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# Off-Shell Effects in Pion-Deuteron Absorption*

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The cross section for the process $\pi^+ + d \to p + p$ is calculated using a $\pi$-$N$ $t$ matrix obtained from a Lagrangian formalism. The $t$ matrix is provided with a suitable cutoff to damp the large (off-shell) momentum components and the total cross section is found to be strongly dependent on the cutoff range parameter taken, i.e., to how fast the $t$ matrix falls off as it is taken off shell. It is argued that this is to be expected physically and that this sensitivity might provide a means of studying the $\pi$-$N$ off-shell $t$ matrix.

Pion absorption on nuclei provides us with two ways of obtaining information about nuclear properties. Two-nucleon absorption may hopefully provide information on nucleon-nucleon correlations, while single-nucleon absorption can provide knowledge of the single-nucleon form factor. The unique feature of these reactions, as compared with conventional nuclear reactions, is the large amount of momentum which must be found to make the reaction go. This is because the mass of the pion is converted to energy. In the case of two-nucleon absorption this momentum is obtained by having the momenta of the two final nucleons almost cancel. This still requires that the two nucleons have large relative momentum—a correspondence with short nucleon-nucleon distances. For single-nucleon absorption the recoiling residual nucleus must carry off momentum equal to that carried by the absorbing nucleon. Thus we are looking at the relative momentum distribution of the nucleon-core system.

To understand either of these processes the basic pion absorption mechanism must be understood. From the above description and from previous work (especially that of Koltun and Reitan[1]) it may be believed that the basic process involves an interaction with two nucleons. Note that the first interaction is a scattering. This is followed by a propagation (off-shell) followed by an absorption.

To learn about the basic mechanisms involved we have studied pion absorption on the deuteron. For this reaction there is no difference between two-nucleon and single-nucleon absorption.

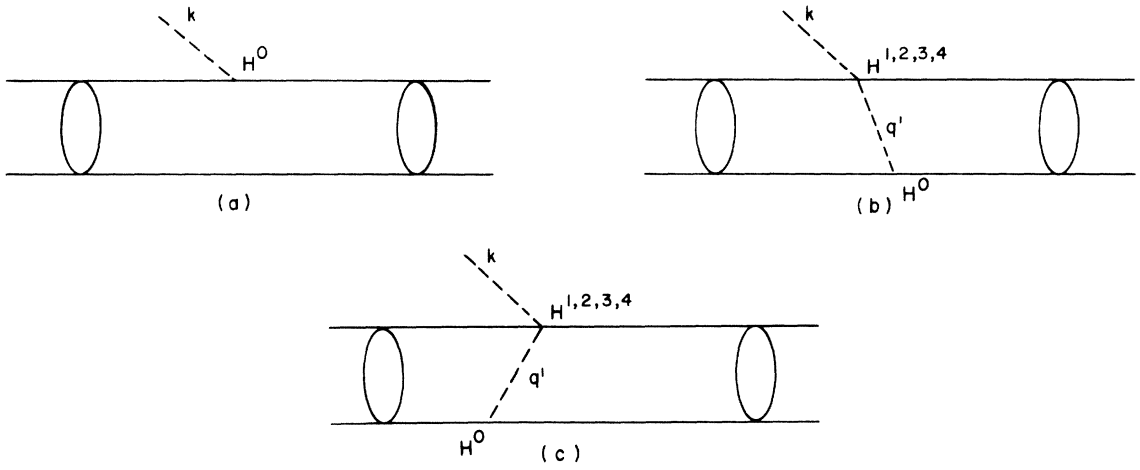


FIG. 1. Graphs representing the types of processes considered in this paper.

The processes considered are shown in the graphs in Fig. 1. The first graph represents direct absorption while the next two describe scattering from one nucleon followed by propagation forward (b) or backward (c) in time to the absorption vertex.

The absorption vertex was taken to be the Galilean-invariant form

$$H^0 = -(4\pi)^{1/2} f \mu^{-1} \vec{\sigma} \cdot [\vec{q}\tau \cdot \varphi - i(2M)^{-1}(\vec{p}\tau \cdot \pi + \tau \cdot \pi \vec{p})], \tag{1}$$

where $\vec{q}$ and $\vec{p}$ are pion and nucleon momentum operators, respectively, and $\varphi$ and $\pi$ are the pion field operators. The $s$-wave scattering-vertex operators are taken to be those used by Koltun and Reitan[1]:

$$H^1 = 4\pi\lambda_1(\omega)\mu^{-1}\varphi \cdot \varphi, \quad H^2 = 4\pi\lambda_2(\omega)\mu^{-2}\tau \cdot \varphi \times \pi. \tag{2}$$

A natural generalization for the $p$-wave operators was given by Klein[2]:

$$H^3 = -4\pi\lambda_3(\omega)\mu^{-3}\vec{q}\varphi \cdot \vec{q}\varphi, \quad H^4 = -4\pi\lambda_4(\omega)\mu^{-3}\vec{\sigma}\tau \cdot \vec{q}\varphi \times \vec{q}\varphi. \tag{3}$$

In the last equation the dot-cross product is understood to apply to coordinate and isospin simultaneously. The $\lambda$'s are chosen at each energy to reproduce the measured pion-nucleon phase shifts.

