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Strong Magnetic Fields Produced by Composition Discontinuities in Laser-Produced Plasmas*

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Magnetic fields in excess of 10^7 G are expected to arise in laser-produced plasmas in which a discontinuity in atomic weight of the target occurs in the focal spot. The upper limit to *B* is controlled by fluid disassembly of the field layer due to magnetic pressure. This may provide a means for direct measurement of these fields in laser-produced plasmas.

The spontaneous generation of magnetic fields of order 10^6 G has been reported to occur¹⁻⁷ in highdensity kilovolt plasmas produced by focusing a Nd laser pulse onto a solid target. Here we show that fields in excess of 10^7 G may be produced in such plasmas if a discontinuity in atomic weight occurs in the focal spot. Such high fields should be directly detectable via the Zeeman splittings of x-ray lines emitted from the plasma.

The magnetic field equation⁵ is

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{\nabla} \times \vec{B}) - \frac{c^2}{4\pi} \nabla \times [\vec{r} \cdot (\nabla \times \vec{B})] - \frac{c}{4\pi e} \nabla \times \left[\frac{1}{N_e} (\nabla \times B) \times \vec{B}\right] + \vec{S}, \qquad (1)$$

where \vec{r} is the resistivity and the source \vec{S} has a thermal and a radiation-pressure part.

$$\vec{\mathbf{S}} = -\frac{ck}{eN_e} \nabla N_e \times \nabla T_e - \frac{c}{eN_e^2} \nabla N_e \times (\nabla \cdot \vec{\mathbf{P}}_R), \quad (2)$$

with the radiation pressure tensor given by^{5,6}

$$\vec{\mathbf{P}}_{R} = -\frac{1}{4\pi} \langle \epsilon \delta \vec{\mathbf{E}} \delta \vec{\mathbf{E}} + \delta \vec{\mathbf{B}} \delta \vec{\mathbf{B}} \rangle + \vec{\mathbf{I}} \frac{1}{8\pi} \langle \delta \vec{\mathbf{E}}^{2} + \delta \vec{\mathbf{B}}^{2} \rangle,$$
(3)

where $\epsilon = 1 - \omega_e^{2}/\omega^2$, and $\delta \vec{E}$ and $\delta \vec{B}$ are the highfrequency electric and magnetic fields of the laser pulse. Now if a discontinuity in target material is arranged, such as in the sandwich target shown in Fig. 1, a very large electron density gradient, ∇N_e , is produced at the composition interface since the average ion charge $Z(N_e \cong ZN_i)$ will in general differ in the two materials. This gradient is along the z axis, whereas ∇T_e and $\nabla \cdot \vec{P}_R$ are approximately along the beam direction at the center of the focal spot. This generates a large source of B field along the y direction.

Since the interface source layer is very thin (typically less then 1 μ m), field diffusion out of the layer initially competes with the source S in Eq. (1). Field diffusion only slows down to speeds less than the fluid disassembly velocity

V when the B field has spread through a layer of thickness of order 10 μ m. We calculate the field values reached during the diffusion-dominated phase.

Thus, Eq. (1) can be solved if we (i) "freeze" the hydrodynamics, (ii) neglect the x dependence



FIG. 1. Field produced by a focused laser pulse incident on a composition discontinuity. Although the critical depths for maximum energy deposition occur at different positions on the two sides, the displacement between them can be made less than the x temperature gradient scale for two materials not too widely spaced in atomic weight.

(13)

of *B* compared to its much faster *z* dependence, and (iii) neglect the *t* and *z* dependence of *T* and σ , and the nonlinear terms in *B* (these latter are of order $\beta^{-1} \leq 1$). For the geometry of Fig. 1, Eq. (1) then becomes

$$\frac{\partial B}{\partial t} \approx \frac{c^2}{4\pi\sigma} \frac{\partial^2 B}{\partial z^2} + S(z), \qquad (4)$$

$$S = -\frac{c}{eN_e} \frac{\partial N_e}{\partial z} \left[\frac{\partial k T_e}{\partial x} + \frac{1}{N_e} \left(\nabla \cdot \vec{\mathbf{p}}_R \right) \cdot \hat{i}_x \right]$$
(5)

which represents the localized source together with diffusion away from the interface due to the finite conductivity $\sigma (\equiv \sigma_{\perp})$.

