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## Flute Instabilities during Fast Magnetic Compression of Collisionless  $\beta$  = 1 Plasma

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End-on framing photographs taken at the Garching 500-kV theta pinch during fast magnetic plasma compression  $\left[n_0=(1.5-4)\times10^{13} \text{ cm}^{-3}\right]$  show distinct radial filaments that are due to flute instabilities of high azimuthal-mode number. The observed wavelengths are of the order  $c/\omega_{b}$ , they decrease with increasing density, but no mass dependence is found. The flute instabilities are responsible for the existence of a plasma halo which surrounds the thermonuclear  $\beta = 1$  plasma column and has a major influence on its behavior.

Fast magnetic compression is as yet the most effective way of heating high- $\beta$  plasmas to thermonuclear temperatures. Recent theoretical works<sup>1,2</sup> indicate that shock-heated plasmas in toroidal systems can be stabilized against the dangerous  $m = 1$  mode by the wall effect. The wall stabilization, however, becomes substantial only for high- $\beta$  plasmas with small compression ratios. This means that the time for energy input is limited to the short period of shock compression. In order to transfer as much energy as possible to the plasma during implosion, it is necessary to have high current densities at the plasma surface and strong acceleration of the ions. High drift velocities in the magnetic piston excite microinstabilities which, on the one hand, lead to strong collisionless electron heating and, on the other, lead to enhanced field diffusion, and hence to broadening of the piston. Apart from microinstabilities, acceleration of the initial plasma produces macroinstabilities which result in a roughening of the plasma boundary and reduce the effective plasma compression.

The development of Rayleigh- Taylor instabilities in magnetically accelerated plasmas has already been observed in "collision-dominated

pinches."<sup>3</sup> Wavelengths and growth rates of these instabilities were discussed from a magnetohydrodynamic point of view and the results agree very well with the observed values. In a collisionless plasma, however, particle interactions are long ranged and a fluid approximation would seem suspect. Up to now, only in the cases of appreciable trapped reversed magnetic fields were flute instabilities in collisionless shocks observed. In consequence of the bias fields these compression experiments lead only to ion temperatures of about 100 eV and rather low  $\beta$  values.<sup>4</sup> To reach keV ion temperatures at high  $\beta$ values—the aim of high- $\beta$  stellarator experiments -- it is necessary to compress magnetic-fieldfree plasmas.

This paper describes observations of macroinstabilities during fast shock compression within an enlarged parameter range, and the scaling of the corresponding wavelengths. The experimental conditions and the regime of plasma parameters we have investigated are similar to those of planned high- $\beta$  stellarator experiments. Therefore the results have a bearing on high- $\beta$ confinement experiments which are based on the wall stabilization of weakly compressed plasmas.



FIG. 1. Schematical setup of the high-voltage theta pinch.  $0_7$ 

The high-voltage theta pinch is shown schematically in Fig. 1. The theta-pinch coil, 43 cm in diameter and 100 cm long, is energized by four Blumlein-type transmission lines that are charged up to 125 kV each with a Marx generator. The rise of the magnetic field in the theta-pinch coil is 30 kG/ $\mu$ sec, the maximum field amplitude 5.5 kG, and the pulse length 0.5  $\mu$ sec. Details of the bank are given by Herppich.<sup>5</sup> The initial plasma is generated by a z-pinch preionization at filling pressures of about 1 mTorr, producing a highly ionized plasma with densities in the range of  $10^{13}$  cm<sup>-36</sup> The following diagnostics are used: magnetic probes for measuring the magnetic field profiles, Thomson scattering of laser light for measuring the electron density and temperature on and outside the axis, a neutron detector for determining the ion temperature, and a highspeed framing camera for end-on photography.

The time development of the compression is shown by the radial magnetic field profiles for an initial density of  $2 \times 10^{13}$  cm<sup>-3</sup> (Fig. 2). The magnetic piston starts to implode with a velocity of about  $5 \times 10^7$  cm/sec, decelerates, and is finally stopped by the pressure of the compressed plasma. (For results concerning the electron and ion heating in and in front of the magnetic piston see Keilhacker et  $al$ .<sup>7</sup>) After compression we have a dense, weakly compressed,  $\beta = 1$  plasma core consisting of hot ions and cold electrons  $(n_e = 10^{14} \text{ cm}^{-3}, E_i = 1.5 \text{ keV}, T_e = 25 \text{ eV})$ . This plasma column is surrounded by a low-density, highelectron-temperature plasma halo  $(n_e \le 8 \times 10^{13})$ cm<sup>-3</sup>,  $T_e \approx 1$  keV) in which the pressure slowly decreases towards the tube wall as is shown by the magnetic field gradients.

