High-Energy Behavior of Non-Abelian Gauge Theories*

H. T. Nieh
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

and

York-Peng Yao
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790, and Department of Physics, University of Michigan,† Ann Arbor, Michigan 48104
(Received 21 March 1974)

The fermion-fermion scattering amplitude in non-Abelian gauge theories is calculated in the approximation of keeping only the leading logarithmic terms at high energy, up to eighth order in the coupling constant. The high-energy behavior of the calculated scattering amplitude is drastically different from the results obtained in other renormalizable field theories, e.g., the φ^3 theory and quantum electrodynamics.

In this Letter we report the results of a calculation on the high-energy behavior of non-Abelian gauge theories.[1] The calculation is carried out for the fermion-fermion scattering up to eighth order in the coupling constant, in the approximation of keeping only the leading logarithmic terms at each order. The details of this work will be published elsewhere.

In the past few years we have witnessed great efforts made to study the high-energy behaviors of various renormalizable field theories. The hope has been that from these studies some general features may be abstracted to make contact with high-energy hadron physics. Among the field theories studied are the φ^3 theory[2] and quantum electrodynamics (QED).[3] The results of a QED calculation by Cheng and Wu[3] are of particular interest: firstly, because a rising total cross section is predicted, in agreement with a recent Pisa–Stony Brook experiment at CERN,[4] and secondly because QED is a *gauge* theory and therefore may be of fundamental importance.

An immediate important question is whether the salient features of QED are also possessed by the more realistic non-Abelian gauge theories. Additional impetus for studying the high-energy behavior of non-Abelian gauge theories comes from recent activities in constructing renormalizable models for weak and electromagnetic interactions,[5] as well as the observation that non-Abelian gauge theories are asymptotically free.[6] These are the motivations for our present study.[7]

We consider the scattering of two fermions $p_1+p_2\rightarrow p_3+p_4$ with $s=(p_1+p_2)^2\rightarrow\infty$ while keeping $t=(p_1-p_3)^2$ fixed. For definiteness, we shall consider the symmetry group SU(2); generalization to other symmetry groups is trivial. The Lagrangian density is

$$\mathcal{L}=\bar{\psi}[\gamma^\mu(i\partial_\mu+g\tfrac{1}{2}\vec{\tau}\cdot\vec{A}_\mu)-m]\psi-\tfrac{1}{4}(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu+g\vec{A}_\mu\times\vec{A}_\nu)^2. \tag{1}$$

To avoid the infrared problem, we introduce a complex scalar doublet and invoke the Higgs mechanism[8] to give masses to the vector mesons.[9] As it turns out, the Higgs particles and the Faddeev-Popov ghosts,[10] to the leading order in ln$s$, do not contribute. The net effect is the same as if one puts in by hand an infrared cutoff to the vector-meson propagator. The calculation is carried out in the Feynman gauge. The results are the following:

(I) The dominant fourth-order diagrams are the two-vector-exchange diagrams [Fig. 1(a)]. The

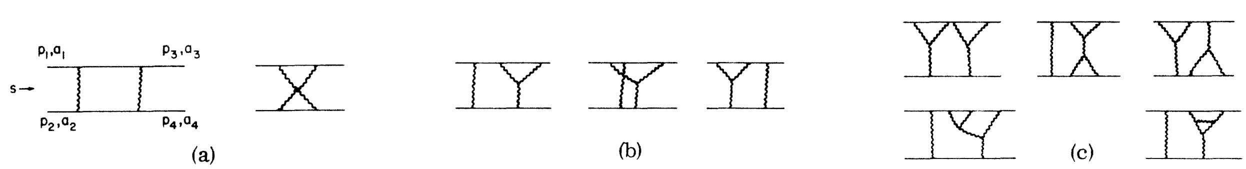

FIG. 1. (a) The dominant fourth-order diagrams. (b) The dominant sixth-order diagrams are these, plus those obtained by $s\leftrightarrow u$ crossing and reflecting through a horizontal plane mirror. (c) The dominant eighth-order diagrams are these, plus those obtained by $s\leftrightarrow u$ crossing, mirror-reflecting, and distinctly permuting the vector legs on the fermion lines.

