Energy Dependence of Factorizable Models for Elastic Scattering

R. Henzi* and B. Margolis*
Department of Physics, McGill University, Montreal, Canada
and
P. Valin*
Department of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 11 February 1974)

We discuss properties of high-energy p-p scattering in terms of models where the eikonal is a function of energy times a function of impact parameter (factorizable models). Using the eikonal determined from the data at one fixed energy, we deduce that the secondary diffraction maximum should rise with increasing energy if the total cross section rises. Further data, from National Accelerator Laboratory energies and up, would be of interest to decide for or against factorizable models.

Some typical general features of factorizable eikonal models are discussed here, with specific reference to proton-proton scattering in the intersecting-storage-rings (ISR) energy range. The eikonal in such models, $i\Omega(b, s)$, is given as the product of a function $g(b)$, depending on impact parameter b, and a function $w(s)$, depending on the square of the c.m. energy, i.e.,

$$\Omega(b, s) = w(s)g(b). \tag{1}$$

Neglecting spin effects, $d\sigma/dt = |i\sqrt{\pi}F(s, t)|^2$ is given in terms of Ω by

$$F(s, t) = \int_0^\infty b\,db(1 - e^{-\Omega(b,s)})J_0(b\sqrt{-t}), \tag{2}$$

where $-t$ is the square of the c.m. momentum transfer and J_0 is the cylindrical Bessel function of order zero. We make the assumptions that spin effects are negligible and that $F(s, t)$ is dominantly real (except perhaps near the diffraction zero).

The main result of our study is concerned with the energy dependence of the secondary diffraction maximum observed in p-p scattering at the ISR. The result is that present experimental information from the ISR on increasing p-p total cross sections[1,2] and diffractive structure[3-5] in elastic scattering at a given *fixed* energy, assuming a factorizable eikonal, implies an increase of about a factor of 2 of the secondary diffraction maximum in $d\sigma/dt$ from the bottom to the top of the ISR energy range. This situation should not change appreciably with the addition of a modest phase to $F(s, t)$. This provides a definite experimental test for the validity of factorizable models.

Before proceeding to the details of our argument, we give a brief historical review concerning the origin of factorizable models. Froissart,[6] in the derivation of his famous bound, gave an intuitive picture of how the amplitude is bounded as the energy increases: Strong interactions at most allow for an *energy-independent* absorptive Yukawa potential with an *energy-dependent* coupling constant, the latter increasing with increasing energy. This is equivalent to a factorizable eikonal with increasing opaqueness Ω. Durand and Lipes,[7] in their version of the Chou-Yang model,[8] have mentioned the possibility that the eikonal in this model might be energy dependent and separable. In the light of the present data, Byers,[9] Kac,[10] and Hayot and Sukhatme[11] more recently have put forward this latter hypothesis explicitly, with the *opaqueness* Ω *being postulated as an increasing function* through the ISR energy range, in order to account for an increasing total cross section. Cheng, Walker, and Wu,[12] on the basis of field theoretical considerations, have given arguments in an attempt to justify an increasing opaqueness. Their model, which is a factorizable one, differs from the Chou-Yang–type model in the form of the impact-parameter–dependent factor $g(b)$ in the eikonal.

Factorizability of the eikonal clearly implies constancy, in impact-parameter space, of the ratio of the eikonals at two different energies. There have been studies concerned with direct determination from data[13-15] of the eikonal or related quantities. Although there are experimental uncertainties, these studies do not seem to support factorizability.

The increase of the secondary diffraction maximum, pointed out above as our main result, manifests itself in any of the studies in Refs. 9–12, which are concerned, to various degrees of accuracy, with accounting for present ISR data. It showed first in the Cheng, Walker, and Wu[12] model, though this model produces a "shoulder" rather than a pronounced diffraction maximum. It was conjectured in Ref. 11 that the increase of the secondary diffraction maximum might hold more generally than in the particular model considered by the authors.

