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[3]B. Richter, Bull. Amer. Phys. Soc. 19, 100 (1974).

[4]A. Pais, Ann. Phys. (New York) 9, 548 (1960). This paper contains several technical ingredients helpful for the derivation of Eqs. (5) and (7), cf. Eqs. (26), (29), (30), and (32).

[5]To pessimize: "···to take the most unfavourable view of " [*Oxford English Dictionary* (Clarendon Press, Oxford, England, 1933), Vol. 7, p. 742]. It has seemed for some time to C. Llewellyn Smith and the author that this term is particularly fitting to describe the treatment of inequalities in which bounds depend on parameters, since only the most unfavorable view of the parameter range determines the content of the inequality.

Are There Anomalous Lepton-Hadron Interactions?

Jogesh C. Pati*
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

and

Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy, and Imperial College, London, England
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It is remarked that the recently observed near constancy of $\sigma(e^+e^- \to \text{hadrons})$ over a large range of center-of-mass energy may reflect the presence of a new class of short-range lepton-hadron interactions. This can be tested by a comparison of e^-p versus e^+p scatterings and a study of the spin, parity, and charge conjugation of the final product in annihilation as well as apparent deviations from scaling in $e^\pm p$ and $\mu^\pm p$ scatterings.

Recent experimental studies[1] of the electron-positron-annihilation cross section into hadrons $[\sigma_h(s)]$ as a function of s, the square of the total center-of-mass energy, seem to reveal a remarkable feature—that it is nearly constant at about 25–30 nb (within 30%) from $s \simeq 9$ to $s \simeq 25$ [in units of $(\text{BeV})^2$]. On the other hand, $\sigma(e^+e^- \to \mu^+\mu^-) \equiv \sigma_\mu(s)$ appears to fall according to the quantum-electrodynamic (QED) s^{-1} law. The near "constancy" of $\sigma_h(s)$ over such a wide region of s does not seem to obtain a simple explanation in terms of the familiar one-photon mechanism.[2] We consider in this note an alternative explanation for the behavior of $\sigma_h(s)$ based on a new class of short-range lepton-hadron interactions (leading to process such as $e^-e^+ \to q\bar{q}$, etc.) which may arise within the class of gauge schemes[3] proposed by us earlier, and point out that this leads to a variety of testable predictions; these should enable one to distinguish our explanation from all others based on the one-photon mechanism.[4]

To make our discussions specific,[5] we concentrate on the possiblity that the interactions in question are due to exchange [see Fig. 1(a)] of heavy exotic[6] spin-1 mesons X (with nonzero baryon and lepton numbers) coupled to electron-quark (and possibly also to muon-quark[7]) currents as follows:

$$\mathcal{L}^X = f(\bar{e}\gamma_\mu q)X_\mu + \text{H.c.} \qquad (1)$$

There could, of course, be a triplet of X's corresponding to three baryonic colors. It is possible that there are vector and axial-vector mesons X_V and X_A coupled to currents $\bar{e}\gamma_\mu q$ and $\bar{e}\gamma_\mu\gamma_5 q$ with strengths f_V and f_A, respectively. For the present, we need not specify the $(\mathcal{P}, \mathfrak{N}, \lambda)$ indices of q.

Let us assume that the effective low-energy

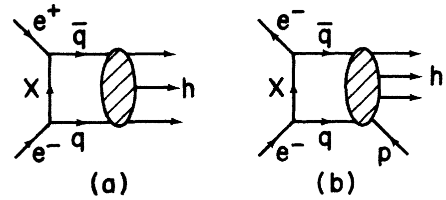

FIG. 1. (a) e^-e^+ annihilation, and (b) ep scattering via X interaction.

strength of the X interaction is characterized by

$$a_X \equiv (f^2/4\pi)(1/m_X^2) \simeq \epsilon\alpha(1\ \text{BeV}^{-2}), \qquad (2)$$

where $\alpha = e^2/4\pi \simeq \frac{1}{137}$ and $\epsilon < \frac{1}{10}$—possibly $\frac{1}{20}-\frac{1}{40}$ (see later). Thus, if $f^2/4\pi \simeq 1-\frac{1}{10}$, X must have a mass in excess of 50–15 BeV. Such an X is not in conflict with the QED properties[8] of e and μ (such as anomalous magnetic moments).

