umes of the metal and molecular phases as 1.4 and 1.7 $cm^3$/mole $D_2$, respectively. In $H_2$ the predicted transition is at 4.2 Mbar. If we had attempted to force fit the potential to give better agreement with the highest pressure Hugoniot points, it would have had to be made less repulsive. This would lead to a further lowering of the free energy of the molecular phase *vis-à-vis* the metal, leading to a higher transition pressure. Therefore the pressure of 4.2 Mbar appears to be a lower bound.

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# Dispersion and Cyclotron Damping of Pure Ion Bernstein Waves

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Ion Bernstein waves are excited by a long wire in the center of a potassium Q-machine plasma column. Because the wave vector is very nearly perpendicular to the magnetic field, we observe pure ion Bernstein waves, in contrast to neutralized ion Bernstein waves which were previously observed. The frequency bands for the cyclotron damping are so narrow that we can distinguish the isotopes of potassium.

There are two different kinds of ion Bernstein waves[1] depending upon the departure from exact perpendicularity for propagation across the magnetic field. We will call pure ion Bernstein waves (PIBW) those which propagate almost perpendicular to the magnetic field and in which the electrons are almost stationary. As we will see, the angular region for which PIBW exist is very narrow, and these waves have not previously been observed. We will call neutralized ion Bernstein waves (NIBW) those for which the electrons are in Boltzmann equilibrium with the wave potential. NIBW propagate in a different and larger angular region, different from 90° but close to it, and they have already been observed.[2-5] The two angular regions are separated by a domain in which the waves are Landau damped by the electrons. These points are easily deduced from the dispersion function of an electrostatic wave in a Maxwellian plasma propagating at an oblique angle:

$$\epsilon(\omega, k_\parallel, k_\perp) = 1 + \sum_{s=e,i} (k_{Ds}^2/k^2)\exp(-\Lambda_s)\sum_{-\infty}^{+\infty} I_n(\Lambda_s)[1 + \xi_{0s}Z(\xi_{ns})], \quad (1)$$

$$k^2 = k_\parallel^2 + k_\perp^2, \quad k_{Ds}^2 = n_{0s}q_s^2/\mathcal{E}_0 KT_s, \quad \Lambda_s = k_\perp^2\rho_s^2/2, \quad \rho_s = v_s/\Omega_s, \quad \Omega_s = q_s B_0/m_s,$$

$$v_s^2 = 2KT_s/m_s, \quad \xi_{ns} = (\omega - n\Omega_s)/k_\parallel v_s;$$

$Z$ is the plasma dispersion function, $v_s$ the thermal velocity, and $\rho_s$ the corresponding mean Larmor radius for the species $s$. For ion waves propagating at a large angle with respect to the magnetic field, the previous expression can be simplified by assuming that (1) the parallel wavelength is very large compared to the ion mean Larmor radius, (2) the frequency is near the ion cyclotron frequency and its harmonics (but not too near), and (3) the electron mean Larmor radius is negligible compared to the perpendicular wavelength. Then using the two inequalities $\xi_{ni} \gg 1$ and $\Lambda_e \ll 1$, one obtains from Eq. (1)

$$\epsilon(\omega, k_\parallel, k_\perp) = 1 - \frac{k_{De}^2}{2k^2}Z'(\xi_{0e}) - \frac{k_{Di}^2}{k^2}\exp(-\Lambda_i)\sum_{-\infty}^{+\infty} I_n(\Lambda_i)\frac{n\Omega_i}{\omega - n\Omega_i}. \quad (2)$$

Note that if $\omega$ is too close to a given harmonic $n\Omega_i$, then $\xi_{ni}$ is kept finite, and the corresponding complex term subsists in Eq. (2), leading to the ion cyclotron damping as already observed by Ohnuma *et al.*[4] The closer the propagation angle to exact perpendicularity, the narrower the frequency interval of cyclotron damping, the width of which is roughly given by $\omega - n\Omega i \lesssim k_\parallel v_i$. Outside these cyclotron damping regions the waves are still sensitive to $k_\parallel$ through the electron term. Depending upon the propagation angle, the term $\xi_{0e} = \omega/(k_\parallel v_e)$ can be large (for very small $k_\parallel$) or very small (for not too small $k_\parallel$) or intermediate. Correspondingly, the electron term in Eq. (2) can be zero (PIBW) or $-k_{De}^2/k^2$ (NIBW) or a complex value (Landau damping due to electrons). One can then distinguish the following $k_\parallel$ domains:

$$k_\parallel\rho_i \ll (m_e/m_i)^{1/2}\omega/\Omega_i \quad \text{(PIBW)}, \quad (3)$$

$$(m_e/m_i)^{1/2}\omega/\Omega_i \ll k_\parallel\rho_i \ll 1 \quad \text{(NIBW)}, \quad (4)$$

$$k_\parallel\rho_i \sim (m_e/m_i)^{1/2}\omega/\Omega_i \quad \text{(Landau-damped IBW)}. \quad (5)$$

Note that for PIBW and NIBW no complex term remains in Eq. (2), leading to undamped modes. The dispersive properties of PIBW and NIBW are very different as illustrated in Fig. 1. PIBW have dispersive properties comparable to the one of electron Bernstein waves. For NIBW nothing in particular appears at the lower hybrid and, whatever the density, there is a cutoff band over each ion cyclotron harmonic.