The details of our analysis are as follows. We consider the partial derivative of $F(s,t)$ with respect to s,

$$\partial F(s,t)/\partial s = [d\ln w(s)/ds]\,\varphi(s,t), \tag{3}$$

$$\varphi(s,t) = \int_0^\infty b\,db\,\Omega e^{-\Omega} J_0(b\sqrt{-t}). \tag{4}$$

Since $d\sigma/dt$, through the ISR energy range, changes slowly with energy, the first partial derivative of $F(s,t)$ with respect to s is the essential quantity determining the energy variation of the differential cross section. Furthermore, knowing $\Omega(b,s)$ at *one* given value of s we can, using (3) and (4), determine some qualitative momentum-transfer characteristics of the modest energy dependence of $d\sigma/dt$. We have

$$\frac{d\sigma}{dt}(s+\Delta s,t) \simeq \frac{d\sigma}{dt}(s,t)\left(1 + 2\,\frac{\Delta F(s,t)}{F(s,t)}\right), \tag{5}$$

$$\frac{\Delta F(s,t)}{F(s,t)} = \Delta s\,\frac{d\ln w(s)}{ds}\,\frac{\varphi(s,t)}{F(s,t)}. \tag{6}$$

Using Eqs. (5) and (6), we can determine the crossover points of $d\sigma(s,t)/dt$ and $d\sigma(s+\Delta s,t)/dt$ in a factorizable model. To do this, we require a knowledge of $\Omega(b,s)$ *at one particular value of s*. To determine $d\sigma(s+\Delta s,t)/dt$ in terms of $d\sigma(s,t)/dt$, $w(s)$ has to be known in addition.

We now consider the evaluation of $\varphi(s,t)$, using the analysis of Henzi and Valin.[13] These authors give the opaqueness $\Omega = -\frac{1}{2}\ln(1 - d^2\sigma_{\text{in}}/db_x\,db_y)$, where $d^2\sigma_{\text{in}}/db_x\,db_y$ is the inelastic differential cross section in impact-parameter space. This provides a choice of $\Omega(b,s)$ yielding a detailed fit to $d\sigma/dt$ for ISR data at each s over a substantial range of t. Any other description providing a detailed fit, we would consider to be adequate. At $s = 1070$ GeV^2, for instance, the resulting forms for $\Omega(b,s)\exp[-\Omega(b,s)]$ and $b\Omega(b,s)\times\exp[-\Omega(b,s)]$ are shown in Fig. 1. The function $\varphi(s,t)$, which follows from this form of $\Omega(b,s)$, has been evaluated numerically and is shown in Fig. 2.

It is to be noted that

$$\varphi(s,t) = \begin{cases} >0 \text{ for } 0 \geq t > t_1 = -0.50\ (\text{GeV}/c)^2, \\ <0 \text{ for } t_1 > t > t_2 = -4.85\ (\text{GeV}/c)^2. \end{cases} \tag{7}$$

The function $\varphi(s,t)$ shows zeros at t_1 and t_2,

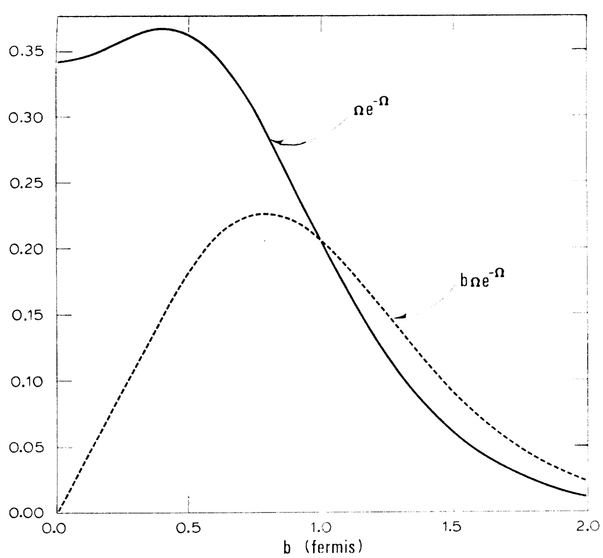

FIG. 1. $\Omega(b,s)\exp[-\Omega(b,s)]$ and $b\Omega(b,s)\exp[-\Omega(b,s)]$ for $s = 1070$ GeV^2, according to the analysis of Ref. 13; ordinate units are dimensionless.

