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Turbulent Heating and Quenching of the Ion Sound Instability*

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Turbulent heating and stabilization of the ion sound instability is investigated by two-dimensional computer simulation. Quasilinear rather than nonlinear effects determine the evolution of the instability. The instability is quenched by flattening of the electron distribution and the formation of a high-energy ion tail.

Numerous stabilization mechanisms have been proposed for the current-driven ion sound instability.[1-5] We have done extensive simulation studies in order to provide a test for these basic predictions.[6] The two-dimensional code has been described previously.[7] Specifically for the purpose of testing nonlinear theories of stabilization, we have made runs in which the ratio between drift velocity and electron thermal velocity was kept constant, in addition to runs with constant current. In the same vein we discuss the case of a current perpendicular to a weak magnetic field ($\Omega_e/\omega_e = 0.04$). The magnetic field (perpendicular to the plane of computation) has a very small effect on wave dispersion, but keeps the electron distribution isotropic. (In the case of a current along a magnetic field, further complicated dynamical effects are added by the formation of an electron runaway tail.)

We find that for a wide range of initial parameters the growth phase of the instability is followed by the decay of the wave energy W, the return of the fluctuation level W/nT_e to the thermal level,[8] and termination of heating in typically $(100-200)\omega_i^{-1}$. Clearly, in the case of constant current, the growth phase of the instability must terminate at the latest when the phase velocity reaches the drift velocity, $u \approx c_s = (T_e/M)^{1/2}$. The runs with constant u/c_s, however, show quenching in much the same way; see Fig. 1. It is seen that in this case the plasma enters a regime in which the macroscopic parameters remain constant.

Nonlinear theories of stabilization generally determine a quasisteady fluctuation level W/nT_e as a function of m/M, u/v_e, and T_e/T_i from the condition that the nonlinear damping just balances the linear growth rate, $\gamma = \gamma^L + \gamma^{NL} = 0$. Actually, for $(\partial/\partial t)\ln(W/nT_e) = \dot{W}/W - \dot{T}_e/T_e = 0$, $\dot{W}/W \equiv 2\gamma$ must be balanced by the electron heat-

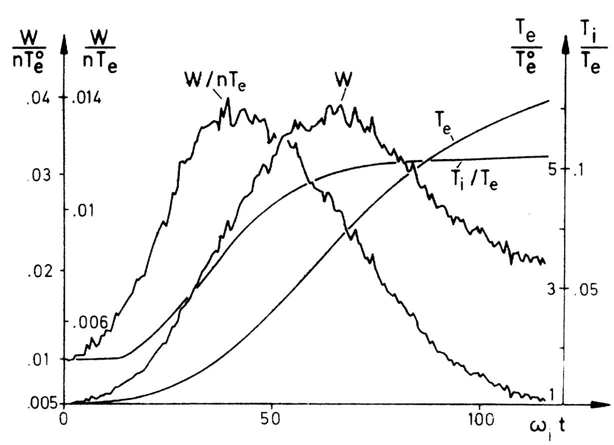

FIG. 1. Wave energy W, fluctuation level W/nT_e, T_e, and T_i/T_e for a typical run. $M/m = 100$, $(T_i/T_e)^0 = 0.02$, $u/v_e = 0.75$.

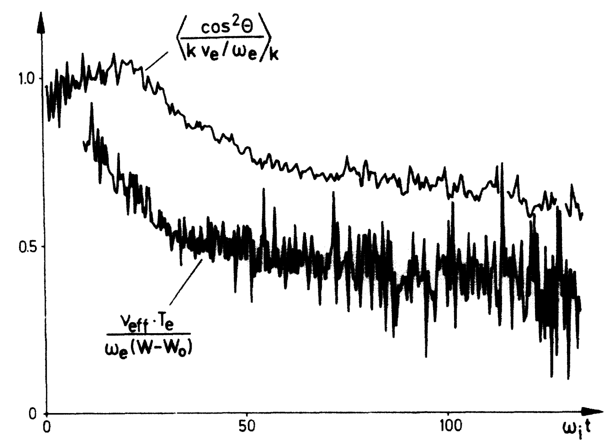

FIG. 2. Ratio between effective collision frequency and nonthermal fluctuation level; spectral form factor. $M/m=100$, $(T_i/T_e)^0=0.02$, $u=v_e^0$.