The angular region of propagation is very narrow for PIBW [Eq. (3)]. This explains why there

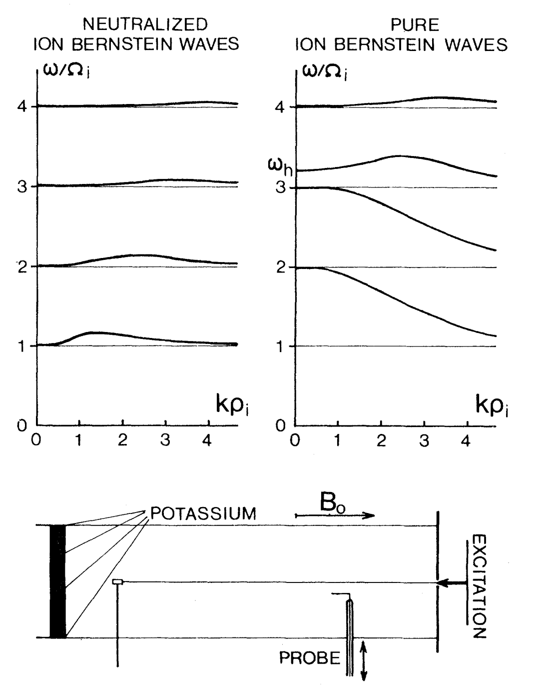


FIG. 1. Comparison between the dispersion curves for neutralized ion Bernstein waves and pure ion Bernstein waves for the same plasma conditions: $\omega_{pi} = 3\Omega_i$ and $T_e = T_i$. $\omega_h = \omega_{pi}^2 + \Omega_i^2$ is the lowest hybrid frequency. The experiment is represented schematically at the bottom.

has not yet been an experimental observation of PIBW. The propagation domain is larger for NIBW, and there is already experimental observation of these waves, in particular by Ault and Ikezi.[2] Finally, Landau damping due to electrons was observed experimentally by Hirose, Alexeff, and Jones.[6]

In the experiment the wave is excited by an antenna of finite length $L$. Although the antenna is perfectly aligned parallel to the magnetic field, because of the finite geometry it excites a set of modes with wave vector $k$ having small $k_\parallel$ components. The wave observed experimentally is a summation of those modes. It can be demonstrated that for the region close to the antenna (at distances much less than $L$), the set of $k_\parallel$ which participates significantly in the wave formation is concentrated around zero $k_\parallel$ in an interval whose width is of the order of $2\pi/L$. We shall present, instead of the algebra, a picture of the electron behavior in the potential of the wave excited by the antenna. Because of the magnetic field the electrons are tied to the field lines, and they can only move parallel to the antenna. The wave makes the potential vary on a given field line synchronously over a distance of the order of $L$ (beyond the antenna extremities the Landau damping strongly damps the oblique waves). The characteristic time for the change of the potential is the wave period $f^{-1}$. The characteristic time for the response of the electron to a perturbation of length $L$ is of the order of the time of flight of thermal electrons, $L/v_e$. Therefore, if $f^{-1} \gg L/v_e$, then the potential is quasistatic for the electrons, and one should expect the antenna to excite NIBW. The experiments quoted were performed in these conditions (Ault and Ikezi,[2] 1970: $f^{-1}=10^{-6}$ sec, $L/v_e=10^{-7}$ sec; Schmitt,[3] 1972: $f^{-1}=10^{-5}$ sec, $L/v_e=10^{-6}$ sec). When $f^{-1}$ and $L/v_e$ are of the same order, no approximation is as yet available, but one should expect some Landau damping due to electrons. Finally, when $f^{-1} \ll L/v_e$, the electrons do not have time to respond to the wave, and one should expect dispersive properties as given for PIBW. The present experiment corresponds to this case ($f^{-1}=2\times10^{-6}$ sec, $L/v_e=10^{-5}$ sec).

Experiments are performed in a single-ended $Q$ machine operating with potassium. The magnetic field is adjustable up to 11 kG. The plasma column has a 6-cm diam, and the density profile is flat over about a 5-cm diam. The antenna consists of a molybdenum wire (1.08 m long and 0.2 mm diam) carefully aligned parallel to the magnetic field and immersed in and located on the center of the plasma column as shown in Fig. 1. The cold plate is negatively biased, and the electrons are reflected by the cold plate (the length of the wire as seen by the electrons should be multiplied by 2).