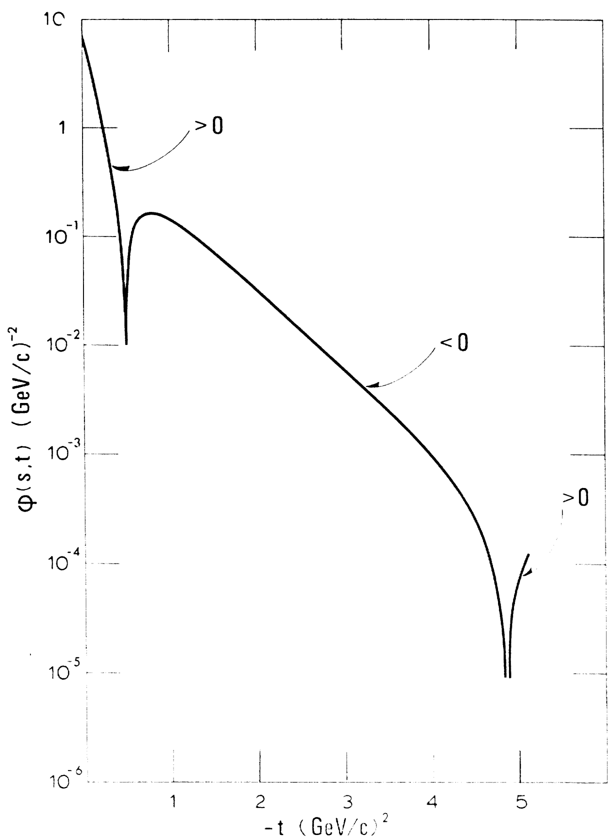

FIG. 2. $\varphi(s,t) = \int_0^\infty b\,db\,\Omega(b,s)\exp[-\Omega(b,s)]\,J_0(b\sqrt{-t})$ for $s = 1070$ GeV^2, according to the input shown in Fig. 1.

these points determining, in the approximation considered, crossovers of $d\sigma(s,t)/dt$ for *any pair of values of* s in the ISR energy range. To discuss the effect of $\varphi(s,t)$ through the entire t range, we assume that the diffraction dip near $t=t_0=-1.4$ (GeV/c)^2 is due to a zero in the real part of $F(s,t)$ and that, as already mentioned above, the finite experimental value of $d\sigma/dt$ in the diffraction dip is accounted for by the imaginary part of $F(s,t)$, the latter being negligible everywhere else. Two cases have now to be distinguished: case I,

$$\mathrm{Re}F(s,t)\equiv\begin{cases}u(s,t)>0, & 0\geq t>t_0,\\ u(s,t)<0, & t_0>t;\end{cases}$$

and case II,

$$\mathrm{Re}F(s,t)=\begin{cases}u(s,t), & 0\geq t>t_0,\\ -u(s,t), & t_0>t.\end{cases}$$

For case I (II), Re$F(s,t)$ has an odd- (even-)order zero at $t=t_0$. The results of Ref. 13, on which the present analysis is based, are in category I.