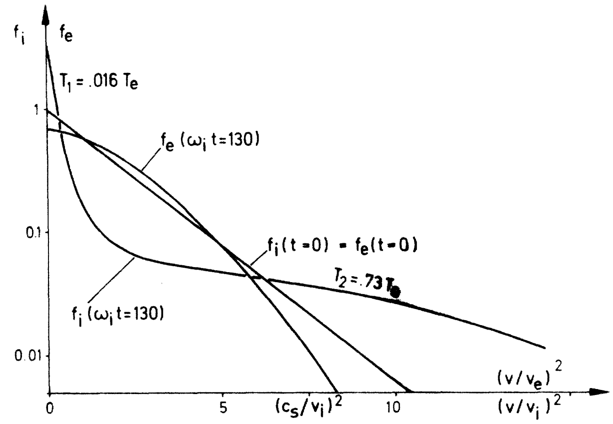

FIG. 3. Normalized electron and ion energy distributions. $M/m=100$, $u=v_e^0$, $(T_i/T_e)^0=0.02$.

ing rate. More generally, for constant $\gamma^L>0$, and an electron heating rate and any nonlinear damping increasing with W/nT_e, saturation at constant W/nT_e would occur, with W and T_e still increasing. Figure 1 shows that such a macroscopic description is not adequate, i.e., changes in the particle and perhaps spectral distribution occur which lead to a reduction of γ^L and, finally, quenching of the instability.

From conservation of wave momentum we find the effective collision frequency[9]

$$\vec{\nu}=(1/V)\sum_{\vec{k}} 2\gamma_{\vec{k}}^e N_{\vec{k}}(\omega_{\vec{k}}/nmu^2)\vec{k}\vec{u}/\omega_{\vec{k}}, \tag{1}$$

where $\gamma_{\vec{k}}^e$ is the total (linear and nonlinear) rate of wave dissipation due to electrons and $N_{\vec{k}}$ is the number of waves $\vec{k}$ in volume V. It follows that we can exclude proposed theories of stabilization by electron trapping or nonlinear Landau damping due to electrons, since for $\gamma_{\vec{k}}^e=0$, $\nu\neq 0$ they do not conserve wave momentum. The collision frequency, Fig. 2, agrees very well with the quasilinear prediction $\nu=\vec{u}\cdot\vec{\nu}\cdot\vec{u}/u^2=eE_0/mu$,

$$\frac{\nu}{\omega_e}=\alpha_2(2\pi)^{1/2}\left\langle\frac{\vec{k}\cdot\vec{u}-\omega_{\vec{k}}}{ku}\,\frac{\vec{k}\cdot\vec{u}}{ku}\,\frac{\omega_e}{kv_e}\right\rangle\frac{W}{nT_e}\equiv\rho\,\frac{W}{nT_e}, \tag{2}$$

where the average is over the spectrum and α_2 takes account of changes in the electron distribution from a Maxwellian,

$$\partial F/\partial w=-\alpha_2(2\pi v_e^2)^{-1/2}w/v_e^2,$$

where

$$F(w)\equiv\int d^2v\,\delta((\vec{k}\cdot\vec{v}/k)-w)f_e(\vec{v}) \tag{3}$$

and $T_e\equiv mv_e^2\equiv m\langle w^2\rangle$. The electron distribution flattens as a result of diffusion, which leads to a reduction of the growth rate and an increase in the effective temperature for wave dispersion, as compared to a Maxwellian of the same energy, $T_e=mv_e^2$,

$$T_e^{\text{eff}}\equiv-\mathrm{P}\int dw\,(mw)^{-1}\partial F/\partial w\equiv\langle 1/mw^2\rangle\equiv\alpha_1^{-2}T_e. \tag{4}$$

