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Nucleation of Vortices in the Superconducting Mixed State: Nascent Vortices*

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Evidence is presented that the mixed-state order parameter at surfaces parallel to the magnetic field is strongly modulated. The minima of this modulation act as the nucleation and denucleation sites for vortices.

To change the number of vortices in a superconducting sample a finite change in magnetic field from the thermal equilibrium value is necessary to overcome any vortex entry or exit barrier. The metastable state in which the field is above the equilibrium value is referred to as a *superheated* state, and that in which it is below as a *supercooled* state. The maximum superheating field of the Meissner state has been calculated for the case of one-dimensional instabilities using the Ginzburg-Landau equations.[1,2] The case where two-dimensional fluctuations of the order parameter are permitted has also been treated,[3,4] and the appearance of large modulations in the order parameter at the surface prior to vortex nucleation has been suggested.[3] A recent calculation[5] of the breakdown of the superheated Meissner state studies the global stability in two dimensions and numerically investigates various Ginzburg-Landau solutions. Solutions involving large periodic variations in the order parameter are found which resemble a structure postulated earlier[6] to explain certain experimental results in the mixed state.

Superheating (or supercooling) in the mixed state is much harder to treat theoretically because of the large variations of the order parameter in the sample interior, but it is the simpler situation experimentally. An early treatment of the barrier to vortex entry (or exit) by Bean and Livingston[7] considered two magnetic forces acting on a vortex located near and parallel to the surface of the superconductor. The first force is directed toward the interior and is the repulsion by the external field. (This force on the vortex lattice keeps it away from the surface and causes the order parameter in the "surface sheath" to be considerably higher than the average value in the bulk.) Very close to the surface another force in the opposite direction, due to the image vortex located outside the surface, tends to dominate. There is thus a distance from the surface at which the vortex energy would be at a maximum. This provides a barrier to vortex penetration.

At higher magnetic fields, the field gradients and magnetic forces are smaller because of the reduced order parameter and a purely magnetic treatment is not a good approximation. The Ginzburg-Landau equations for a normal-state interior have been solved assuming a constant order parameter at the surface.[8] A large order parameter at the surface is still found (and the material near the surface remains superconducting to $H_{c3} = 1.69H_{c2}$). There has been no complete Ginzburg-Landau treatment for the mixed state in the presence of a surface. However, Fink[9] has treated

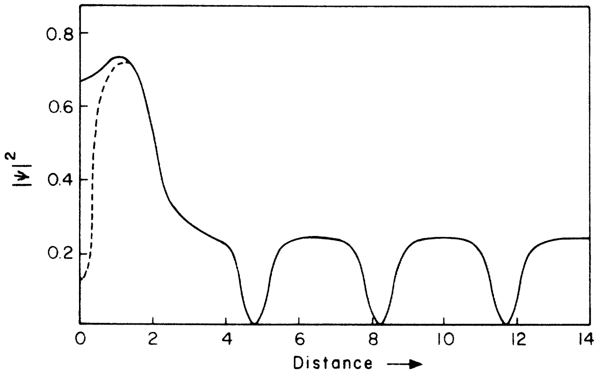

FIG. 1. The order parameter $|\psi|^2$ as a function of distance into the superconductor in units of a coherence length for $H = 0.6H_{c2}$ and $\kappa = 4$. The dashed line indicates the order-parameter minima (nascent vortices) at the surface.

the problem by joining the extrapolated solution for the surface superconducting state[8] to a solution for the mixed state in an infinite superconductor. The solid curve in Fig. 1 is adapted from this calculation.

We will discuss a modification of this picture which seems a reasonable extrapolation of Kramer's calculations[3,5] and which enables us to explain a wide range of experimental results quite naturally. Much of Fink and Kessinger's results appear to remain valid in spite of the proposed modification. We will first state the proposed nature of the order parameter at the surface and its behavior at the onset of vortex entry (nucleation) and exit (denucleation). We will then give the experimental evidence for this model.