The electron density variation N_e is modeled by choosing

$$\frac{1}{N_e} \frac{\partial N_e}{\partial z} = \left| \frac{\Delta N_e}{N_{e0}} \right| \frac{1}{l_N} \exp\left(-\frac{z^2}{l_N^2}\right).$$
(6)

Similarly, a local temperature-gradient scale in the z direction is defined by

$$l_T = T_e / (\partial T_e / \partial x). \tag{7}$$

Note that l_T and l_N can be either positive or negative, and usually $|l_T| \gg |l_N|$. The solution to (4) with B(t=0)=0 then follows as

$$B = -2\tau_{B}\left(\frac{ckT_{e}}{el_{N}l_{T}}\right)\left|\frac{\Delta N_{e}}{N_{e0}}\right|\left[1+Kl_{T}\frac{\delta_{rad}}{N_{e}kT_{e}}\right]$$

$$\times\left\{\left(1+\frac{t}{\tau_{B}}\right)^{1/2}\exp\left(-\frac{z^{2}}{l_{N}^{2}(1+t/\tau_{B})}\right)-\exp\left(-\frac{z^{2}}{l_{N}^{2}}\right)+\frac{z\pi^{1/2}}{l_{N}}\left[\Phi\left(\frac{z}{l_{N}(1+t/\tau_{B})^{1/2}}\right)-\Phi\left(\frac{z}{l_{N}}\right)\right]\right\},\tag{8}$$

where

$$\tau_{B} = \pi \sigma l_{N}^{2} / c \tag{9}$$

is characteristic field diffusion time, the strong-field $(\Omega_e \tau_e \gg 1)$ perpendicular conductivity σ is⁸

$$\sigma = 7.2 \times 10^7 T_e^{3/2} / Z \ln\Lambda$$
(10)

with T_e in degrees Kelvin, and $\Phi(y) = 2\pi^{-1/2} \int_0^y dx \exp(-x^2)$. The radiation-pressure part of (8) was written⁵

$$\hat{i}_{x} \cdot (\nabla \cdot \vec{\mathbf{P}}_{R}) \cong K \mathscr{E}_{rad}, \tag{11}$$

where \mathscr{E}_{rad} is the radiation energy density in the focal spot and K the absorption coefficient.⁹ Radiation reflection⁵ also contributes to (11) but has been neglected.

The solution (8) has features shown in Fig. 2. The field diffuses a distance

$$z_{B} \cong l_{N} (t/\tau_{b})^{1/2} = c (t/\pi \sigma)^{1/2}$$
(12)

away from the interface in a time t after the laser pulse is initiated. The value of B at the origin (z = 0), and for times $t \gg \tau_B$, becomes

$$B_{0} \cong \frac{8.7 \times 10^{-3} T_{e}^{-7/4} t^{1/2}}{l_{T} (Z \ln \Lambda)^{1/2}} \left| \frac{\Delta N_{e}}{N_{e0}} \right| \left(1 + K l_{T} \frac{\mathcal{E}_{rad}}{N_{e} k T_{e}} \right).$$

After some time the field (13) can grow to a magnitude such that $\beta \cong 1$. If this occurs the field layer then acts as a piston and drives a shock into the adjacent plasma. Fluid expansion of the *B* layer then follows on a time scale $\sim z_B/V_A$ and competes with further growth in *B*. Only very short pulses (for which $\tau_L < z_B/V_A$) could give rise to fields in excess of this $\beta = 1$ limit, i.e., for most cases the maximum *B* field is

$$B_{M} \cong 6 \times 10^{-8} (N_{e} T_{e} + N_{i} T_{i})^{1/2}.$$
(14)

Now typically we are interested in Nd pulses of energy > 50 J, focal spot radii < 50 μ m, and



FIG. 2. Diffusion of magnetic flux away from its source near the origin.

pulse times ≤ 1 nsec incident on a high atomic weight target.³ For $T_e \sim 5 \times 10^7 \,^{\circ}$ K, $Z \sim 20$, $\ln \Lambda \sim 10$, the field diffuses a distance $z_B \sim 45\sqrt{t}$ cm, e.g., $z_B = 4.5 \,\mu$ m for $t = 10^{-10}$ sec.