We observe[10] that $f_e(v)$ reaches a self-similar form $f_e(v)=C\exp[-(v/v_0)^x]$ with $x=3.6-4$, $\alpha_2\approx 0.33$, $\alpha_1^2\approx 1.6$ (Fig. 3). In the three-dimensional case the effects of changes in $f_e(v)$ are weaker but still appreciable. For $x=5$ (quasilinear prediction[1]) we have $\alpha_2=0.445$, $\alpha_1^2=1.45$. For $u/c_s\gg 1$ most of the energy delivered to the plasma goes into electron heating, $\dot{T}_e\approx\nu mu^2$. The energy extracted from the electrons,[9]

$$-\frac{\delta E_e}{\delta t}=\frac{1}{V}\sum_{\vec{k}}2\gamma_{\vec{k}}^e N_{\vec{k}}\omega_{\vec{k}}\approx\left\langle\frac{\omega_{\vec{k}}}{\vec{k}\cdot\vec{u}-\omega_{\vec{k}}}\right\rangle n\dot{T}_e, \tag{5}$$

goes into wave growth and ion heating. The observed electron and ion heating rates agree very well with these predictions. In particular, varying the initial temperature ratio $(T_i/T_e)^0$ between 0.02 and 0.5 leaves the final temperature ratio nearly unaffected.

For quenching of the instability we must turn to the interaction of the waves with the ions. We observe in all cases that the ions develop a two-temperature distribution, Fig. 3. The bulk of the distribution is heated relatively little and accelerated by the applied electric field at a rate which

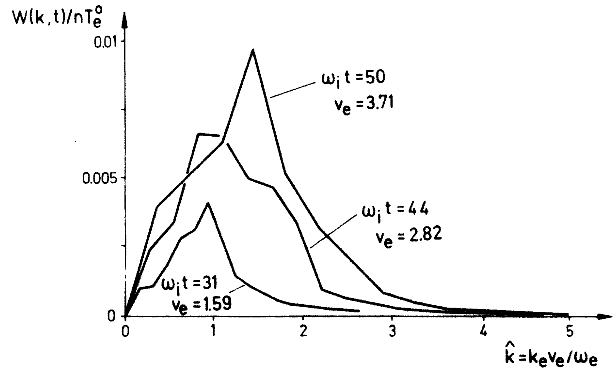

FIG. 4. Evolution of the wave spectrum $W=\int d^2\hat{k} \times W(\hat{k},t)$, $\hat{k}=kv_e/\omega_e$. $M/m=1600$, $u/v_e=0.75$ $(T_i/T_e)^0 = 0.02$.

is close to free acceleration. At small ion temperatures a substantial fraction of the space-averaged bulk temperature is connected with the (reversible) sloshing motion of the ions, as can be determined from the observed difference between space-averaged and local ion temperature, $n(T_i - T_i^L) = \langle(\omega_i/\omega)^2\rangle W \approx 2W$. The remaining irreversible bulk heating rate is small but about 2 orders of magnitude larger than expected from nonlinear Landau damping. We must note that the wave spectrum contains a substantial fraction of waves with large kv_e/ω_e and small phase velocity $v_{\rm ph} < c_s$ which can interact directly with the bulk of the distribution. We observe that tail formation sets in only after the temperature ratio increases to a value such that a nonnegligible but small fraction of particles in the space-averaged ion distribution fall into the linear phase-velocity range. Then the tail starts extending to larger velocities, up to about $2c_s$, and the number of particles in the tail increases.

The process of tail formation is coupled with the evolution of the spectrum, Fig. 4. In the course of time it moves to larger and larger values of kv_e/ω_e. This appears to be connected simply with the increase in v_e, whereas the cutoff at large and small kv_e/ω_e is due to damping on ion bulk and tail, respectively. Higher-order processes which, e.g., would lead to a Kadomtsev spectrum[1] evidently play no role. The time scale for tail formation, $\tau_D \approx (10-20)\omega_i^{-1}$, agrees very well with the quasilinear estimate as do the bulk and tail heating rates. The early tail-formation process is not inconsistent with a trapping picture either. The popular trapping estimates of the fluctuation level can always be made to agree since W is proportional to the fourth power of the trapping width. But from a trapping picture we would expect too that the tail would be formed in roughly the same time, after sloshing brings particles up to the phase velocity, and then extend to about $2c_s$.