We assume that in thermal equilibrium in the mixed state both the order parameter and the magnetic field at the surface are periodically modulated in the direction perpendicular to the magnetic field. (One would expect at least a small amount of such modulation *a priori* since the existence of the vortex structure in the interior would have some effect on the current density and order parameter at the surface. Because the current density at the surface is very close to the critical value, even a "gentle" perturbation by the vortex lattice might be expected to have a large effect.) The dashed line in Fig. 1 represents a minimum of this assumed modulation of the order parameter. The modulation would probably have the periodicity of the interior vortex lattice, at least for H well below H_{c2}. The order parameter at the modulation minima does not necessarily decrease to zero but is apparently quite low. We call these lines of reduced surface order parameter and increased magnetic field penetration "nascent vortices." These vortexlike structures need not have quantized fluxoid,

$$\oint (m\vec{v} + 2e\vec{A}/c)\cdot d\vec{l},$$

or zero order-parameter minima since they are *at* the surface and there is no complete path around them in superconducting material.

If the applied field is increased somewhat from its equilibrium value, no vortices will pass through the surface barrier to enter the interior, but the increased magnetic pressure on the interior vortex lattice will cause it to move away from the surface. The spacing of the nascent vortices will not change because the interior lattice spacing is unchanged. The nascent vortices will adjust to the increased field by an increase of shielding current and consequent lowering of order parameter. At some increase of the field above equilibrium, the order parameter at the nascent-vortex minimum reaches zero. Now, for the first time, the magnetic pressure on the nascent vortex can displace a net amount of fluxoid deep into the sample interior. The quantization of fluxoid requires a zero order-parameter core. For even a very slight further increase in field a vortex will nucleate from the nascent-vortex site, move through the surface sheath, and join the interior vortex lattice. Since, however, we are considering a sample large in the direction perpendicular to the surface, the situation immediately after the nucleation of a vortex (or a layer of vortices) is essentially the same as just before the nucleation. The order parameter at the surface will still be essentially zero at the nascent-vortex minimum.

If the applied field is decreased somewhat from its equilibrium value, vortices will not immediately exit through the surface barrier. The currents in the nascent vortices will decrease and the order parameter will start to increase. The interior vortex lattice will experience a reduced magnetic pressure and will move closer to the surface. If, now, the interior vortex lattice moves toward the surface rapidly enough as a function of the decrease in magnetic field, this *decrease* in field can cause an *increase* in current at the surface and therefore cause the order parameter to decrease at the nascent-vortex minima.[10] This, in fact, happens to the extent that when the greatest degree of supercooling

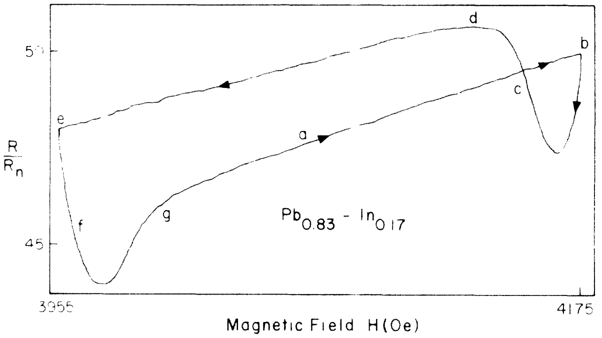

FIG. 2. A recorder tracing of the hysteresis loop of the microwave absorption ($f = 35$ GHz) at $H \simeq 0.8H_{c2}$ and $T = 1.7$°K. The magnetic field is parallel to the surface and perpendicular to the microwave currents.

exists, and vortices start to exit from the sample, the nascent-vortex minima are again at zero order parameter and are presumably the denucleation sites.