To investigate the origin of this plasma halo the discharge was photographed axially through an eccentric plate glass window by a high-speed framing camera (Fig. 3). The time sequence of the photographs taken at intervals of 100 nsec is



FIG. 2. Radial magnetic field profiles for a deuterium plasma with initial density of  $2 \times 10^{13}$  cm<sup>-3</sup>.

referred to the external magnetic field  $B_e$ . A large number of radial streaks that are due to flute instabilities of high azimuthal-mode numbers are clearly visible behind the luminous front, which moves inwards with roughly the piston velocity.

The imaging optics allow sharp imaging of the entire length of the plasma  $(l = 100 \text{ cm})$ . Noteworthy are the clear contours of the radial streaks, even after the decrease of the external



FIG. 3. End-on framing pictures during the fast magnetic plasma compression of a deuterium plasma  $(n_0)$  $=2\times10^{13}$  cm<sup>-3</sup>).

field. A slight axial twist would already cause smearing and eradicate the filaments. The scaling of the mode number and wavelength of the observed flutes was investigated by means of discharges in deuterium and hydrogen varying the initial density in the range  $(1.5-4) \times 10^{13}$  cm<sup>-3</sup>.

The mode number  $m$  and the wavelength  $\lambda_f$  can be determined from framing pictures by, for example, counting the streaks in each case direct behind the luminous front at a fixed time  $(t = 400$ nsec). In Fig. 4 the wavelength  $\lambda_i$  is plotted versus  $n_0$  on a log-log scale for discharges in deuterium and hydrogen. Assuming that  $\lambda_f \propto n_0^a m_i^b$  $(m_i = \text{ion mass})$  yields  $a = -\frac{1}{2} \pm \frac{1}{4}$ ,  $b \approx 0$ . The wavelength of the flutes thus decreases with increasing initial density, and the following approximation is obtained:

$$
\lambda_f \propto 1/\sqrt{n_0}.
$$

Shonk and Morse' developed a simplified numerical model in which a high-density cylinder of electrons and ions is accelerated in a low-density background plasma. The density plots show the development of high- $m$  modes similar to those of the experiment, i.e., the wavelengths are about  $c/\omega_{\scriptscriptstyle{{{{\mathcal{p}}}}_i}}$  ( $\omega_{\scriptscriptstyle{{{\mathcal{p}}}_i}}$  is the ion plasma frequency of the background plasma). The measured wavelengths are of the order  $c/\omega_{bi}$  and the density dependence agrees well with the result of the calculation, but no mass dependence is found experimentally. (It is not clear what local density should be taken for comparing the experimental results quantitatively with the particle-in-cell calculations. In the experiment the density, measured by laser light scattering, varies across the shock front, and the wavelength  $\lambda_f$  corresponds to a value between  $0.3c/\omega_{pi}$  and  $0.8c/\omega_{pi}$ .)

The measured density dependence of the wavelengths is consistent with experiments in the lowdensity region of  $\sim 10^{11}$  cm<sup>-3</sup>,<sup>7</sup> where no magnetic field gradients behind the piston, and correspondingly no halo, were observed, In that case the wavelengths are of the order of half the tube circumference and no high- $m$ -number flutes can develop.

The observed flute instabilities are therefore considered to be the cause of the outside plasma, which exerts a major influence on the production and the behavior of the  $\beta = 1$  inner plasma column (e.g. , compression velocity, damping of plasma oscillations, and final plasma radius'). Besides microinstabilities, responsible for enhanced resistivity and strong electron heating, macroinstabilities thus also decisively govern the physical



FIG. 4. Wavelength  $\lambda_f$  of the flutes versus the initial plasma density  $n_0$  (crosses, deuterium; circles, hydrogen) .

processes during the fast magnetic compression of collisionless high- $\beta$  plasmas.

