the following empirical equation for our data:

$$V_{sc} = V_{sc_0}(T)\frac{\ln(d/d_0)}{(d-D)/d_0} \tag{4}$$

with the values of $V_{sc_0} = 390$, 345, and 200 cm/sec and with $D = 2.4$, 2.5, and 3.2 layers, $d_0 = 3.6$, 4.0, and 5.7 layers at $T = 1.50$, 1.60, and 1.80°K, respectively. These curves are also plotted in Fig. 3. We note that the critical velocity is a maximum at thicknesses where third-sound attenuation is a minimum.[10]

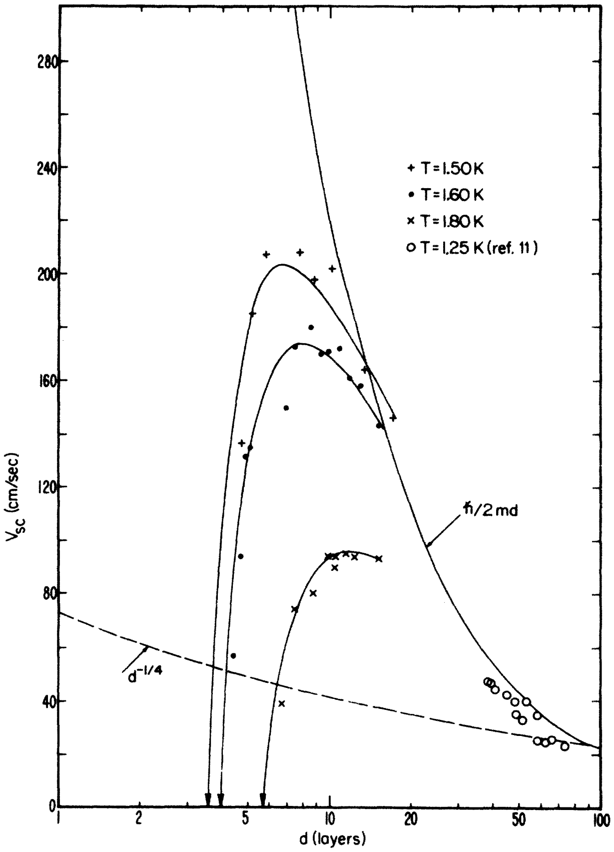

FIG. 3. Our measured critical velocities at three different temperatures. The lines drawn through the experimental points are from Eq. (4). The circles and the curve $\hbar/2md$ are from Pickar and Atkins (Ref. 11). The dashed curve is the Leiden condition (Ref. 2). The arrows at the abscissa give the superfluid onset thickness.

The open circles of Fig. 3 are the measured critical velocities on saturated films by Pickar and Atkins.[11] The curve $\hbar/2md$ is one suggested by them and is based on a model in which the critical velocity is associated with the appearance of a vortex line and its image. The dashed line in Fig. 3 is the empirical expression $d^{-1/4}$ which in no way describes our results.

Figure 3 shows, as discussed in the introductory paragraph, that for the critical velocity on the thick-film side of the peak where the results are relatively independent of temperature, a Feynman condition, $V_{sc} \propto d^{-1}$, is obeyed. A major factor in arriving at this description is the way in which the results of Pickar and Atkins for saturated films compare with our results on unsaturated films.[11] On the thin-film side of the peak where the data show a strong temperature dependence, thermal fluctuations have been used to explain critical velocities.[4,12-14] It is also clear that the peak value is increasing with decreasing temperature and that it would be of considerable interest to obtain its value below 1°K which we are preparing to do. If the $\hbar/2md$ dependence is correct, then one can estimate what the upper bound on the critical velocity in films should be. Superflow has been observed in a film 2.1 layers thick below 0.8°K.[7] If the maximum in V_{sc} occurs at twice this thickness then this upper bound is 520 cm/sec.

We would like to acknowledge helpful discussions with S. J. Putterman, R. J. Donnelly, and J. P. Hulin.

