Proton Magnetic Form Factor in Asymptotically Free Field Theories

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Asymptotically free field theories of the strong interactions make specific predictions for the breakdown of scaling in deep inelastic electroproduction. I propose to use the Bloom-Gilman local duality connection between inelastic structure functions and elastic form factors to predict the high-q^2 behavior of the magnetic form factor. The results agree with experiment but are not a conclusive test of asymptotic freedom.

Two virtues of asymptotically free gauge field theories of the strong interactions[1] are that they are *not* free field theories and that they make predictions which are *not* asymptotic. The prediction[1] for inclusive electroproduction is that the fixed-q^2 moments of the structure functions decrease logarithmically:

$$\int_0^1 x^n F(x, q^2) d(x) = \sum_{i=1}^{I} A_n{}^i \left(\frac{\ln(q^2/\Lambda^2)}{\ln(q_0{}^2/\Lambda^2)}\right)^{-d_n{}^i}, \quad n \text{ even}, \quad n \geqslant =, \quad q^2 \geqslant q_0{}^2. \tag{1}$$

The exponents $d_n{}^i$ are explicit model-dependent functions of n. The unknown coefficients $A_n{}^i$ conceal our ignorance of strong-interaction dynamics. The quantity Λ^2 sets the scale, and is a free parameter to be determined by experiment. Equation (1) is correct to first order in renormalization-group-improved perturbation theory. For it to be a good approximation the reference momentum transfer $q_0{}^2$ must be large enough for the running coupling constant $\bar{g}$ to be small.[2] If the model of strong interactions is color SU(3) with three quartets of quarks,[3] the effective expansion parameter is

$$\frac{\bar{g}^2}{4\pi^2} = \frac{g^2}{4\pi^2}\left(1 + \frac{25 g^2}{48\pi^2}\ln\frac{q^2}{\Lambda^2}\right)^{-1} \simeq \frac{12}{25\ln(q^2/\Lambda^2)} \quad (q^2/\Lambda^2 \text{ large}). \tag{2}$$

For $q^2/\Lambda^2 \gtrsim 5$, the right-hand side is 0.3 and one can expect perturbation theory to start making sense. We restrict ourselves to this twelve-quark model in what follows. This is no great loss of generality, since the results will depend much more on the arbitrary parameter Λ than on the group-theory details (as long as asymptotic freedom is maintained).

The available experimental information on electroproduction[4] does not extend over a large enough range of q^2 for the logarithmic effects of Eq. (1) to be significant. On the other hand, the elastic magnetic form factor of the proton is known with precision over a larger range (up to $q^2 = 20$ GeV^2). We propose, as an indirect way of hunting the logarithm, to use the local duality hypothesis of Bloom and Gilman[5] to predict the high-q^2 behavior of the elastic form factor from the expression, Eq. (1), for the structure function.

Bloom and Gilman, carrying the spirit of finite-energy sum rules to its extreme, propose the following connection between the conventionally defined F_2 structure function and the elastic form factors[5]:

$$\int_{q^2/(q^2+W_t{}^2)}^{1} (dx/x^2) F_2(x, q^2) = G^2(q^2), \tag{3}$$

where x is the "improved" scaling variable

$$x = q^2/(q^2 + W^2), \tag{4}$$

and W_t is some fixed value of the final hadron mass, close to the pion-nucleon threshold. The form factor $G(q^2)$ is related to the familiar[6] magnetic form factor via

$$G^2(q^2) = \frac{\mu^{-2} + q^2/4m^2}{1 + q^2/4m^2} G_M{}^2(q^2), \tag{5}$$

where μ is the nucleon magnetic moment, and we have used the "form-factor scaling law" $G_M(q^2) = \mu G_E(q^2)$. We are concerned with large q^2, so that reasonable deviations from this law are irrelevant, and $G_M \simeq G$.

To predict the form factor via Eq. (3) it is enough to know the threshold behavior ($x \to 1$) of the structure function. The predictions of asymptotic freedom are particularly simple in this region. The large-n moments of $F(x, q_0{}^2)$ at a measured $q_0{}^2$ are sensitive only to the behavior of the structure function near $x = 1$. Conversely, the reconstruction of $F(x, q^2)$ near threshold from its moments [as predicted by Eq. (1)] only requires the knowledge of the high-n moments. In the model we are considering, and for high n, the dominant[7] moments are simply[1]

$$d_n \approx \tfrac{8}{25}\ln(n+2), \tag{6}$$

an approximation which is already better than 5% at $n=6$. Let α_0 be the leading power behavior of $F_2(x,q_0^2)$ as $x\to 1$:

$$\lim_{x\to 1} F_2(x,q_0^2) = a(1-x)^{\alpha_0}. \tag{7}$$

We now compute from Eq. (1) the leading power behavior $\alpha=\alpha_0+\delta\alpha$ at a slightly higher $q^2=q_0^2+\delta q^2$. This is simply done by writing

$$I_n(\alpha_0)\equiv\int_0^1 x^n(1-x)^{\alpha_0}dx = n!\,\Gamma(\alpha_0+1)/\Gamma(n+\alpha_0+2), \tag{8}$$

$$I_n(\alpha)\equiv\int_0^1 x^n(1-x)^{\alpha_0+\delta\alpha}dx = \frac{n!\,\Gamma(\alpha_0+\delta\alpha+1)}{\Gamma(n+\alpha_0+\delta\alpha+2)} = I_n(\alpha_0)\left[\frac{\ln(q^2/\Lambda^2)}{\ln(q_0^2/\Lambda^2)}\right]^{-d_n}, \tag{9}$$

and comparing the two expressions for $I_n(\alpha)$ at some large finite n. Using the moments of Eq. (6), we find

$$\delta\alpha(q^2,q_0^2,\Lambda^2) = \tfrac{8}{25}\ln\left[\frac{\ln(q^2/\Lambda^2)}{\ln(q_0^2/\Lambda^2)}\right]. \tag{10}$$

In practice this approximation turns out to be excellent over a wide range of q^2.[8] Combining Eqs. (3), (5), and (10), we obtain

$$G_M(q^2)\simeq\left\{\int\frac{W_t^2\,dq^2}{(q^2)^2(1+q^2/W_t^2)^{\alpha_0+\delta\alpha}}\right\}^{1/2} g_0,\quad q^2\geq q_0^2, \tag{11}$$

where g_0 is chosen to normalize[9] G_M at $q^2=q_0^2$. The asymptotic behavior of the form factor is of the form $(\ln q^2)^{-\ln q^2}$.[10]

We know from the electroproduction data that some sort of approximate scaling has already set in at $q^2=5$ GeV^2. We choose this to be our reference point q_0^2, and proceed to compare Eq. (11) with the proton data. In so doing, three remarks are in order.

(i) The sensitivity of Eq. (11) to reasonable changes of W_t in the vicinity of the pion-nucleon threshold is very small for $q^2>5$ GeV^2. We choose W_t to equal the mass of the first pion-nucleon resonance, the value advertised by Bloom and Gilman.

(ii) Equation (11) is very sensitive to the presence of the $\delta\alpha$ term predicted by the asymptotically free theories ($\delta\alpha=0$ for exact scaling).

(iii) For the success of an eventual fit to be at all meaningful, it is necessary that the error in the input α_0 be considerably smaller than the slowly varying function $\delta\alpha$ in the q^2 range under consideration. In the example below, $0<\delta\alpha<0.30$. The form $F_2(x)=a(1-x)^{\alpha_0}$ provides a good fit to the Stanford Linear Accelerator Center–Massachusetts Institute of Technology (SLAC-MIT) data[4] for $1>x>0.5$ and values of q^2 of the order of 5 GeV^2. The result ($\alpha_0=3.004\pm0.009$) justifies our analysis.

In Fig. 1, we compare Eq. (11) to the data,

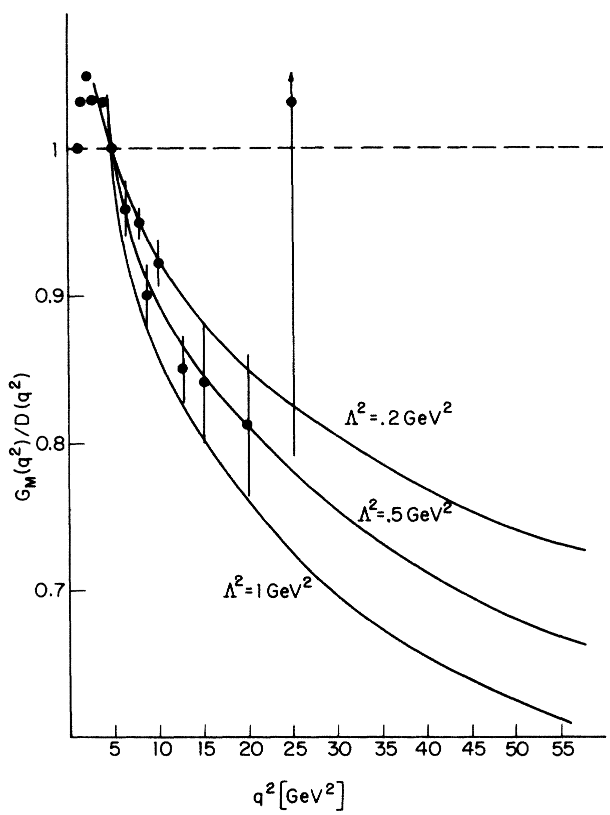

FIG. 1. Magnetic form factor of the proton normalized to the dipole fit. Comparison between the prediction of Eq. (11) and the data (Ref. 6).