Case I and $\Delta s>0$.—It follows from Eqs. (5) and (6) for $d\sigma(s+\Delta s,t)/dt$, combined with the behavior (7) of $\varphi(s,t)$, given that $w(s)$ increases with increasing s (to account for the increasing total cross section), that for $0\leq t<t_1$, the differential cross section rises as s increases. At $t=t_1$, $d\sigma(s+\Delta s,t)/dt$ crosses below $d\sigma(s,t)/dt$ and for $t_1>t>t_0$, the differential cross section falls, merging roughly with the differential cross section at other energies at their more or less common diffraction dip near $t=t_0$. The differential cross section $d\sigma(s+\Delta s,t)/dt$ rises again with increasing Δs as one approaches the secondary maximum and finally crosses again below $d\sigma(s,t)/dt$ at or near $t=t_2$.

Case II.—For this case the *same* situation holds, since when $u(s,t)\to -u(s,t)$, $\varphi(s,t)\to -\varphi(s,t)$. There still remains the alternative to be considered, that the diffraction dip might be due to a relative minimum of Re$F(s,t)$, with Re$F(s,t_0)>0$, as for instance in the work of de Groot and Miettinen.[16] This case is qualitatively not different from case II and similar results for the energy behavior of $d\sigma/dt$ are expected.

In order to get some quantitative estimate for the increase of the secondary diffraction maximum through the ISR energy range, we use for the energy dependence $w(s)$ of the eikonal

$$w(s)\propto s^{\epsilon}, \tag{8}$$

where[12] $\epsilon=0.082$. Taking, furthermore, $s=1070$ GeV^2 and $\Delta s=2380$ GeV^2, which corresponds to the full ISR energy range, we find at $t=-1.8$ (GeV/c)^2, where the secondary diffraction maximum of $d\sigma/dt$ occurs, from Eqs. (5) and (6), an increase of a factor of about 2.

Such an increase is substantial and can be checked against present and future measurements. There is no indication in the present ISR data of such energy dependence. However large $-t$ measurements are not available at the bottom of the ISR energy range. Such measurements then are of interest, both at the ISR and at the National Accelerator Laboratory.

We note finally that the above considerations are of importance through the full ISR energy range only if Regge effects from lower-lying trajectories have disappeared. The behavior of the differential cross sections at small $-t$ values, specifically the small shrinkage rate, indicates that this is so. It is expected then that this will also be the case for the range of t values considered in this study.

*Work supported in part by the National Research Council of Canada and the Quebec Department of Education.

[1]U. Amaldi *et al.*, Phys. Lett. 44B, 112 (1973).

[2]S. R. Amendolia *et al.*, Phys. Lett. 44B, 119 (1973).

[3]G. Barbiellini *et al.*, Phys. Lett. 39B, 663 (1972).

[4]A. Böhm *et al.*, to be published. These data at larger $-t$, in which a diffraction dip is apparent at $t=-1.4$ (GeV/c)^2, are available in *Proceedings of the Sixteenth International Conference on High Energy Physics, The University of Chicago and National Accelerator Laboratory, 1972,* edited by J. D. Jackson and A. Roberts (National Accelerator Laboratory, Batavia, Ill., 1973). Here, we base our work on these data as presented by J. C. Sens, in Proceedings of the Conference on Recent Advances in Particle Physics, New York, 15–17 March, 1973 (to be published).

[5]Sens, Ref. 4.

[6]M. Froissart, Phys. Rev. 123, 1053 (1961).

[7]L. Durand and R. Lipes, Phys. Rev. Lett. 20, 637 (1968).

[8]T. T. Chou and C. N. Yang, Phys. Rev. 170, 1591 (1962).

[9]N. Byers, in Proceedings of the Eleventh International School of Subnuclear Physics "Ettore Majorana," Erice, 6–26 July 1973 (to be published).

[10]M. Kac, Oxford University Report No. 33/73, 1973 (unpublished).

[11]F. Hayot and U. P. Sukhatme, University of Michigan Randall Laboratory Report No. UM HE 73-31, 1973 (unpublished).

[12]H. Cheng, J. K. Walker, and T. T. Wu, Phys. Lett. 44B, 97 (1973).