---

[*] Work supported in part by the Office of Naval Research, Contract No. N00014-69-A-0200-4014, and the National Aeronautics and Space Administration, Contract No. NAS 7-100.

[1] K. R. Atkins, *Liquid Helium* (Cambridge Univ. Press, Cambridge, England, 1959), p. 198.

[2] W. M. Van Alphen, G. J. Van Haasteren, R. De Bruyn Ouboter, and K. W. Tacomis, Phys. Lett. 20, 474 (1966).

[3] R. P. Feynman, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland, Amsterdam, 1955), Vol. 1.

[4] J. S. Langer and J. D. Reppy, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland, Amsterdam, 1955), Vol. 6.

[5] A more complete description will be published elsewhere: K. Telschow, I. Rudnick, and T. Wang, to be published.

[6] M. Chester and L. C. Yang, Phys. Rev. Lett. 31, 1377 (1973).

[7] J. H. Scholtz, E. O. McLean, and I. Rudnick, Phys. Rev. Lett. 32, 147 (1974).

[8] The value used [27 (layers)^3 K] is that reported in Ref. 7 and agrees with E. S. Sabisky and C. H. Anderson, Phys. Rev. Lett. 30, 1122 (1973). The latter also find that for films thicker than about 10 atomic layers, α is a function of d. We have made such corrections using their $\alpha(d)$ for MgO which is expected to be similar to glass. E. S. Sabisky and C. H. Anderson, Phys. Rev. A 7, 790 (1973).

[9] This agreement is direct experimental evidence against the Doppler-shift expression $\pm\frac{1}{2}(\langle\rho_s\rangle/\rho)V_s$ advanced by D. Goodstein and P. G. Saffman, Proc. Roy. Soc., Ser. A 325, 447 (1971). This is discussed more completely in Ref. 5.

[10] T. Wang and I. Rudnick, J. Low Temp. Phys. 9, 425 (1972). The thicknesses plotted there must be multiplied by 0.677 to correct them for the proper value of Van der Waals constant; see Refs. 7 and 8.

[11] K. A. Pickar and K. R. Atkins, Phys. Rev. 178, 389 (1969). In their experiment third-sound pulses were seen traveling parallel and antiparallel to the film flow at all times. Thus their film may not have been flowing at critical velocity in the measurement region. Consequently their reported values for V_{sc} may be low. Also, these authors omitted the factor ρ_s/ρ in the Doppler-shift expression. If the correct expression [Eq. (1)] is used then the data they present are consistent with V_{sc} being independent of temperature for their saturated films.

[12] S. V. Iordanskii, Zh. Eksp. Teor. Fiz. 48, 708 (1965) [Sov. Phys. JETP 21, 467 (1965)].

[13] J. S. Langer and M. E. Fisher, Phys. Rev. Lett. 19, 560 (1967).

[14] R. J. Donnelly and P. H. Roberts, Phil. Trans. Roy. Soc. London, Ser. A 271, 41 (1971).

Attenuation of 9.4-GHz Acoustical Waves in Helium Films

Ch. Frénois, J. Joffrin, P. Legros, and A. Levelut
Laboratoire d'Ultrasons,[*] *Université Paris VI, 75230 Paris-Cédex 05, France*
(Received 21 February 1974)

Using a standard acoustical technique, we present new data concerning the attenuation of 9.4-GHz phonons in thin helium films. The mechanical resonance of the film is observed in the temperature range 1.4 to 2.1 K. A comparison with the bulk case is made.

Interest has grown recently concerning the lifetime and velocity of excitations in helium at different temperatures. The propagation of heat pulses in bulk[1] or in films[2] has provided new data for thermal phonons and has proved to be well suited for observing the change in their propagation characteristics. In addition, Brillouin scattering experiments[3] and the propagation of acoustical waves[4] have given some insight on the processes which govern the lifetime of phonons of low frequencies ($\nu < 1$ GHz).

In this Letter we present a new experiment which fills the gap between these two extreme frequency ranges and which also proves to be well suited for measuring several properties of thin helium films. Using standard acoustical