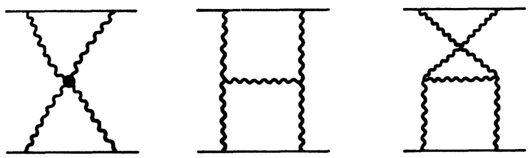

FIG. 2. A set of sixth-order diagrams, of which each has a leading behavior of s^2. This power cancels in the sum.

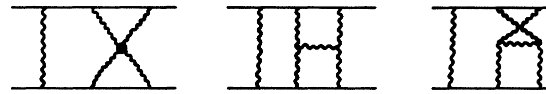

FIG. 3. A set of eighth-order diagrams, of which each has a leading behavior comparable to those of Fig. 1(c). However, this behavior is canceled in the sum.

leading diagrams in sixth and eighth orders are those that are obtained by adding vector-meson lines to give radiative effects to the vector-fermion-fermion vertices of the dominant fourth-order diagrams, and by permuting the vector legs on the fermion lines in all possible distinct ways. The relevant diagrams in g^6 and g^8 are shown in Figs. 1(b) and 1(c). We expect that the dominant diagrams in still higher orders are obtained through the same procedure.

(II) In the sixth and eighth orders, there are diagrams which individually increase as a power of s. Such power dependence on s disappears, however, when these diagrams are combined (Fig. 2). Invariably, the cancelation occurs as a result of the correlation between the three-vector and four-vector vertices, a correlation reflecting the gauge symmetry and renormalizability of the theory. A more important class of cancelations first appears in eighth order (Fig. 3), where each individual diagram has the same leading behavior as those of Fig. 1(c). When collected together, the leading $\ln s$ terms disappear. Beyond this we have not observed any striking cancelations of the logarithmic terms as is the case in QED.

(III) Let us write the scattering amplitude in the form

$$\langle p_3, p_4|(S-1)|p_1, p_2\rangle = iN(2\pi)^4\delta(p_1+p_2-p_3-p_4)T, \tag{2}$$

where N is a normalization factor. The invariant amplitude T can be decomposed into an isospin-nonflip part (T^{nf}) and an isospin-flip part (T^{f}):

$$T = \delta_{a_1a_3}\delta_{a_2a_4}T^{\mathrm{nf}} + \vec{\tau}_{a_1a_3}\cdot\vec{\tau}_{a_2a_4}T^{\mathrm{f}}, \tag{3}$$

where a_i is the isospin index for the fermion with momentum p_i. We find from our calculation that[11]

$$T^{\mathrm{nf}} = i3\pi(16\pi^2)^{-1}(g/2)^4(s/m^2)K(t)[1 - 4(g^2/16\pi^2)\ln^2 s + \tfrac{5}{2}(g^2/16\pi^2)^2\ln^4 s \mp \dots], \tag{4}$$

and

$$T^{\mathrm{f}} = 4(16\pi^2)^{-1}(g/2)^4(s/m^2)\ln s\, K(t)[1 - \tfrac{2}{3}(g^2/16\pi^2)\ln^2 s + \tfrac{19}{60}(g^2/16\pi^2)^2\ln^4 s \mp \dots], \tag{5}$$

where (μ^2 being the vector-meson mass)

$$K(t) = \int\frac{d\alpha_1\, d\alpha_2\,\delta(1-\alpha_1-\alpha_2)}{\alpha_1\alpha_2(-t)+\mu^2-i\epsilon} = \frac{1}{\pi}\int\frac{d^2q_\perp}{(q_\perp{}^2+\mu^2-i\epsilon)[(q_\perp+k)^2+\mu^2-i\epsilon]}, \quad -t = k_\perp{}^2, \tag{6}$$

is the effective potential due to two-particle exchange. We note the following features of Eqs. (4) and (5): (a) The series alternate in signs. (b) The nonflip amplitude is imaginary, while the flip amplitude is real. (c) The series are power series in $g^2\ln^2 s$; the origin of this behavior will be discussed later.

We first note the striking qualitative differences, in the approximation of keeping only the leading logarithmic terms in each order of perturbation calculation, between the high-energy behavior of non-Abelian gauge theories on the one hand and that of QED and the φ^3 theory on the other. Recall that the dominant diagrams in QED and the φ^3 theory are the ladder diagrams, the summation of which results in an *absolute series* in $g^2\ln s$. In contrast, the dominant diagrams in non-Abelian gauge theories are those in Fig. 1, resulting in an *alternating series* in $g^2\ln^2 s$. The underlying reason for this difference has to do with the origin of the $\ln s$ factors. For QED and the φ^3 theory, the leading $\ln s$ factors arise from integrations in longitudinal phase space.[12] In non-Abelian gauge theories, however, because of the form of the three-vector coupling, there is not enough transverse momentum cutoff for those diagrams in Figs. 1(b) and 1(c) and, as a consequence, integrations in longitudinal and transverse phase space both contribute $\ln s$ factors.

