normalizing both to the usual dipole fit

$$D(q^2) = [1 + q^2/(0.71 \text{ GeV}^2)]^{-2}\mu.$$

Predictions are shown for different choices of Λ^2. The meaning of Λ^2 can be read in Eq. (2) that relates it to the effective perturbation-theory expansion parameter. A value $\Lambda^2 = 1$ GeV^2 (0.2 GeV^2) corresponds to scaling being approximately true (to within ~30%) at $q^2 = 5$ GeV^2 (1 GeV^2).

The agreement of the data and the prediction is satisfactory. We can safely state that there is no contradiction with the combined hypothesis of asymptotic freedom and strong local duality. Unfortunately, we cannot conclude that the logarithmic deviations from exact scaling predicted by asymptotically free theories have been observed, since we rely so heavily on the Bloom-Gilman hypothesis.

With Λ^2 known, we can predict the deviations from scaling in high-q^2 electroproduction. For instance, the structure function $F_2(x=0.2, q^2 = 40 \text{ GeV}^2)$ would be down relative to its value at $q^2 = 5$ GeV^2 ($q^2 = 1$ GeV^2) by an amount of ~10% (40%) for $\Lambda^2 = 1$ GeV^2 (0.2 GeV^2).

I am indebted to S. L. Glashow and H. D. Politzer for very useful discussions, and to L. R. Sulak for his help in analyzing the SLAC-MIT data.

After the completion of this work we learned that D. J. Gross and S. B. Treiman have recently treated the same problem.[11]

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[1]H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973).

[2]H. D. Politzer, "Setting the Scale for Predictions of Asymptotic Freedom" (to be published).

[3]Three of the twelve quarks are charmed.

[4]G. Miller *et al.*, Phys. Rev. D 5, 528 (1972).

[5]E. D. Bloom and F. J. Gilman, Phys. Rev. D 4, 2901 (1971).

[6]P. N. Kirk *et al.*, Phys. Rev. D 8, 63 (1973).

[7]At high n all leading contributions behave similarly and there is no need for a summation in Eq. (1).

[8]For a mathematically more satisfactory derivation of Eq. (8) see D. J. Gross, Phys. Rev. Lett. 32, 1071 (1974). For the exact treatment, see A. De Rújula, S. L. Glashow, and H. D. Politzer, to be published.

[9]Equation (3) does not contain an arbitrary normalization and one could, in principle, determine the absolute scale of the form factor. However, the coefficient a of the leading power-behavior Eq. (7) is very sensitive to the presence of nonleading terms, and the prediction of the absolute scale requires the complete solution of the inverse-moment problem. Alas, the unknown coefficients A_n^i reassume a role in the complete solution and asymptotic freedom loses a lot of predictive power.

[10]A similar prediction has been obtained recently in a different approach: J. Kogut, private communication; J. Kogut and L. Susskind, Phys. Rev. D (to be published).

[11]D. J. Gross and S. B. Treiman, Phys. Rev. Lett. 32 1145 (1974) (this issue).

Hadronic Form Factors in Asymptotically Free Field Theories*

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The breakdown of Bjorken scaling in asymptotically free gauge theories of the strong interactions is explored for its implications on the large-q^2 behavior of nucleon form factors. Duality arguments of Bloom and Gilman are invoked to relate the form-factor behavior to the threshold properties of deep inelastic structure functions. For very large q^2 the form factors are predicted to fall faster than any inverse power of q^2.

In a recent paper[1] by one of us the breakdown of Bjorken scaling was investigated in the context of asymptotically free gauge theories of the strong interactions.[2,3] An asymptotic extrapolation formula was obtained that expresses the deep inelastic structure function $F_2(\omega, q^2)$ at one value of q^2 in terms of the function $F_2(\omega, q'^2)$ at another value, q'^2, of the momentum-transfer variable —provided both q^2 and q'^2 are large enough to be in the "asymptotic" region. It was shown that deviations from scaling are expected to be especially large in the vicinity of threshold $[\omega \equiv (2\nu + m^2)/q^2 \approx 1]$.[4] Here we wish to explore the consequences which this breakdown of scaling would suggest for a related topic: the q^2 dependence of elastic and transition form factors of nucleons.