[13]R. Henzi and P. Valin, to be published.
[14]U. Amaldi, J. Phys. (Paris), Colloq. 34, C1-241 (1973).
[15]A. W. Chao and C. N. Yang, Phys. Rev. D 8, 2063 (1973).
[16]E. H. de Groot and H. I. Miettinen, Rutherford High Energy Laboratory Report No. RL 73-003, 1973 (unpublished).

Solitary Waves in Nonlinear Field Theories

Philip B. Burt
Department of Physics and Astronomy, Clemson University, Clemson, South Carolina 29631
(Received 28 January 1974)

Solitary wave solutions of field equations of the form $\partial_\mu\partial^\mu\varphi + m^2\varphi + \alpha\varphi^{2p+1} + \lambda\varphi^{4p+1} = 0$, with $p \neq 0, -\frac{1}{2}, -1$, are developed. For vanishing interaction the solutions reduced to positive- or negative-frequency plane-wave solutions of the Klein-Gordon equation.

Solitary wave solutions of nonlinear field equations have been studied in several areas of physics recently.[1,2] In this note solitary wave solutions of a class of field equations for systems with polynomial self-interactions are developed. These solutions reduce to positive- or negative-frequency plane-wave solutions of the Klein-Gordon equation in the limit of vanishing coupling constants.

The field equations considered here have the form

$$(\partial_\mu\partial^\mu + m^2)\varphi + \alpha\varphi^{2p+1} + \lambda\varphi^{4p+1} = 0, \tag{1}$$

with $p \neq 0, -\frac{1}{2}, -1$. For $p = \frac{1}{2}$ Eq. (1) is a scalar analog of the massive Yang-Mills field equation, while for $\alpha = 0$, $p = \frac{1}{2}$, it becomes the field equation of the familar $\lambda\varphi^4$ theory.

Solitary wave solutions of Eq. (1) are

$$\varphi_{p\vec{k}}^{(\pm)} = A^{(\pm)} e^{\mp i\check{k}\cdot\check{x}} \left[\left(1 - \frac{\alpha A^{(\pm)2p} e^{\mp 2ip\check{k}\cdot\check{x}}}{4m^2(p+1)}\right)^2 - \frac{\lambda A^{(\pm)4p} e^{\mp 4ip\check{k}\cdot\check{x}}}{4m^2(2p+1)}\right]^{-1/2p}, \tag{2}$$

where

$$\check{k} = (k_0, \vec{k}), \tag{3}$$

$$\check{k}\cdot\check{k} = k_0^2 - k^2 = m^2, \tag{4}$$

and A is a constant. While the solutions given in Eq. (2) can be verified by differentiation, the steps involved are rather lengthy. However, the result can be established in the following way. Let

$$y = \check{k}\cdot\check{x}. \tag{5}$$

Then, using Eqs. (3)–(4), Eq. (1) becomes

$$d^2\varphi/dy^2 + \varphi + \alpha\varphi^{2p+1}/m^2 + \lambda\varphi^{4p+1}/m^2 = 0. \tag{6}$$

Multiplying by $d\varphi/dy$ and integrating, one has

$$\tfrac{1}{2}(d\varphi/dy)^2 + \tfrac{1}{2}\varphi^2 + \alpha\varphi^{2p+2}/(2p+2)m^2 + \lambda\varphi^{4p+2}/(4p+2)m^2 = \tfrac{1}{2}B, \tag{7}$$

where B is a constant. This equation can be separated to give

$$\int dz[B - z^2 - \alpha z^{2p+2}/(p+1)m^2 - \lambda z^{4p+2}(2p+1)m^2]^{-1/2} = \pm y. \tag{8}$$

For the special case $B = 0$, let

$$q = z^{2p}, \tag{9}$$

$$dq = 2pz^{2p-1}dz. \tag{10}$$

Equation (8) becomes

$$\int dq\, q^{-1}[1 + \alpha q/(p+1)m^2 + \lambda q^2/(2p+1)m^2]^{-1/2} = \pm 2ipy. \tag{11}$$