Consider the second-order effective interaction mediated by X [Fig. 1(a)] leading to e^-e^+ annihilation. First note that the X interaction, as postulated, can contribute *only to semileptonic processes* in order f^2 (i.e., $e^-e^+ \to$ hadrons and possibly also to $\mu^-\mu^+ \to$ hadrons), but not to pure leptonic[9] (i.e., $e^-e^+ \to e^-e^+$ and $e^-e^+ \to \mu^-\mu^+$, etc.) or to pure nonleptonic processes. Secondly, as long as $m_X^2 \gg s$, and therefore $m_X^2 \gg t$, where t is the effective[10] square of momentum carried by the X line [Fig. 1(a)], one may treat the effective current-current interaction mediated by X as a local four-fermion interaction which (after Fierz reshuffling) is given by

$$\mathcal{L}_{\text{eff}}^X = (f^2/m_X^2)\tfrac{1}{4}[4S - 2V + 2A - 4P]. \qquad (3)$$

S, V, A, and P correspond to the local interactions $\{\bar{q}(x)\Gamma_i q(x)\}\{\bar{e}(x)\Gamma_i e(x)\}$ with $\Gamma_i = 1$, γ_μ, $\gamma_\mu\gamma_5$, and γ_5, respectively. Note that if there were primary vector *and* axial-vector interactions (as mentioned) and if $f_V = f_A$ with $m_{X_V} = m_{X_A}$, there would be no net S and P terms. For algebraic simplicity, consider first the contributions from the V *term only*. This will lead to an amplitude for $e^-e^+ \to$ hadrons given by $A_X = (f^2/m_X^2) \times [\bar{v}_e\gamma_\mu u_e]\langle h|V_\mu^X(0)|\rangle$, where $V_\mu^X = -\frac{1}{2}(\bar{q}\gamma_\mu q)$ is the vector hadronic current in the V term of (3). The one-photon exchange leads to the familiar annihilation amplitude $A_\gamma = (e^2/s)[\bar{v}_e\gamma_\mu u_e]\langle h|V_\mu^{em}(0) \times|0\rangle$. The contribution of A_X and A_γ to the annihilation cross section is

$$\sigma_h(s) = \frac{4\pi s}{3}\left\{\frac{\alpha^2\rho_{\gamma\gamma}(s)}{s^2} + \frac{2\alpha a_X\rho_{\gamma X}(s)}{s} + a_X^2\rho_{XX}(s)\right\}, \qquad (4)$$

where the functions $\rho_{\gamma\gamma}(s)$, $\rho_{\gamma X}(s)$, and $\rho_{XX}(s)$ represent the hadronic tensors[11] for the current correlations $(V_\mu^{em}V_\nu^{em})$, $(V_\mu^{em}V_\nu^X + V_\mu^X V_\nu^{em})$, and $(V_\mu^X V_\nu^X)$, respectively. For s in the asymptotic region,[2] $\rho_{\gamma\gamma}(s) = \text{Tr}(Q_\gamma^2)$, where Q_γ is the quark-charge matrix; similar expressions would hold for $\rho_{\gamma X}(s)$ and $\rho_{XX}(s)$. Thus, for sufficiently high s, one may write

$$\sigma_h(s) = \tfrac{4}{3}\pi\alpha^2\rho_{\gamma\gamma}(s)[s^{-1} + 2\delta' + \delta^2 s] \qquad (5)$$
$$[\text{units: } (\text{BeV})^2],$$

where $\delta = \epsilon\rho_{XX}(s)/\rho_{\gamma\gamma}(s)$ and $\delta' = \epsilon\rho_{\gamma X}(s)/\rho_{\gamma\gamma}(s)$, both of which are constants of order ϵ. The sign and the precise magnitude of the interference term δ' depend upon the SU(3) structure[12] of V_μ^X *vis-à-vis* V_μ^{em}.

The contributions from the A, S, and P terms in Eq. (3) *cannot interfere*[13] with A_γ, A_X, or each other, since they lead to final hadronic states differing in parity or in charge-conjugation property or both. From dimensional considerations and the scaling hypothesis we expect that the contributions of A (and also S and P, if present) to $\sigma_h(s)$ at high s should be proportional to s (like the V contribution), so that these simply *add* constructively to the $\delta^2 s$ term in Eq. (5). If we allow the next-to-leading contribution in the light-cone expansion of the various density correlations, they contribute terms independent of s to $\sigma_h(s)$, which only alter the coefficient δ'. Thus the *form (5)* is expected to represent the net contribution of one-photon plus V, A, (S and P) terms with *redefined* δ' and δ^2, both of which are still constants of order ϵ.