First, let us recall some results from Ref. 1. One starts with the asymptotic relation[2,5] for the moments of the structure function $\nu W_2 = mF_2$:

$$\int_1^\infty d\omega\, F_2(\omega, q^2)\omega^{-N-2} \underset{q^2\to\infty}{\longrightarrow} C_N[\ln(q^2/\mu^2)]^{-A_N}, \qquad (1)$$

where the C_N are unspecified constants and μ is an unspecified scale parameter. The exponents A_N are related to the calculable[2,5] anomalous dimensions of the dominant operators of spin $N+2$ in the Wilson expansion for a product of currents. With

$$t = \ln(q^2/\mu^2), \quad t' = \ln(q'^2/\mu^2), \quad t > t',$$

one now derives the asymptotic extrapolation formula

$$F_2(\omega, t) = \int_1^\omega \frac{d\omega'}{\omega'} F_2\left(\frac{\omega}{\omega'}, t'\right) T\left(\omega', \frac{t}{t'}\right), \qquad (2)$$

where ($\sigma > 0$)

$$T\left(\omega, \frac{t}{t'}\right) = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} ds \left(\frac{t}{t'}\right)^{-A_s} \omega^{s+1}. \qquad (3)$$

Equation (2) is supposed to become exact when t and t' are in the asymptotic region, i.e., where the effective "coupling constant" $\bar{g}(t')$ is sufficiently small compared to unity.

The contributions of several operators (singlet and nonsinglet) should be added together in Eq. (1), each with its own anomalous dimension.[6] But the threshold behavior of F_2 is governed by the moments of large N, and the operators all have the same large-N behavior,

$$A_N \underset{N\to\infty}{\longrightarrow} G[4\ln(N+2) - 0.69], \qquad (4)$$

where G is determined by the structure of the strong gauge group and by the quark representation under the group. If the strong gauge group is SU(3)$'$, and if the theory contains three quark triplets, then one finds that $G = \frac{4}{27}$. This is the value that we shall adopt for later numerical purposes.

Suppose that the reference structure function, at t', has the following behavior near threshold:

$$F_2(\omega, t') \sim (\omega - 1)^d, \quad \omega \to 1. \qquad (5)$$

Then the structure function at $t > t'$ has itself the rather simple threshold behavior given by

$$\frac{F_2(\omega, t)}{F_2(\omega, t')} \underset{\omega\to 1}{\longrightarrow} \left(\frac{t}{t'}\right)^{0.69G} \frac{\Gamma(d+1)}{\Gamma(d+1+P)} (\ln\omega)^P, \qquad (6)$$

where

$$P = 4G\ln(t/t'). \qquad (7)$$

The threshold property expressed by Eq. (5) seems in fact to obtain (with $d \simeq 3$) in the q'^2 region relevant to the Stanford Linear Accelerator Center–Massachusetts Institute of Technology (SLAC–MIT) experiments. Accepting this, we now wish to consider the implications of Eq. (6) for the question of nucleon electromagnetic form factors. Consider the contribution to the structure function F_2 coming from some resonance of mass M_r. For simplicity we take the width to be infinitely narrow. This contribution is

$$F_2^{\,r} = \nu G^2(q^2)\delta(\nu - \nu_r), \qquad (8)$$

where $\nu_r = \frac{1}{2}(M_r^{\,2} - m^2 + q^2)$. This equation defines the electromagnetic transition form factor connecting the nucleon and the resonance. If the "resonance" in question is the nucleon itself ($M_r^{\,2} = m^2$), then we are dealing with the diagonal electric and magnetic form factors and

