Compatibility of Hydrodynamic Equations with the Generalized Multiperipheral Picture*

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A way of resolving the difference between the Fermi-Landau and multiperipheral pictures of particle production is suggested. The basic observation is that if the Fermi-Landau fluid is already one-dimensional in the formation stage, then the entropy will be greatly reduced compared to the standard treatment. This allows the possibility for logarithmic growth with energy of the multiplicity, in agreement with the multiperipheral picture.

A question of fundamental importance to the development of a theory of strong-interaction physics is why elementary particles are copiously produced at very high energies ($\gtrsim 10^3$ GeV). Although this question has been asked for more than thirty years,[1] a universally compelling answer has not yet been given. One answer was proposed almost twenty years ago by Fermi,[2] and subsequently generalized by Landau,[3] to the effect that a very high-energy hadron-hadron collision produced a highly excited fluid of "prematter" from which elementary particles were formed. Though specific ideas of the formation process differed, general agreement was reached that the multiplicity should grow like a power of s, the most probable value[4] being $\frac{1}{4}$:

$$n \propto s^{1/4}. \tag{1}$$

The attractive feature of the more sophisticated models leading to Eq. (1) is the possibility of obtaining the equations governing the behavior of the excited fluid from a polynomial field theory[4]; the precise nature of the field theory determines in a direct way the multiplicity growth. The connection to a field theory then relates the copious particle production to the fundamental properties of Lorentz invariance, unitarity, and crossing symmetry expected from any field theoretic formulation.

In the past several years, a more phenomenological approach has been taken towards understanding particle production. Motivated to a large extent by the ideas of the multiperipheral model of Fubini and co-workers,[5] a detailed picture of multiparticle scattering has been produced from high-energy data taken under controlled conditions.[6] This picture includes such important ideas as transverse-momentum damping, Regge factorization, and Feynman scaling. Taken together these ideas imply the existence of an isomorphism between the description of multiparticle production and the coordinate description of a classical many-body system.[7] This isomorphism, which is usually labeled as the Feynman fluid analogy, provides a generalization of the Amati-Bertocchi-Fubini-Stanghellini-Tonin multiperipheral picture. To a large extent this picture has been constructed inductively from the data, and it should be no surprise if its basic properties differ from the earlier field-theoretic picture.

Indeed it is the case that the generalized multiperipheral picture requires the multiplicity to increase logarithmically with energy,

$$\bar{n} \propto \ln s, \tag{2}$$

as opposed to the power behavior of Eq. (1). The logarithmic growth is a direct consequence of the assumption of transverse-momentum damping, and the requirement that a factorizable Regge singularity determines the leading energy behavior of cross sections. It is therefore clear that the difference between (1) and (2) represents a difference in principle between the field-theoretic and multiperipheral approaches. Moreover, because of the numerical similarity between $\ln s$ and $s^{1/4}$ over experimentally accessible energies, present data do not favor one behavior over the other.

The main intent of this note is to suggest a way of reconciling the above discrepancy. The suggestion is that the experimentally observed transverse-momentum cutoff is not properly taken into account in the hydrodynamic approach. In this approach there are three stages: the collision and formation of the highly excited fluid; the adiabatic expression of the fluid; the condensation of the fluid into particles. Regardless of the complications of the last two stages, the assumption is that the total entropy S remains constant. Further it is assumed that the multiplicity is proportional to the entropy. The origin of the power growth in multiplicity is therefore in the power growth of the entropy with energy in the very first stage, before the hydrodynamic equations are applied. For this reason the Fermi statistical model which assumes no expansion stage gives the same multiplicity as the hydrodynamic model. This fact will also allow us to investigate the effects on multiplicity of various assumptions about the formation stage without having to explicitly consider the way in which the fluid expands or condenses into particles. We retain the same physical assumptions during the last two phases as are contained in the hydrodynamic model.

The basic observation to be made is that *if at the formation stage, the fluid is already damped in transverse momentum,* then the entropy will be *greatly reduced* compared to the assumption of no damping. An example will make this idea clear. For the calculation of S and $\bar{n}$ we can consider Fermi-like independent-emission models,

$$\sigma_n(P)=\frac{V^n}{n!}\prod_{k=1}^{n}\int\frac{d^3p_k}{E_k}f(p_k)\delta^4\left(\sum p_k-P\right), \qquad (3)$$

where V can in principle depend upon $\sqrt{s}$, and $f(p_k)$ is an arbitrary function to be specified below. A standard way to treat such phase-space integrals (3) is to form the Laplace transform[9]

$$\sigma_n(\beta)=\int e^{-\beta\cdot P}\sigma_n(P)d^4P. \qquad (4)$$

The total cross section $Q=\sum_n\sigma_n(\beta)$ then has the form of a grand canonical partition function. The four-vector β^μ is a generalization of the inverse temperature. Calculations of integrals of the form (3) are treated in detail in the literature and will not be reproduced here.[10] For us the essential points to recall are the following: (i) In such models an entropy S can be defined analogous to the $\langle-\ln\rho\rangle$ in statistical mechanics, where ρ gives the density of states; (ii) for a large class of matrix elements including the class (3), $\bar{n}\propto S$; (iii) in the simple case that V is independent of s and particles are made independently $[f(p_k)=1]$, $\bar{n}\propto s^{1/3}$; (iv) where the additional information is added that particles are produced with a transverse momentum damping $f=f(p_T)$, then $\bar{n}\propto\ln s$. Thus the knowledge of transverse-momentum damping dramatically and necessarily lowers the rate at which the entropy (or multiplicity) increases with energy.