Clearly, neither the assumptions for trapping nor for linear Landau damping, which describes the onset of trapping, are fully satisfied, as we do have a wide two-dimensional spectrum; but, at least at the time of tail formation, the energy in the wave motion is a substantial fraction of the unperturbed ion energy. Once the tail is formed, however, or if we start with less extreme temperature ratios, the assumptions for quasilinear interaction with the ions should be satisfied. We are confirmed in this by measuring the (irreversible) ion heating rate $\dot{T}_i^L$ which is found to be proportional to W and in agreement with linear Landau damping.

The quasilinear rates of dissipation have been estimated from the slopes of the electron and ion distributions. It is found that the linear growth rate reflects the behavior of the wave energy. Landau damping on the ion tail leads to quenching of the instability. By the same token we can attempt to explain the maximum fluctuation level as a dynamical maximum at which wave growth is overtaken by electron heating. Quasilinear

TABLE I. Dependence on the mass ratio. $(T_i/T_e)^0 = 0.02$. † indicates that the value is still increasing at the end of the computation time t^f.

u	M/m	$W^{\max}/nT_e^0$ (10^{-2})	$(W/nT_e)^{\max}$ (10^{-2})	$(n_2/n)^f$	$(T_i/T_e)^f$	$(T_e/T_e^0)^f$	$\omega_i t^f$
v_e^0	100	3.7	1.4	0.2	0.134	4.5	140
	400	6.6	1.25	0.13	0.093	9.0	137
	1600	9.0	1.04	0.08	0.046†	14†	114
$0.75v_e$	100	3.9	1.35	0.20	0.106	6.18	117
	1600	2.7†	1.0	0.05	0.022†	38.5†	64.5

theory gives reasonable estimates and no nonlinear limitation appears necessary. In particular, we can explain the observed scaling laws to which we now must turn.

We can estimate the tail parameters from conservation of wave energy and momentum. It follows that[2] $T_2 \approx T_e$, $n_2/n \propto (m/M)^{1/4}$, and that the resulting critical drift velocity u/v_e scales as $(m/M)^{1/4}$; see Table I. One striking feature of turbulent-heating experiments has always been the relative constancy of the fluctuation level, $W/nT = 10^{-3}-10^{-2}$, under widely different experimental conditions. It can be explained by the self-regulatory effects of quasilinear diffusion.

In conclusion, we have shown that the ion sound instability has no steady saturated nonlinear state but evolves into a damped state, and that quasilinear effects are sufficient to explain our simulation experiments. At the observed fluctuation levels other nonlinear effects such as nonlinear Landau damping or resonance broadening appear to be rather weak. Our conclusions seem to hold also for a wide range of experiments which develop similar fluctuation levels and effective collision frequencies and for which losses can be neglected during the rapid-heating phase.[9] In agreement with our simulation in experiments to termination of heating, even before the current reaches its maximum,[9] ion tail formation[9] and flattening of the electron distribution[11] are observed.

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Thermal Self-Focusing of Electromagnetic Waves in Plasmas*

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An intense electromagnetic wave propagating in a collisional plasma is found to be unstable to a thermal self-focusing instability by a self-consistent solution of the hydrodynamic, heat conductivity, and wave propagation equations. The results are applied to ionospheric modifications and proposed power transmission experiments, and to laser-plasma interactions.

Thermal self-focusing can occur when an intense electromagnetic wave propagates through a plasma, because the wave-induced heating leads to a temperature increase which in turn causes a hydrodynamic expansion. The concomitant increase in the index of refraction concentrates the radiation in the heated region, further increasing the heating. Although this phenomenon has been recognized for a number of years,[1,2] the nonlocal interaction between hydrodynamic motion, thermal conductivity, and wave propagation has prevented the theory[1] developed for self-fo-