$$G^2 = \frac{G_E^{\,2} + (q^2/4m^2)G_M^{\,2}}{1 + (q^2/4m_2)}. \qquad (9)$$

Accepting that the second term dominates the first term in the numerator of Eq. (9) at large q^2, we have

$$G^2 \underset{q^2\to\infty}{\longrightarrow} G_M^{\,2}[1 + O(1/q^2)]; \qquad (9')$$

with $M_0 > M_r$, it now follows from positivity that

$$\int_{1+m^2/q^2}^{1+M_0^2/q^2} d\omega\, F_2(q^2, \omega) > G^2(q^2). \qquad (10)$$

As $q^2 \to \infty$, the integration is restricted to the threshold of ω, and we may therefore adopt Eq. (6) for $F_2(q^2, \omega)$. In this way we find

$$G^2(q^2) < \text{const}\left(\frac{t}{t'}\right)^{0.69G} \times \frac{\Gamma(d+2)}{\Gamma(d+2+P)}\left(\frac{s_0}{q^2}\right)^{d+1+P}, \qquad (11)$$

where $s_0 = M_0^{\,2} - m^2$. Notice that the parameters d and q'^2 depend on the choice of reference structure function, according to Eq. (5). As Eq. (6) shows, d increases with q'^2.

In deriving Eq. (10) we have *assumed* that the limit $q^2 \to \infty$ is uniform in ω, even in the vicinity of threshold, $\omega = 1 + O(1/q^2)$; and we are then led to the prediction that the elastic and transition form factors fall faster than any inverse power of q^2—indeed, faster than $(q^2)^{-2G\ln\ln q^2}$. The uniformity assumption would be false if the corrections to Eq. (1), which are of order $[\ln(q^2/\mu^2)]^{-1}$, appear with coefficients that increase sufficiently rapidly for large N.[7]

The inequality of Eq. (11) becomes an equality if we invoke the notion of full local duality advocated by Bloom and Gilman[8] for electroproduction, who find a best fit to the SLAC-MIT data with $s_0=(1.23\ \text{GeV})^2$. Using this local duality, we then have for the nucleon form factor

$$\frac{G_M^2(q^2)}{G_M^2(q'^2)} \approx \left(\frac{\ln(q^2/\mu^2)}{\ln(q'^2/\mu^2)}\right)^{0.69G} \times \frac{\Gamma(d+1)}{\Gamma(d+2+P)}\left(\frac{s_0}{q^2}\right)^P\left(\frac{q'^2}{q^2}\right)^{d+1}, \quad (12)$$

provided both q'^2 and q^2 are large enough. The same formula holds for transition form factors to resonances. As noted earlier, all the revelant twist-two operators have the same anomalous dimensions for large N, irrespective of isospin and other quantum numbers. It follows that the asymptotic extrapolation formula of Eq. (12) holds separately for the nucleon isovector and isoscalar magnetic form factors and similarly for the corresponding axial-vector form factors.

The analysis given above can be repeated for the longitudinal structure function $F_L(q^2, \omega)$, which receives contributions from the nucleon electric form factor $G_E(q^2)$. In asymptotically free theories of the type discussed in Ref. 2, the moments of F_L are asymptotically smaller by one power of $\ln(q^2/\mu^2)$ than the corresponding moments of F_2. Moreover, the N dependence of the ratios of the coefficients can be explicitly computed.[9] Using the results thus obtained, and combining them with an analysis of the sort described in Ref. 1, one finds near threshold ($\omega \to 1$) that $F_L/F_2 \to (1-\omega)[\ln(q^2/\mu^2)]^{-1}$. On the basis of the duality arguments employed here one now finds that $G_E^2/G_M^2 \to [\ln(q^2/\mu^2)]^{-1}$ as $q^2 \to 0$. Thus the "scaling" of electric and magnetic form factors (to within logarithms) emerges as a prediction of asymptotically free theories based on spin-$\frac{1}{2}$ constituents.

