New Measurement of the Σ^- Magnetic Moment*†

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We report a new and more precise measurement of the magnetic moment of the Σ^- hyperon. Our result of $(-1.48\pm 0.37)\mu_N$ is in agreement with the sign given by simple SU(3) theory, but differs somewhat from the value of $-0.9\mu_N$ predicted by the theory with no mass breaking.

In an earlier publication[1] we reported the first measurement of the magnetic dipole moment of the Σ^- hyperon. Our earlier determination placed limits on the magnitude of the Σ^- moment but was insensitive to its sign. We have now accumulated considerably more data and have made a more comprehensive analysis of all of our data. We present here the first indication of the sign of the Σ^- magnetic moment as well as a much more precise value than that previously reported.

As in our earlier experiment, K^- mesons produced by the slow extracted proton beam of the Brookhaven National Laboratory alternating-gradient sychrotron were brought to rest in targets of Pb, separated ^{208}Pb, and Pt. The Σ^- hyperons were produced in these targets by nuclear K^- capture. The resulting K^-, Σ^-, and π^- x rays were observed using two 50-cm^3 true coaxial Ge(Li) detectors which exhibited instrumental resolutions of 1.10 and 0.98 keV full width at half-maximum at 292 keV, the energy of the $K^-(n=9\to n=8)$ atomic transition in Pb.

Since the Σ^- has spin $\frac{1}{2}$, its atomic states are split by the resulting fine structure and a measurement of this splitting yields a value for the magnetic moment. Theoretical predictions for the Σ^- magnetic moment have been made from SU(3) symmetry considerations.[2-4] From these, relations are predicted between the magnetic moments of members of the baryon octet. Specifically, with no mass breaking, the magnetic moments are

$$\mu(\Xi^-)=\mu(\Sigma^-)=-[\mu(p)+\mu(n)]\approx -0.9\mu_N, \quad (1)$$

where $\mu_N=e\hbar/2m_pc$, one nuclear magneton.

The Σ^- magnetic moment can be written as a sum of two components,[5]

$$\mu=(g_0+g_1)\mu_\Sigma$$

where μ_Σ is one sigma magneton, $e\hbar/2m_\Sigma c$, g_0 is the Dirac factor (-1 in this case), and g_1 is the anomalous term. The fine-structure splitting of an atomic level with principal quantum number n and orbital quantum number l is given by[5]

$$\Delta E_{n,l}=(g_0+2g_1)\frac{(\alpha Z)^4}{2n^3}\frac{m}{l(l+1)}, \quad (2)$$

where Z is the nuclear charge, m the reduced mass of the Σ^--nucleus system, and α is the fine-structure constant. For the $\Sigma^-(n=12)$ state one can calculate the relative intensities of the three $E1$ transitions to the $n=11$ state. Circular transitions are defined as those between orbits with $l=n-1$ and the first noncircular transitions as those between orbits with $l=n-2$. If we label the spin-up to spin-up ($\Delta j=\Delta l=\Delta n=1$) transition a, spin-down to spin-up ($\Delta j=0$, $\Delta l=\Delta n=1$) transition b, spin-down to spin-down ($\Delta j=\Delta l=\Delta n=1$) transition c, the following intensity ratios for $n=12$ to $n=11$ transitions are obtained assuming a statistical population of the fine-structure states[5]:

$$\begin{aligned}&\text{circular,} \quad a{:}b{:}c=252{:}1{:}230;\\ &\text{noncircular,} \quad a{:}b{:}c=209{:}1{:}189.\end{aligned} \tag{3}$$

In our analysis, the contribution from b was ignored. The energy difference between the two prominent transitions a and c is then equal to the difference in splitting of the two levels as given by Eq. (2). Since the intensity ratio and fine-structure splitting are different for circular and noncircular transitions the contribution of the latter must be included in the analysis.

The contribution from the first noncircular transition was estimated from the data in the following manner. Figure 1 shows schematically the circular transition (labeled γ), the most intense noncircular one (labeled α), and a competing $\Delta n=2$ transition labeled β. Since the energies of the transitions α and γ are not well resolved experimentally, one needs to determine α/γ. From the data we can in principle determine the intensity ratio $\beta/(\alpha+\gamma)$ since the transition β is well resolved in energy. Because the $\Sigma(11\to10)$ was observed in Pb, the $\Sigma(12\to10)$ should not be so broadened by the strong-interaction effect as to be unobservable. We found an upper limit on $\beta/(\alpha+\gamma)$ of 0.07. The intensity ratio α/β calculated for electric dipole radiation is 2.3 for the transition depicted in Fig. 1. We