Another distinctive feature of the results (4) and (5) is the factorizability of the t dependence. In QED and the φ^3 theory, the result is actually a series in $g^2K(t)\ln s$, which sums to a moving Regge pole in t. Here, the series will sum to a modified s dependence which is fixed in t.

As we have only a limited number of terms in the series, it is futile to speculate on the possible forms of the eventual sums. The fact that the series alternate in signs makes it difficult even to make tentative physical interpretations of the results. It is possible that we may have a case here where the leading-logarithm approximation does not reflect the true high-energy behavior of the theory. For example,[13] to the order considered, T^{nf} can be put in the form

$$T^{\mathrm{nf}} = i3\pi(16\pi^2)^{-1}(g/2)^4(s/m^2)K(t)\left[1+4(g^2/16\pi^2)\ln^2 s+\tfrac{27}{2}(g^2/16\pi^2)^2\ln^4 s+\dots\right]^{-1}, \tag{7}$$

which may become insignificant compared to the nonleading logarithmic contributions.

What is significant from this study is that the high-energy behavior of non-Abelian theories can be drastically different from that of other renormalizable field theories, e.g., QED and the φ^3 theory. In view of the possibility that non-Abelian gauge theories may provide a framework for constructing realistic models, the difficult problem of finding the high-energy behavior of such theories is of great importance. We have taken the initial step toward understanding this problem. No doubt, much remains to be done. What is urgently needed, we feel, is probably a fresh new method of attacking the problem.

We wish to thank Professor C. N. Yang for his encouragement, and Professor B. McCoy and Professor T. T. Wu for telling us their unpublished results. One of us (Y.-P.Y.) would like to express his gratitude to Professor C. N. Yang and members of the Institute for Theoretical Physics for their hospitality.

---

*Research supported in part by the National Science Foundation under Grant No. GP-32998X.

†Permanent address: Research supported in part by the U. S. Atomic Energy Commission.

[1]C. N. Yang and R. Mills, Phys. Rev. 96, 191 (1954).

[2]B. W. Lee and R. F. Sawyer, Phys. Rev. 127, 2266 (1962); S. J. Chang and T. M. Yan, Phys. Rev. Lett. 25, 1586 (1970); B. Hasslacher, D. K. Sinclair, G. M. Cicuta, and R. L. Sugar, *ibid.* 25, 1591 (1970).

[3]H. Cheng and T. T. Wu, Phys. Rev. Lett. 24, 1456 (1970), and earlier papers quoted therein.

[4]S. R. Amendolia *et al.*, Phys. Lett. 44B, 119 (1973).

[5]For a review, see, e.g., B. W. Lee, in *Proceedings of the Sixteenth International Conference on High Energy Physics, The University of Chicago and National Accelerator Laboratory, 1972,* edited by J. D. Jackson and A. Roberts (National Accelerator Laboratory, Batavia, Ill., 1973).

[6]D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); G. t'Hooft, unpublished.

[7]B. McCoy and T. T. Wu are currently engaged in calculating the vector-vector scattering in non-Abelian gauge theories. Their calculation, up to sixth order, is in qualitative agreement with our finding reported here.

[8]P. W. Higgs, Phys. Lett. 12, 132 (1964), and Phys. Rev. Lett. 13, 508 (1964); F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Lett. 13, 585 (1965).

[9]For the Feynman rules, see, e.g., G. t'Hooft, Nucl. Phys. B35, 167 (1971).

[10]L. D. Faddeev and V. N. Popov, Phys. Lett. 25B, 29 (1967).

[11]We primarily work in α space to extract the high-energy behaviors of diagrams. However, to bring out the physics more clearly, we occasionally work directly in momentum space. Needless to say, these two methods give the same result.

[12]In the radiative graphs, renormalization effects also lead to ln$s$ dependence. However, in QED they cancel when gauge-invariant sets are grouped together. See, e.g., Y.-P. Yao, Phys. Rev. D 8, 2316 (1970).

[13]This is suggested by a positivity requirement.