It is important to note that the A (and S, P) contributions will lead to C even, $J^P = 1^+$ (and also $0^\pm$ states if S and P are present), which cannot be obtained from vector-current contributions. The relative proportion of these states should be at least an order of magnitude higher than what is expected from the two-photon contribution, especially at high s.

If $\rho_{\gamma\gamma}(s)$ may be treated as a constant for $s \gtrsim 10$ (say), the variation of $\sigma_h(s)$ will be characterized by the variation of $\chi(s) \equiv s^{-1} + \delta^2 s$. Clearly this function is approximately a constant within 20–30% for values of $\delta \simeq \frac{1}{20}-\frac{1}{30}$ and s varying from 10 to 40. The percentage variation of $\sigma_h(s)$ is further reduced if the coefficient δ' of the constant term is positive; thus qualitatively values of $\delta \simeq \frac{1}{20}-\frac{1}{40}$ (i.e., $\epsilon \approx \frac{1}{20}-\frac{1}{40}$) appears reasonable to account for the present data. We stress that a characteristic feature of the X mechanism is that $\sigma_h(s)$ should fall very slowly (much slower than $1/s$) up to the minimum point at $s = 1/\delta$ (rather than really being a constant), and then *rise* again.[14] The approximate "constancy" (within a factor of 2) should persist until $s \simeq 100$ or so, unless one experiences opening of new channels (like color[4]).

The X interaction should, in general, contribute to η, $\pi^0 \to e^+e^-$ decay and also possibly to $\eta \to \mu^+\mu^-$ decay[15] (if X is coupled to muon-quark currents) in order f^2. The observed partial decay width for the latter is roughly consistent with two-photon intermediate-state contribution. With $\epsilon \simeq \frac{1}{20}-\frac{1}{40}$ (as needed from the annihilation experiments), the X contribution to $\mu^+\mu^-$ decay is expected to be of order $(f^2/4\pi)(1/m_X^2) = \epsilon\alpha(1/\text{BeV}^2)$ which is roughly comparable to the two-photon contribution.

Turning to the elastic ep-scattering amplitude, the contributions T_γ and T_X from the one-photon and the V part of the X interaction [see Eq. (3)] are given by $T_\gamma = (e^2/s)t_\gamma$, $T_X = (f^2/m_X^2)t_X$, where

$$t_Y = \bar{u}_p[F_1{}^Y(s)\gamma_\mu + F_2{}^Y(s)\sigma_{\mu\nu}q_\nu]u_p(\bar{u}_e\gamma_\mu u_e) \qquad (Y=\gamma, X).$$

Since isoscalar and isovector form factors of the nucleon are similar, we expect $F_i{}^X(s) \propto F_i{}^\gamma(s)$; thus the net effect of adding the contribution of the V part of the X interaction to the one-photon contribution is to multiply the familiar expressions $|F_i{}^\gamma(s)|^2/s^2$ in the cross section by a factor $\varphi_{\text{el}}(s) = 1 + \Delta_{\text{el}}'s + \Delta_{\text{el}}{}^2 s^2$, where $\Delta_{\text{el}} = \epsilon F_i{}^X(s)/F_i{}^\gamma(s) \sim O(\epsilon)$ and $\Delta_{\text{el}}' = 2\Delta_{\text{el}}$ [the ratio $F_i{}^X(s)/F_i{}^\gamma(s)$ is expected to be a constant of order unity]. Since ϵ and therefore Δ_{el} are much less than unity, the X contribution will be important only at high $|s| \gtrsim 1/\Delta_{\text{el}}$ in units of $(\text{BeV})^2$. This remark is not altered by including the contributions from the A (S and P) parts of the X interaction as well.