The conclusion we draw is that if transverse-momentum damping is present already at the formation stage in the Fermi-Landau picture of particle production, then the entropy, and hence particle multiplicity, will grow less fast than has previously been assumed. If this is the case, then the logarithmic growth found in the generalized multiperipheral picture is not necessarily incompatible. The inclusion of transverse-momentum damping at the formation state is also bound to cause a quantitative change in the way in which the fluid expands and condenses into particles. We conjecture that it might in fact be possible to arrange for the resulting spectra to be compatible with the multiperipheral picture.

In practice, the idea of transverse-momentum damping in the hydrodynamic model would be imposed on the prematter distribution function directly, not through the Fermi-like independent-emission model (3). In so doing one would note the following. Since entropy is conserved, the multiplicity growth with s depends only upon the velocity of sound c which is determined by the equation of state.[4] The exact value of c depends upon the detailed shape of the prematter distribution functions. If this distribution function is essentially one-dimensional, as is implied by having transverse-momentum damping, then $p=\epsilon$ and we have $\bar{n}\sim s^0$. This again illustrates that the particle multiplicity grows less fast than $s^{1/4}$. We believe the correct interpretation of s^0 is in fact $\ln s$, since in this one-dimensional model, what one should obtain is a rapidity plateau whose height is independent of energy; the kinematic bounds on the width of the plateau then supply the $\ln s$ growth.

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[3]L. Landau, Izv. Akad. Nauk SSSR, Ser. Fiz. 17, 51 (1953); reprints of the major articles in English translation are contained in *Collected Papers of L. D. Landau*, edited by D. Ter Haar (Gordon and Breach, New York, 1965).

[4]The exact power depends upon the equation of state. For $p=c^2\epsilon$, where p denotes pressure, ϵ the energy density, and c the velocity of sound, then $\bar{n}\propto s^{(\nu-1)/2\nu}$ where $c^2=1/(2\nu-1)$. Moreover in one dimension the hydrodynamics with this velocity of sound corresponds formally to an interaction Langrangian $L_{\rm int}\sim\varphi^{2\nu}$. These results of G. A. Milekhin [in *Proceedings*

of the International Conference on Cosmic Rays, Moscow, 1959, edited by G. B. Zhdanov (U.S.S.R. Academy of Sciences, Moscow, 1960), Vol. I, p. 220] can be found in the review article by E. L. Feinberg, Tr. Fiz. Inst., Akad. Nauk SSSR 29, 155 (1965) [in *Quantum Field Theory and Hydrodynamics, Proceedings of the P. N. Lebedev Physics Institute, Vol. 29,* edited by D. V. Skobel'tsyn (Consultants Bureau, New York, 1967), p. 151]; see also E. L. Feinberg, Phys. Rep. 5C, 237 (1972).

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[6]See, for example, D. Horn and F. Zachariasen, *Hadron Physics at Very High Energies* (Benjamin, Reading, Mass., 1973).

[7]For a review see R. Arnold, ANL Report No. ANL/HEP 7317 (unpublished). The analog picture has been discussed in the literature by a number of people. See, e.g., R. P. Feynman, in *Particle Physics,* AIP Conference Proceedings No. 6, edited by M. Bander, G. L. Shaw, and D. Y. Wong (American Institute of Physics, New York, 1972), p. 183; A. H. Mueller, Phys. Rev. D 4, 150 (1971); K. Wilson, Cornell University Report No. CLNS-131, 1970 (unpublished); J. D. Bjorken, in *Particles and Fields—1971,* AIP Conference Proceedings No. 2, edited by A. C. Melissinos and P. F. Slattery (American Institute of Physics, New York, 1971), p. 110; M. Bander, Phys. Rev. D 6, 164 (1972), and Phys. Rev. Lett. 30, 460 (1973), and Phys. Rev. D 7, 2256 (1973); R. C. Arnold, Phys. Rev. D 5, 1724 (1972), and ANL Report No. ANL/HEP 7241 (unpublished); T. D. Lee, Phys. Rev. D 6, 3617 (1972); R. C. Arnold and G. H. Thomas, Phys. Lett. 47B, 371 (1973); G. H. Thomas, Phys. Rev. D 8, 3043 (1973); R. C. Arnold, S. Fenster, and G. H. Thomas, Phys. Rev. D 8, 3138 (1973); R. C. Arnold and G. H. Thomas, Phys. Rev. D (to be published).

[8]For the multiperipheral approach cf. Refs. 3 and 7; the hydrodynamic approach has been compared with recent accelerator data by P. Carruthers and Minh D.-V., Phys. Lett. 41B, 597 (1972), and 44B, 507 (1973). See also F. Cooper and E. Schonberg, Phys. Rev. Lett. 30, 880 (1973).

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[10]For the treatment of an explicit example and further references, see D. Sivers and G. H. Thomas, Phys. Rev. D 6, 1961 (1972).