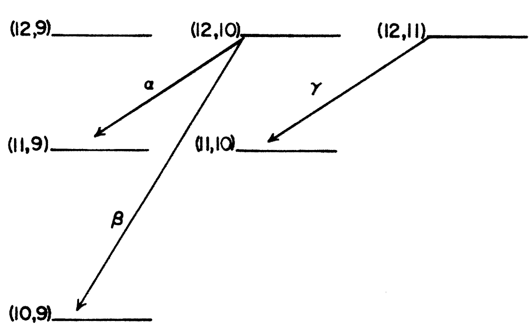

FIG. 1. Schematic diagram of circular and noncircular transitions considered in the analysis of the Σ x-ray data. Levels are labeled by (n, l) values. The fine-structure splittings are not shown.

obtain for the transitions in Pb

$$\frac{\alpha}{\alpha+\gamma}=\frac{\beta}{\alpha+\gamma}\,\frac{\alpha}{\beta}\leqslant 0.16.$$

The data were analyzed with a least-squares fitting program in the following manner. The instrumental response of the system at the energy corresponding to the $\Sigma(12\to11)$ transition was obtained from the adjacent $K^-(9\to8)$ transition and from radioactive sources. The widths of the Σ^- components were held fixed for a given fit and the centers and separations of the noncircular transitions were related to those of the circular transitions by the Dirac equation. The separation between the components was varied and the corresponding χ^2 determined as a function of this separation and thus of the Σ^- magnetic moment. As a check, similar χ^2 analyses were performed on single lines from radioactive sources and on computer-simulated Gaussians. In these tests, the χ^2 function was centered about zero, but a finite interval existed over which χ^2 changed by one unit. This was a reflection of statistical uncertainties in the width of the single Gaussian.

The error in the magnitic moment was partly determined by the range of values over which χ^2 changed by one unit. The final error reflects this uncertainty as well as those associated with the instrumental width and the contribution of noncircular transitions. Using our estimate of the intensity ratio $\alpha/(\alpha+\gamma)$, we varied the relative yield of the first noncircular transition between 0.05 and 0.15 for Pb and 0.10 and 0.20 for Pt. Each set of data was analyzed and the individual curves of χ^2 versus Σ^- magnetic moment were added together to yield the result shown in Fig. 2. The region of μ shown between the two vertical lines in this figure represents uncertainties due to the instrumental width but not those due to the uncertainties in the contributions from noncircular transitions. Because of the uncertainties in the contributions of noncircular transitions we increased the error by 20%, based on the analyses of the Σ^- x-ray data in which different fractions of noncircular contributions were used.

From Fig. 2 we note an indication of a preference for a negative sign for the Σ^- magnetic moment, the χ^2 difference between the two minima being 0.95 for one χ^2 curve and 0.82 for the other. The value obtained for $\mu(\Sigma^-)$ is

$$\mu(\Sigma^-)=(-1.89\pm0.47)\mu_\Sigma$$

or, equivalently,

$$\mu(\Sigma^-)=(-1.48\pm0.37)\mu_N.$$

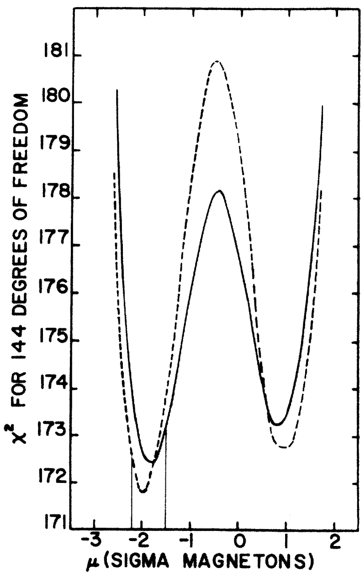

FIG. 2. χ^2 response from fitting the $\Sigma^-(12 \rightarrow 11)$ transitions. The curves represent a compilation of data from natural Pb (5.4×10^8 K^- stops), ^{208}Pb (8.3×10^7 K^- stops), and Pt (3.6×10^8 K^- stops). The solid curve was calculated for instrumental resolution that was 1 standard deviation greater than the most probable; the dotted curve for 1 standard deviation less. Included in our quoted error (see text) but not shown above is the uncertainty introduced by contributions from noncircular transitions.

Our value for $\mu(\Sigma^-)$ differs by 1.6 standard deviations from the SU(3) value given in Eq. (1). We note, however, that our result is in agreement with the recent measurement of $\mu(\Xi^-)$ by Cool *et al.*,[6] who obtained

$$\mu(\Xi^-) = (-2.2 \pm 0.8)\mu_N.$$

Since the mass of the Σ^- is 27% greater than that of the proton, the presence of an appreciable mass correction term in the magnetic moment would not be surprising. However, as mentioned by Cool *et al.*,[6] there is at present no fully acceptable method for calculating such a correction. Certainly at the present level of experimental precision no definitive disagreement with the value predicted by Eq. (1) is implied by our result.

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