A calculation of the surface resistance for $H_{c2} < H < H_{c3}$ and microwave currents parallel to the dc magnetic field gave excellent agreement with experiment when the parameters of the surface sheath given by Fink and Kissinger[8] were used in a simple two-layer model.[11] A similar calculation was done for the surface resistance and reactance at all magnetic fields for the microwave current perpendicular to the magnetic field, which is again parallel to the surface. The conductivity of the sample interior was taken as that of the mixed state below H_{c2}.[12] Agreement with experiment was *not* found. The data could be fitted by the *ad hoc* choice of a highly anistropic complex conductivity for the surface sheath,[13] but such a conductivity is inconsistent with a relatively thick uniform layer of high order parameter.[8] If, however, we assume that there exists at the sample surface a layer of nascent vortices and assume this layer has just the conductivity of the interior vortex lattice,[12] but again take Fink and Kessinger's parameters[8] for the total surface sheath, we get excellent agreement with experiment. Specifically, if we adjust the thickness of the nascent-vortex layer to fit the real part $R(H)$ of the surface impedance exactly, the imaginary part $X(H)$ is correctly given to within a few percent at lower fields and to within about 10% at H_{c2}. This latter discrepancy is not unexpected since near H_{c2} the average order parameter of the nascent-vortex lattice could not be as low as that of the interior lattice or surface superconductivity above H_{c2} would be precluded. By decreasing the nascent-vortex layer density by about 25% in the region of H_{c2}, exact fits for both $R(H)$ and $X(H)$ can be achieved. The thickness of the layer needed to fit $R(H)$ is between 1 and 2 coherence lengths and decreases slightly with increasing H as expected for a single layer of vortices. The nascent-vortex layer would not have a significant effect in the case where the microwave currents flow parallel to the magnetic field.[1]

The existence of nascent vortices is most strikingly manifest in aspects of the "intrinsic hysteresis" due to the surface barrier.[14] In Fig. 2 we show a recorder tracing of $R(H)$ for the orientation where the microwave current is perpendicular to the dc magnetic field, which is parallel to the sample surface. The general form of this hysteresis (a) is the same in a number of type II superconductors, (b) is not much affected by sample surface conditions (though our samples are generally smooth on the scale of a penetration depth when viewed on a scanning electron microscope), and (c) decreases rapidly in amplitude when the magnetic field is rotated out of the sample surface by 1 or 2 degrees.

We interpret this loop as follows: Along the section $a \rightarrow b$ the magnetic field is rising, vortices are nucleating, and the nascent vortices have zero order-parameter minima. The sample is continuously in a state of maximum superheating, and the superheating field has pushed the interior vortex lattice as far from the surface as possible. At point b the applied field is reduced. The surface-sheath shielding currents decrease *everywhere,* and the order parameter values at all the nascent-vortex minima become nonzero in *unison* (regardless of sample inhomogeneities).

A very small number of superconducting carriers is extremely effective in reducing the power absorption. (In the two-fluid model for the parameters of $Pb_{0.83}$-$In_{0.17}$, 5% superconducting carriers in normal material reduces the real part of the resistivity by 70%.) Thus when the order parameter values at all the nascent-vortex minima become simultaneously nonzero, even slightly, we expect an extremely steep drop in the absorption.

As the field is further reduced the interior vortex lattice moves closer to the sample surface in response to the decreased magnetic pressure, thus tending to increase R. From b to c the curve is retraceable.

Throughout the region $c \rightarrow d$ the interior vortex lattice is moving toward the surface rapidly enough with decreasing H to increase the surface current. The nascent vortices are forced to zero order parameter and immediately start to act as denucleation sites. This happens over a range of fields because of sample inhomogeneities. The region $c \to d$ is not retraceable and would exhibit an abrupt drop in R if the field were increased slightly. From d to e vortices are exiting over all of the sample surface, thereby decreasing R. The sample is in a state of maximum supercooling.

At e the field is increased, the interior vortex lattice moves back, and the nascent-vortex minima become nonzero in unison, again giving a sharp drop in R. The region $f \to g$ is analogous to $c \to d$, but in this case nascent-vortex minima are decreasing to zero order parameter as vortices *enter* the sample.

A similar hysteretic pattern has been observed in penetration depth experiments in films at much lower frequencies.[15] The results of susceptibility measurements "can be described by a single row of vortices pinned by the boundary of the specimen."[16] Tunneling studies in mixed state with the magnetic field parallel to the junction show a hysteresis and evidence for "normal material within . . . (a coherence length) of the surface."[17] We have done surface-impedance experiments with superconductors plated with normal metal and also studied the effect of a dc transport current on the surface impedance. All of these results are readily explainable in terms of nascent vortices.

We have chosen to keep our discussion in terms of a simple Ginzburg-Landau picture where we tacitly associate microwave absorption with low or zero order parameter and a consequent large number of "normal electrons." The absorption process by vortices is actually more complicated. If the order-parameter minima decreased to a critical but nonzero value where the currents flowing in the core were critical, sufficient loss would probably occur to explain our results. The discussion in this paper would then need only slight alteration.

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