The initial and final nucleon-nucleon interactions may be calculated with any reasonable form of nucleon-nucleon potential. We chose to consider the Hamada-Johnston[3] potential and the boundary condition model (BCM).[4] Figure 2 shows the results of this type of calculation. As can be seen the direct term alone gives much too small a cross section. The addition of $s$-wave rescattering is seen to improve the situation greatly at low energies. Of course, one cannot expect the resonance to appear in such a formalism since it is believed to come from $p$-wave scattering. The inclusion of $p$-wave scattering from Eqs. (3) leads to the excessively large result shown. To realize the problem one must remember that the forms in Eqs. (3) are only valid at low $q$ while they are being used over all momenta in the intermediate integrals which arise. The most important contributions come from momenta which are too large for the expressions in Eq. (3) to be valid.

To translate to coordinate space, the forms presented correspond to zero-range pion-nucleon interactions. These lead to Yukawa-like forms in the nucleon-nucleon coordinate. The largest contribution arises from $p$-wave scattering—$p$-wave absorption leading to a $d$-wave contribution represented by a Hankel function of order 2 in the effective mass of the propagating pion:

$$[1 + 3/\mu' r + 3/(\mu' r)^2] \exp(-\mu' r)/\mu' r, \tag{4}$$

$$\mu' = [(3\mu^2 - k^2)/4]^{1/2}. \tag{5}$$

By choosing the pion-nucleon system to have a finite range the singularity of this term (and others)

will be reduced. We have chosen to do this by using a form taken by Landau and Tabakin,[5] Eisenberg, Hufner, and Moniz,[6] Myhrer and Koltun,[7] and others. This form corresponds to a separable representation of the pion-nucleon $t$ matrix given by

$$\tilde{H}^3 = -4\pi\lambda_3(\omega)(k^2+\beta^2)^2\mu^{-3}\left[\frac{\bar{q}}{q^2+\beta^2}\varphi\right]\cdot\left[\frac{\bar{q}}{q^2+\beta^2}\varphi\right],$$
$$\tilde{H}^4 = -4\pi\lambda_4(\omega)(k^2+\beta^2)^2\mu^{-3}\vec{\sigma}\tau\cdot\left[\frac{\bar{q}}{q^2+\beta^2}\varphi\right]\times\left[\frac{\bar{q}}{q^2+\beta^2}\varphi\right]. \quad (3')$$

The results of this calculation are given in Fig. 3. As may be seen, the fit is quite good for $\beta \approx 300$ MeV/$c$ and the results are very sensitive to the value of $\beta$.

The calculation is made only through second order and lacks such effects as breakup of the deuteron by inelastic pion scattering. It is also possible that the effect of inelasticity in the nucleon-nucleon channel is strong enough that a coupled-channel approach is necessary, although this would only seem to be true above 140 MeV pion kinetic energy.

The effect observed here is an explicit demonstration of what was seen in several calculations of absorption on a single nucleon bound to a nucleus.[8] Here it was noted that the use of a Kissling-er optical potential [which comes from an interaction Hamiltonian of the form given in Eq. (3)] gives much too large a cross section. If a different off-shell form is taken the magnitude of the calculated cross section changes greatly.

It is not difficult to see why this process is so sensitive to the off-shell character of the pion-nucleon amplitude. If the initial and final states of the nucleons are taken to be on shell then the intermediate pion must propagate with an energy $\omega/2$ (it must give half its energy to each nucleon). This puts its intermediate momentum at

$$q^2 = \left(\frac{\omega}{2}\right)^2 - \mu^2 = \frac{\mu^2+k^2}{4} - \mu^2 = \frac{k^2}{4} - \frac{3\mu^2}{4}. \quad (6)$$

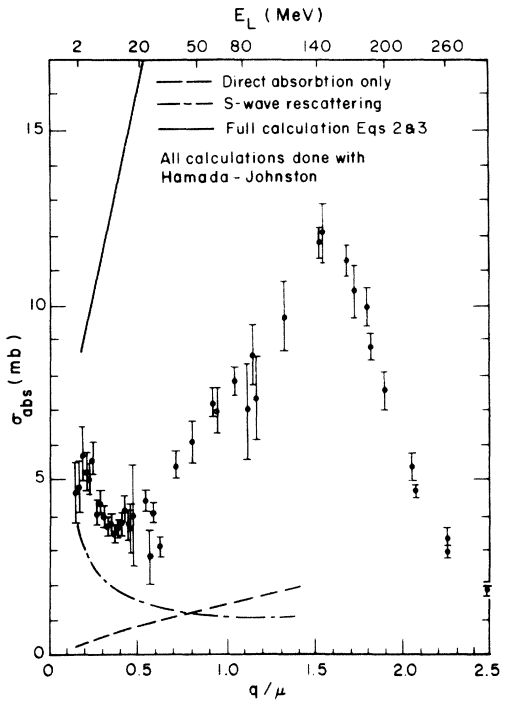


FIG. 2. Comparison of direct absorption, $s$-wave rescattering plus absorption, and $s$- and $p$-wave rescattering plus absorption using Eqs. (2) and (3) with the $\pi^+ + d \to p + p$ total cross-section data. See Ref. 10 for a compilation from which the data were taken.