Beyond the major uniformity and duality assumptions that have gone into Eq. (12), there are several practical delicacies that arise in any attempt to use this equation for the regions of q^2 that are presently accessible to experiment. We have to start at some reference q'^2 where the structure function $F_2(\omega, q'^2)$ is known in the vicinity of $\omega = 1$; and we have to suppose that q'^2 is large enough to be regarded as asymptotic. It is an open question whether present experiments have penetrated this asymptotic region. Certainly the present evidence is too sparse to determine, for Eq. (5), whether the exponent d is varying with q^2 over the limited range covered. To

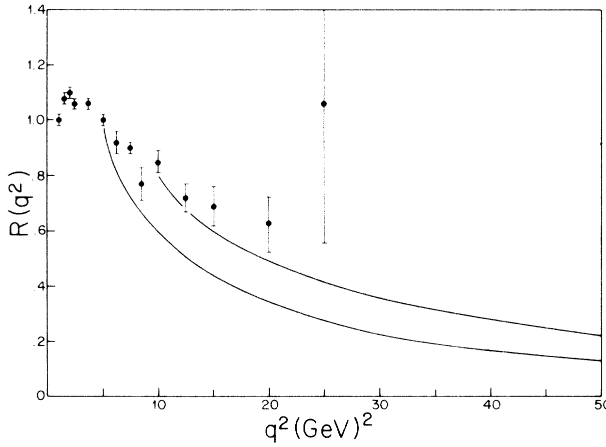

FIG. 1. The quantity $R(q^2) \equiv (1+q^2/q_0^2)^4 G_M^2(q^2)/G_M^2(0)$, $q_0^2 = 0.71\ \text{Gev}^2$, as a function of q^2, with scale parameter $\mu^2 = 0.5\ \text{Gev}^2$. The upper curve corresponds to $q'^2 = 10\ \text{Gev}^2$; the lower curve, to $q'^2 = 5\ \text{Gev}^2$.

be sure, Eqs. (6) and (7) suggest that d should not vary rapidly with q^2 and we shall accept the experimental[10] result, $d=3$, as applying to any q'^2 in the region of a few GeV^2. From Eq. (12) we see, for any limited range in the vicinity of the reference q'^2, that the form-factor variation with q^2 comes chiefly from the factor $(q^2)^{-d-1}$ on the right-hand side of the equation. With $d=3$ this implies $G_M \sim (q^2)^{-2}$, which is the celebrated dipole behavior—in rough agreement with experiment up to q^2 values of a few GeV^2. Actually, in this region the behavior is more accurately represented by

$$G_M \sim (1+q^2/q_0^2)^{-2},$$

with $q_0^2 = 0.71\ \text{Gev}^2$. This departure from Eq. (12), together with the other departure of order $(q^2)^{-1}$ represented by the approximation in going from Eq. (9) to Eq. (9′), might well arise from higher-order corrections to Eq. (5), i.e., from a next term of order $(\omega-1)^{d+1}$. Indeed, the successful duality fits of Bloom and Gilman suggest that this is so. For practical purposes we incorporate these effects into Eq. (12) by replacing the factor $(q'^2/q^2)^{d+1}$ with the factor $(q'^2+q_0^2)^{d+1}/(q^2+q_0^2)^{d+1}$, $d=3$. For $q^2 > q'^2 \gg q_0^2$, this amounts to a correction of order $(q^2)^{-1}$.

Results for $\mu^2 = 0.5\ \text{GeV}^2$ and two different choices of q'^2 are shown in Fig. 1, which also displays the data points.[11] Each curve would move upward with decreasing μ^2. In view of the sensitivity to μ^2 and q'^2, detailed comparison with experiment is premature at present, except for the general trends indicated.