The investigation of fast magnetic compression of plasmas with initial densities in the  $10^{13}$ -cm<sup>-3</sup> region shows that weakly compressed high- $\beta$ plasmas with thermonuclear temperatures can be produced. Behind the magnetic piston, however, there remain, during the entire compression phase, magnetic field gradients extending to the tube wall. Strong currents are required to maintain these gradients. The high-density plasma column is accordingly surrounded by an outside plasma in contact with the mall. Framing pictures show that flute instabilities are responsible for the existence of this plasma halo.

According to theory it should be possible to achieve stable toroidal confinement at small compression ratios with the aid of the conducting wall. Up to now it is not clear how a diffuse magnetic field profile and the low-density, high-electron-temperature plasma halo affect the stable confinement of the central plasma column. If large experiments with wall stabilization of high- $\beta$ plasmas are considered, this question must be clarified, and flute instabilities should be investigated in detail over a wider density range. The development of these flutes will be investigated in a toroidal experiment (belt pinch) up to initial densities of  $10^{14}$  cm<sup>-3</sup>, and the long-time behavior of the plasma halo and its influence on the confinement of a weakly compressed, high- $\beta$  plasma can be discussed in an enlarged parameter range. For a better theoretical understanding of flute instabilities driven by radia1 plasma acceleration a two-dimensional  $(r, \varphi)$  computer program using a hybrid model of electron fluid and ion particles in a so-called "particle in cell" calculation is<br>being developed.<sup>10</sup> being developed.

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## Nucleation of Vortices in the Superconducting Mixed State: Nascent Vortices\*

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Evidence is presented that the mixed-state order parameter at surfaces parallel to the magnetic field is strongly modulated. The minima of this modulation act as the nucleation and denucleation sites for vortices.

To change the number of vortices in a superconducting sample a finite change in magnetic field from the thermal equilibrium value is necessary to overcome any vortex entry or exit barrier. The metastable state in which the field is above the equilibrium value is referred to as a superheated state, and that in which it is below as a supercooled state. The maximum superheating field of the Meissner state has been calculated for the case of one-dimensional instabil-'ities using the Ginzburg-Landau equations.  $^{\rm 1.5}$ The case where two-dimensional fluctuations of the order parameter are permitted has also been treated,  $34$  and the appearance of large modulations in the order parameter at the surface prior to vortex nucleation has been suggested.<sup>3</sup> A recent calculation<sup>5</sup> of the breakdown of the superheated Meissner state studies the global stability in two dimensions and numerically investigates various Ginzburg-Landau solutions. Solutions involving large periodic variations in the order parameter are found which resemble a structure postulated earlier<sup>6</sup> to explain certain experimental results in the mixed state.

Superheating (or supercooling) in the mixed state is much harder to treat theoretically because of the large variations of the order parameter in the sample interior, but it is the simpler

situation experimentally, An early treatment of the barrier to vortex entry (or exit) by Bean and Livingston' considered two magnetic forces acting on a vortex located near and parallel to the surface of the superconductor, The first force is directed toward the interior and is the repulsion by the external field. (This force on the vortex lattice keeps it away from the surface and causes the order parameter in the "surface sheath" to be considerably higher than the average value in the bulk.) Very close to the surface another force in the opposite direction, due to the image vortex located outside the surface, tends to dominate. There is thus a distance from the surface at which the vortex energy would be at a maximum. This provides a barrier to vortex penetration.

At higher magnetic fields, the field gradients and magnetic forces are smaller because of the reduced order parameter and a purely magnetic treatment is not a good approximation. The Ginzburg-Landau equations for a normal-state interior have been solved assuming a constant order parameter at the surface.<sup>8</sup> A large order parameter at the surface is still found (and the material near the surface remains superconducting to  $H_{\alpha}$ = 1.69 $H_{c2}$ ). There has been no complete Ginzburg-Landau treatment for the mixed state in the presence of a surface. However, Fink<sup>9</sup> has treated



FIG. 3. End-on framing pictures during the fast magnetic plasma compression of a deuterium plasma  $(n_0$ <br>= 2×10<sup>13</sup> cm<sup>-3</sup>).