normalizing both to the usual dipole fit

$$D(q^2)=[1+q^2/(0.71\ \mathrm{GeV}^2)]^{-2}\mu.$$

Predictions are shown for different choices of Λ^2. The meaning of Λ^2 can be read in Eq. (2) that relates it to the effective perturbation-theory expansion parameter. A value $\Lambda^2=1\ \mathrm{GeV}^2$ ($0.2\ \mathrm{GeV}^2$) corresponds to scaling being approximately true (to within $\sim 30\%$) at $q^2=5\ \mathrm{GeV}^2$ ($1\ \mathrm{GeV}^2$).

The agreement of the data and the prediction is satisfactory. We can safely state that there is no contradiction with the combined hypothesis of asymptotic freedom and strong local duality. Unfortunately, we cannot conclude that the logarithmic deviations from exact scaling predicted by asymptotically free theories have been observed, since we rely so heavily on the Bloom-Gilman hypothesis.

With Λ^2 known, we can predict the deviations from scaling in high-q^2 electroproduction. For instance, the structure function $F_2(x=0.2, q^2=40\ \mathrm{GeV}^2)$ would be down relative to its value at $q^2=5\ \mathrm{GeV}^2$ ($q^2=1\ \mathrm{GeV}^2$) by an amount of $\sim 10\%$ (40%) for $\Lambda^2=1\ \mathrm{GeV}^2$ ($0.2\ \mathrm{GeV}^2$).

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After the completion of this work we learned that D. J. Gross and S. B. Treiman have recently treated the same problem.[11]

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[1]H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973).

[2]H. D. Politzer, "Setting the Scale for Predictions of Asymptotic Freedom" (to be published).

[3]Three of the twelve quarks are charmed.

[4]G. Miller *et al.*, Phys. Rev. D 5, 528 (1972).

[5]E. D. Bloom and F. J. Gilman, Phys. Rev. D 4, 2901 (1971).

[6]P. N. Kirk *et al.*, Phys. Rev. D 8, 63 (1973).

[7]At high n all leading contributions behave similarly and there is no need for a summation in Eq. (1).

[8]For a mathematically more satisfactory derivation of Eq. (8) see D. J. Gross, Phys. Rev. Lett. 32, 1071 (1974). For the exact treatment, see A. De Rújula, S. L. Glashow, and H. D. Politzer, to be published.

[9]Equation (3) does not contain an arbitrary normalization and one could, in principle, determine the absolute scale of the form factor. However, the coefficient a of the leading power-behavior Eq. (7) is very sensitive to the presence of nonleading terms, and the prediction of the absolute scale requires the complete solution of the inverse-moment problem. Alas, the unknown coefficients A_n^i reassume a role in the complete solution and asymptotic freedom loses a lot of predictive power.

[10]A similar prediction has been obtained recently in a different approach: J. Kogut, private communication; J. Kogut and L. Susskind, Phys. Rev. D (to be published).

[11]D. J. Gross and S. B. Treiman, Phys. Rev. Lett. 32 1145 (1974) (this issue).

Hadronic Form Factors in Asymptotically Free Field Theories*

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The breakdown of Bjorken scaling in asymptotically free gauge theories of the strong interactions is explored for its implications on the large-q^2 behavior of nucleon form factors. Duality arguments of Bloom and Gilman are invoked to relate the form-factor behavior to the threshold properties of deep inelastic structure functions. For very large q^2 the form factors are predicted to fall faster than any inverse power of q^2.

In a recent paper[1] by one of us the breakdown of Bjorken scaling was investigated in the context of asymptotically free gauge theories of the strong interactions.[2,3] An asymptotic extrapolation formula was obtained that expresses the deep inelastic structure function $F_2(\omega, q^2)$ at one value of q^2 in terms of the function $F_2(\omega, q'^2)$ at another value, q'^2, of the momentum-transfer variable —provided both q^2 and q'^2 are large enough to be in the "asymptotic" region. It was shown that deviations from scaling are expected to be especially large in the vicinity of threshold $[\omega\equiv(2\nu+m^2)/q^2\approx 1]$.[4] Here we wish to explore the consequences which this breakdown of scaling would suggest for a related topic: the q^2 dependence of elastic and transition form factors of nucleons.