What *is* qualitatively significant is the experimental indication of a falloff below the dipole formula at large q^2.[12] If Bjorken scaling holds exactly, then the Bloom-Gilman analysis suggests that G_M should fall exactly like $(q^2)^{-2}$. The more rapid falloff observed to set in around $q^2 \approx 5$ would then imply either a failure of the duality arguments, or a breakdown of exact Bjorken scaling. It is this latter interpretation that we have adopted here, in the context of asymptotically free gauge theories. The interesting circumstance arises that one can get at a possible breakdown of exact scaling indirectly, through the elastic form factor, as well as directly, through the deep inelastic structure functions. The breakdown effects are magnified for the form factor, as one sees by comparing the q^2 dependence of Eqs. (6) and (12). If is clearly worthwhile to pursue both approaches, out to the largest possible values of q^2.

After this work was completed we learned that a similar analysis was carried out by A. De Rújula,[13] and also that Kogut and Susskind have reached the same conclusions about the asymptotic behavior of form factors and have made extensive use of Eq. (2).[14]

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[1]D. J. Gross, Phys. Rev. Lett. 32, 1071 (1974).

[2]D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973), and Phys. Rev. D 8, 3633 (1973), and 9, 980 (1974).

[3]H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).

[4]Since we shall invoke duality arguments in this paper, it is convenient to use the Bloom-Gilman scaling variable instead of the more conventional $\omega = 2\nu/q^2$.

[5]H. Georgi and H. D. Politzer, Phys. Rev. D 9, 416 (1974).

[6]The anomalous dimensions of the singlet, normal-parity operator and of the nonsinglet operators were calculated in Refs. 2 and 5. The anomalous dimensions of the abnormal-parity singlet operators, which contribute to W_3, were calculated by one of us (D. J. Gross, unpublished); again, for large N, the results are as in Eq. (4).

[7]There are some indications that the corrections to Eq. (1) may indeed behave as $\bar{g}^2 \ln^2 N$ (relative to unity for the leading term). For Eq. (6) this would produce a relative correction of order $(\ln\ln\omega)^2/\ln(q^2/\mu^2)$—which behaves like $\ln(q^2/\mu^2)$ for $\omega = 1 + O(1/q^2)$. If this situation really persists, one would then have to imagine summing over the corrections to Eq. (1) of all orders in $\bar{g}^2$, in order to test the validity of our basic assumption. This will be a difficult task. For the present we stick to our assumptions.

[8]E. D. Bloom and F. J. Gilman, Phys. Rev. D 4, 2901 (1971).

[9]M. Calvo, private communication (to be published).

[10]G. Miller *et al.*, Phys. Rev. D 5, 528 (1972).

[11]P. N. Kirk *et al.*, Phys. Rev. D 8, 63 (1973).

[12]As has already been noted, with exact Bjorken scaling the duality argument implies that G_M falls like $(q^2)^{-(d+1)/2}$. Given that $d = 3$ in the SLAC-MIT region this leads to the simple dipole behavior; but nothing in the argument explains why $d = 3$. However, the dipole behavior is sometimes thought to follow from the assumption that the nucleon is a bound state of three quarks; this is the picture embodied in the dimensional rules of S. Brodsky and G. Farrar, Phys. Rev. Lett. 31, 1153 (1973). This picture has been shown to emerge in nongauge theories for which the dimensions are taken to be canonical (C. G. Callan and D. J. Gross, to be published). However, in asymptotically free gauge theories there appear to be logarithmic corrections. These have not so far been calculated. For consistency with the results obtained in the present paper, one has to suppose that these corrections sum up to give the modifications to the dipole formula which are embodied in Eq. (12). As seen, these modifications become very substantial indeed at large q^2, even though compounded out of logarithms.

[13]A. De Rújula, Phys. Rev. Lett. 32, 1143 (1974) (this issue).

[14]J. Kogut and L. Susskind, "Parton Models and Asymptotic Freedom" (to be published), and "The Scale-Invariant Parton Model" (to be published).