Very little diffusive broadening of the composition jump occurs. In a time t the interface broadens via ion diffusion to a thickness l_N given by $\lambda_i (t/\tau_i)^{1/2}$, where the scattering time τ_i for ions of charge Ze on side 1 by ions of charge $(Z + \delta Z)e$ on side 2 is⁸ (for the simplest case δZ $\ll Z$)

$$\tau_{i} \cong \frac{17}{\ln\Lambda} \left(\frac{m_{i}}{m_{p}}\right)^{1/2} \frac{T_{i}^{3/2}}{N_{i}Z^{4}}.$$
 (15)

For example, if $T_i = 10^7 \,^{\circ}\text{K} (T_i < T_e)$, $ZN_i = N_e = 10^{21} \text{ cm}^{-3}$, $Z \sim 20$, $\ln \Lambda \sim 10$, $m_i / m_p \sim 10^2$, then for t = 0.3 nsec, $l_N \cong 0.2 \ \mu\text{m}$.

For a jump with gradient scale $l_N \sim 1 \ \mu m$ (note also $l_N \gg \lambda_D \sim 10^{-6}$ cm), the field diffusion time τ_B [Eq. (9)] is 4×10^{-12} sec, so that the large-*t* formula (13) applies for B_0 throughout most of the pulse duration. For example, if $T_e = 5 \times 10^7$, $l_T = 4 \times 10^{-3}$, Z = 20, $\ln \Lambda \sim 10$, we find

$$B_{0} \approx 4.6 \times 10^{12} t^{1/2} \left(1 + \frac{K l_{T} \mathcal{S}_{rad}}{N_{e} k T_{e}} \right) \left| \frac{\Delta N_{e}}{N_{e0}} \right|, \qquad (16)$$

$$B_{H} = 4.2 \times 10^{-4} N_{e}^{-1/2}.$$

The field B_0 reaches its maximum value B_{M} in a time

$$t_{M} \cong 10^{-32} N_{e} \left(1 + \frac{K l_{T} \mathscr{E}_{R}}{N_{e} k T_{e}} \right)^{-2} \left| \frac{\Delta N_{e}}{N_{e0}} \right|^{-2}.$$
(17)

Since for most cases T_e increases to its maximum on the same time scale as the laser pulse width, τ_L , an optimum situation involves matching τ_L with t_M . For a sample case $|\Delta N_e/N_{e0}|^2 = 0.1$ and $\mathcal{S}_R \cong 0$, these formulas show that B_0 would rise to a value $B_M \cong 1.3 \times 10^7$ G in $t_M = 0.1$ nsec at the critical depth where $N_e = 10^{21}$ cm⁻³. However, heating also penetrates into the overdense regions beyond the critical depth for laser energy deposition. If similar temperatures occur in the region $N_e = 10^{22}$, B_M becomes 4.2×10^7 G which is reached in $t_M = 1$ nsec. For such fields, Zeeman splitting of x-ray lines is about equal to their Doppler broadening $[\hbar\Omega_e = (1.2 \times 10^{-8} \text{ eV 1 G})B]$, and should become detectable.

Emission from the region of maximum B(z), i.e., B_0 , could be selectively observed by seeding the interface with a small amount of material, e.g., Al, for which hydrogenic and He lines have been identified. Smearing of the splitting would then arise only from the x and t dependence of B_0 . However the effect may still appear as a splitting instead of a broadening in time-integrated profiles since the x-ray lines are principally emitted when T (and therefore B) is large.

Finally we note that the question of electrostatic instability and anomalous conductivity arises for the current layer associated with the *B* field. However, for highly charged ions [use of parameters below Eq. (15)] the ion collision frequency τ_i^{-1} is comparable to the ion plasma frequency at the critical depth $[\omega_i = \omega_e (Zm_e/m_i)^{1/2} \sim 2 \times 10^{13}]$. Thus, ion sound instability is collisionally damped and the classical conductivity (10) is expected to apply.

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