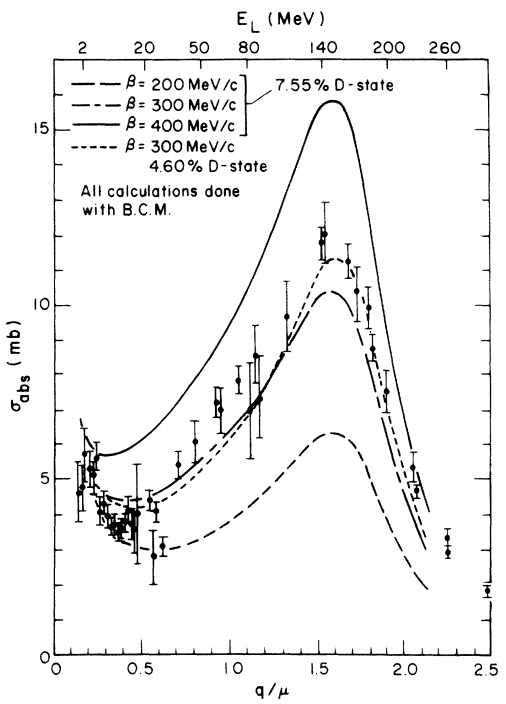


FIG. 3. Comparison of $s$- and $p$-wave rescattering plus absorption using Eqs. (2) and (3') for three values of the off-shell range parameter $\beta$. Calculations using the Hamada-Johnston potential give results which differ from the 7.55% BCM by only 10–15%.

Thus knowledge of the behavior of both vertices is required far off shell. It is assumed that Eq. (1) represents the absorption vertex correctly, although at the higher pion momenta this can be questioned.

It may be asked why calculations which use the pion-nucleon amplitude on shell[9] work as well as they do. The answer may lie in the following considerations. The on-shell replacements substitute $\vec{k}\cdot\vec{q}' \to \vec{k}\cdot\hat{q}'k$ in an integral which involves other functions of $\vec{q}'$. We have just argued that the replacement should be something like

$$\vec{k}\cdot\vec{q}' \to \vec{k}\cdot\hat{q}'q'\frac{(k^2+\beta^2)}{(q'^2+\beta^2)},$$

so that we need to compare the constant $k$ to $q'(k^2+\beta^2)/(q'^2+\beta^2)$ over the important range of integration. The magnitudes of these two functions are in fact very similar in this region. Since the constant off-shell form agrees neither with the field-theory limit at small momenta nor with the Chew-Low form (which is cut off at high momenta), we regard the fits obtained using this form as fortuitous.

Angular distributions have also been calculated and the results show some improvement over those of Ref. 9, especially at energies below the 3-3 resonance. There would appear to be some hope of determining off-shell nucleon-nucleon information from the measured angular distributions.

There remains a problem which is always present in this general approach, namely double counting. It is not clear how much of the process in Figs. 1(a) and 1(b) is already included in the one-pion-exchange portion of the nucleon-nucleon interaction. This uncertainty leads to a corresponding uncertainty in the off-shell $\pi$-$N$ $t$ matrix determined in this manner. A correction for this effect is possible in principle.

In conclusion we wish to suggest that pion absorption should be sensitive to the off-shell pion-nucleon $t$ matrix and, in fact, is probably the best way to study this quantity.

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# $L$ Dependence in Angular Distributions of the Two-Proton Transfer Reaction $^{48}$Ca($^{16}$O, $^{14}$C)$^{50}$Ti†

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The reaction $^{48}$Ca($^{16}$O, $^{14}$C)$^{50}$Ti to the lowest $0^+$, $2^+$, $4^+$, and $6^+$ states of $^{50}$Ti has been studied at 56 MeV. Rapid oscillations in the angular distribution persist to extreme forward angles ($\theta_L = 3°$). In the present data, the forward-angle structure is found to be an unambiguous indicator of the transferred angular momentum. The data are reproduced by distorted-wave Born-approximation calculations. A qualitative understanding is obtained in the strong-absorption model. It is believed that such behavior will occur in many heavy-ion–induced transfer reactions.

We report a study of the two-proton transfer reaction $^{48}$Ca($^{16}$O, $^{14}$C)$^{50}$Ti at 56 MeV bombarding energy. Rapid angular oscillations in the differential cross sections are found to persist down