As regards inelastic ep scattering [Fig. 1 (b)], consider first the contribution from the vector part of the X interaction. We expect the corresponding structure function $[W_i{}^{XX}(q^2,\nu)]_{i=1,2}$ (where $s=q^2$) and the mixed structure functions $W_i{}^{X\gamma}(q^2,\nu)$, which arise from interference of X and one-photon contributions, scale if the electromagnetic structure functions $W_i{}^{\gamma\gamma}(q^2,\nu)$ also scale and that they are proportional to each other [i.e., $W_i{}^{XX}(q^2,\nu) \propto W_i{}^{\gamma\gamma}(q^2,\nu) \propto W_i{}^{X\gamma}(q^2,\nu)$]. Thus, the net effect of including the contribution from the V part of the X interaction is to multiply the photon-generated functions $W_i{}^{(\gamma,\gamma)}(s,\nu)/s^2$ in the inelastic cross section by factors $\varphi_i(s) = 1 + \Delta_i{}'s + \Delta_i{}^2 s^2$, where $\Delta_i = \epsilon W_i{}^{XX}(s,\nu)/W_i{}^{\gamma\gamma}(s,\nu)$ and $\Delta_i{}' = \epsilon W_i{}^{X\gamma}(s,\nu)/W_i{}^{\gamma\gamma}(s,\nu)$, and both Δ_i and $\Delta_i{}'$ are constants of order ϵ. The "structure functions" deduced experimentally with conventional expressions should thus correspond to $\overline{W}_i(s,\nu) \equiv W_i{}^{\gamma\gamma}(s,\nu)\varphi_i(s)$, which do not scale if one *assumes* $W_i{}^{\gamma\gamma}(s,\nu)$ to scale. We refer to this as *apparent violation of scaling*, which tends to be significant for $|s| \gtrsim 1/\Delta_i = O(1/\epsilon)$. For example, if $\Delta_i{}'$ is *positive*, and $\Delta_i \simeq \Delta_i{}' \simeq \epsilon/2 \simeq \frac{1}{80}$ (say), the functions $W_i(s,\nu)$ (for fixed ω) should fall by about 15% as s decreases from -4 to -25 $(\text{BeV})^2$; eventually they should rise again for increasing $|s|$. Note that the fact that s is timelike for the e^-e^+ annihilation but spacelike for ep scattering provides a mechanism (via the linear terms δ's and Δ_i's) for the X effect to be significant for the former at present energies but not yet for the latter.

The contribution of the A (and S and P) terms may change the complexion of these remarks somewhat through the appearance of new structure functions, although the qualitative remark that the X effect will not be important until some appropriately high value of $|s| \sim 1/\epsilon$ should still hold. Note that these contributions, along with the associated interference terms to both elastic and inelastic scattering, can be distinguished from pure photon and V contributions by angular correlation and polarization measurements.

We now stress that if the photon and the X mechanisms are both operative, one expects to see a difference between e^-p and e^+p scatterings, because the photon and the V parts of the X interaction contribute with opposite signs for e^-p versus e^+p, while the A (S and P) terms contribute with the *same* sign. Hence, their interference term, which in general must exist and is relatively important in the regions $s \sim 1/\Delta_{\text{el}}$ and $s \sim 1/\Delta_i$, should lead to a measurable difference[15] between e^-p and e^+p processes. We therefore urge a *model-independent test of* X from comparisons of e^-p with e^+p.

The remarks made here about e^-e^+ annihilation and $e^\pm p$ scatterings apply to $\mu^-\mu^+$ annihilation and $\mu^\pm p$ scatterings as well, if the X's are coupled to muon-quark currents. The constants δ, Δ_i, etc. may, however, be different in the two cases, if the quark indices (in the corresponding currents) are different.[7]

Before concluding, we should remark that the X mesons with interactions as introduced here must arise in the class of gauge theories which attempt to unify[3,16] baryons and leptons and their gauge interactions. In some of the models within this class the X's are coupled to both $\bar{e}n$ and $\bar{\nu}_e p$ currents; in this case, the X's must be sufficiently heavy (so that $f^2/m_X^2 < G_{\text{Fermi}}$) and they are not relevant for the annihilation experiments at present energies. On the other hand, there are alternative possibilities[15] (for example, ν_e and ν_μ being linked to charmed quarks χ and χ' through X's) which permit the X's to be "light" as considered here. We have ignored such questions of a realistic model of all interactions in this note in view of the dramatic experimental feature of $\sigma_h(s)$. The X mechanism provides a simple explanation of the data and is consistent with known phenomena; it also offers several distinctive tests.

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[1]B. Richter (Report of SPEAR data), in Proceedings of the Conference on Lepton Induced Reactions, Irvine, California, 1973 (to be published).

[2]See J. D. Bjorken, SLAC Report No. 1318, 1973 (unpublished), for a review of this subject.

[3]J. C. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973), and Phys. Rev. Lett. 31, 661 (1973), and International Centre for Theoretical Physics Report No. IC/74/7, 1974 (unpublished), and Phys. Rev. D (to be published).

[4]Some possibilities of this kind involving color are mentioned in the last paper of Ref. 3.

[5]Short-range interactions in question (leading to $e^-e^+ \to q\bar{q}$, etc.) may arise in a variety of ways—for example, via exchange of heavy spin-0 or spin-1 mesons in the s, t, or u channel, all of which lead to identical behavior for $\sigma_h(s)$ for large s. They may, however, be distinguished from each other by a combination of other tests discussed in the text.

[6]The possible relevance of the X particles postulated by us earlier in the context of baryon-lepton unification (Ref. 3) for e^-e^+ annihilation was first pointed out in the discussion following the report by Richter (Ref. 1).

[7]The $(\mathcal{P}, \mathfrak{N}, \lambda)$ index of q for the muon-quark current may be different from that of the electron-quark current, which is forced in a number of gauge models.

[8]The X contribution $(\delta\kappa_X^{\,l})$ to the anomalous magnetic moments of leptons via the (naive) triangle graph is $(f^2/12\pi^2)(m_q m_l/m_X^2)$, where $l = e$ or μ (assuming $m_X \gg m_q$). This has the right sign and magnitude with $\epsilon \simeq 1/50$ (say) not to spoil the agreement between experiment and QED theory provided m_q is small $\lesssim 500$ MeV [small m_q is suggested at least from chiral SU(2) considerations]. The X interaction [Eq. (3)] also contributes to hfs splitting; the change is nearly $10 g_q^A$ parts per million for $\epsilon \simeq 1/50$ [$g_q^A \bar{u}_p \gamma_\mu\gamma_5 u_p \equiv \langle p|\bar{q}\gamma_\mu\gamma_5 q|p\rangle$; thus g_q^A is of order 1 or $\sin^2\theta_{\rm Cabibbo}$ depending upon whether q is the (unrotated) $\mathfrak{N}$ or λ field]. Reduction of errors in κ^l and hfs splitting should thus be important in testing the validity of the X interaction. We thank G. Feinberg for pointing out the importance of the X effect on hfs splitting.

[9]Pure leptonic processes $e^-e^+ \to e^-e^+$ and $e^-e^+ \to \mu^-\mu^+$ receive contributions from the X mechanism in fourth order, which (within a renormalizable theory) would be of order $(f^2/4\pi)^2(1/m_X^2)$. Form factors at the lepton-quark–X-meson vertices may however produce extra convergence factors for the box diagram, in which case the contribution will be suppressed by additional factors of order s/m_X^2 (compare similar suppression for two-photon contribution to e^-p scattering). Without such suppression, the present validity of QED tests for leptonic processes may limit $f^2/4\pi \lesssim \frac{1}{5}$.

[10]For convergent loop integrals (which applies to Fig. 1 within a renormalizable scheme) the effective range of integration for t is much less than m_X^2 if $s \ll m_X^2$.

[11]The familiar restriction on the form of the hadronic tensor, i.e., $T_{\mu\nu}^{em}(q) = \int d^4x\, e^{iq\cdot x}\langle 0|V_\mu^{em}(x/2)V_\nu^{em}(-x/2)|0\rangle = \rho_{\gamma\gamma}(q^2)(q_\mu q_\nu - g_{\mu\nu}q^2)$, which follows from current conservation, also applies (at least to a good approximation) when $V_\mu^{em}V_\nu^{em}$ is replaced by $V_\mu^X V_\nu^X$ and $V_\mu^{em}V_\nu^X + V_\mu^X V_\nu^{em}$.

[12]The isospin and SU(3) structures of V_μ^X are determined by the $(\mathcal{P}, \mathfrak{N}, \lambda)$ indices of q in Eq. (1).

[13]The only exception is the A-P interference (involving, for example, a pion final state), which is proportional to the mass of the electron.

[14]Of course the linear growth of $\sigma_h(s)$ is expected to be damped at sufficiently high $s \sim m_X^2$ because of momentum dependence of propagator and other effects.

[15]Possible relevance of the X mechanism to $\eta \to \mu^+\mu^-$ decay was remarked upon by M. Gell-Mann, in Proceedings of the Conference on Lepton Induced Reactions, Irvine, California, 1973 (to be published).

[16]The two-photon contribution to this difference is considerably smaller (by order of magnitude) than the X contribution.

[17]See the last paper in Ref. 3.