The wave is excited by driving the wire with a sinusoidal voltage of known frequency. The excitation amplitude is typically 0.05 to 0.1 V ($KT_i = KT_e = 0.2$ eV). The dc bias of the wire is close to the plasma potential. The wave is detected by a small probe (of length 4 cm and diameter 0.2 mm), parallel to the magnetic field and movable radially. The detected signal is mixed with the excitation signal, and the product is plotted for a given frequency as a function of distance from the wire. Figure 2 shows an example of raw data for two sets of successively increasing frequencies. The first set, Fig. 2(a), covers the interval between the first and the second ion gyroharmonics. The wave appears to propagate for frequencies up to the second harmonic, which

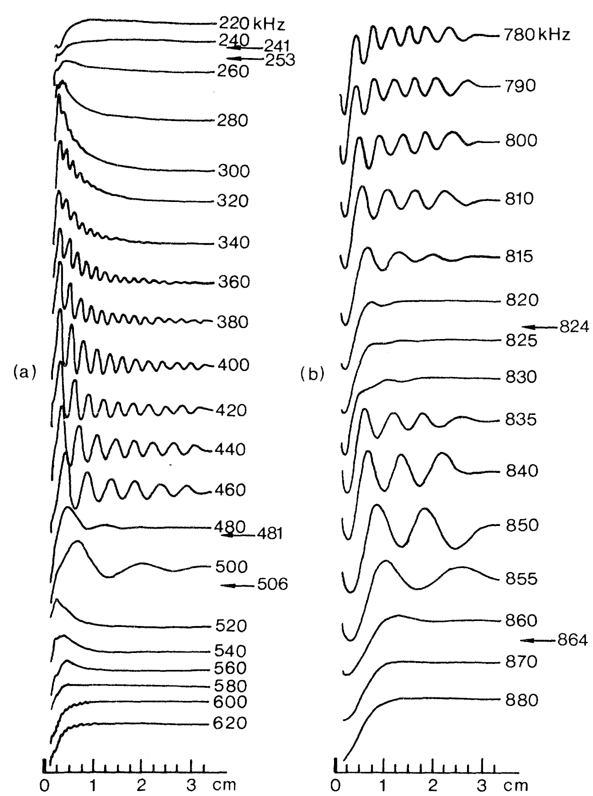


FIG. 2. Example of raw data as obtained in the experiment. (a) $n_0 = 1.5\times10^9$ cm$^{-3}$ and $B_0 = 6480$ G. (b) $n_0 = 8\times10^8$ cm$^{-3}$ and $B_0 = 11\,080$ G. The arrows indicate the ion gyroharmonics in kHz.

is a characteristic of PIBW. When the wave is not Landau or cyclotron damped (as we will see later), the damping is very weak, the amplitude varying as $r^{1/2}$ because of the cyclindrical geometry. Figure 2(b) gives more details around the second harmonic [but for different plasma parameters than in Fig. 2(a)]. There are two successive frequency domains where the wave is damped. Since in the experiment the observed wave propagates at an angle very close to exact perpendicularity, the cyclotron damping is confined to very narrow frequency domains around the harmonics. The two domains in Fig. 2(b) correspond to the two isotopes of potassium, $K^{39}$ and $K^{41}$, which exist naturally with 93% and 7% abundances, respectively. The same feature appears around the higher harmonics. It would be interesting to compare the half-width $\Delta f$ of the cyclotron absorption peak which here is of the order of 8 kHz, to the one estimated, taking for the wave a $k_{\parallel}$ component of $2\pi/L$ ($\Delta f \sim k_{\parallel}v_i/2\pi \sim 1$ kHz). Such a discrepancy is easily explained by a broadening of the absorption peak due to several imperfections in the experiment: the nonexact uniformity of the magnetic field, the time-varying component for the magnetic field (a consequence of the residual oscillatory current delivered by the rectifier), and finally the presence of a slight radial static electric field in the plasma.

Measuring the wavelength for each frequency we obtain experimentally the dispersion curves shown in Fig. 3, which agree very well with the theoretical dispersion curves of PIBW. In Fig. 3(a) the small systematic error observed at low frequency is explained by the fact that the conditions for the existence of PIBW become questionable (electron time of flight, $8\times10^{-6}$ sec; wave period, $4\times10^{-6}$ sec). For low frequencies the electrons begin to respond to the wave potential, lowering the dispersion curve and producing a Landau damping [see Fig. 2(a)]. Note that in Fig. 3(a) the isotope $K^{41}$, despite its small relative density, produces a jump in the dispersion curve which is observed experimentally. In Fig. 3(a) the lower hybrid frequency is 1200 kHz and does not appear in the result. The lower hybrid frequency is visible in Fig. 3(b), which corresponds to a lower density. Note that above the second harmonic we observe the forward wave and the cutoff band as predicted for PIBW. It was verified that the measured hybrid frequency was equal to the value calculated using the ion plasma frequency obtained from probe measurements.

In conclusion we can say that because in our experiment we are able to excite waves with an almost perpendicular wave vector, we have excited and observed pure ion Bernstein waves which have not been observed before. Another consequence of the good geometry is that we have observed very localized ion cyclotron damping which permits the separate cyclotron damping (therefore heating) of the two potassium isotopes.

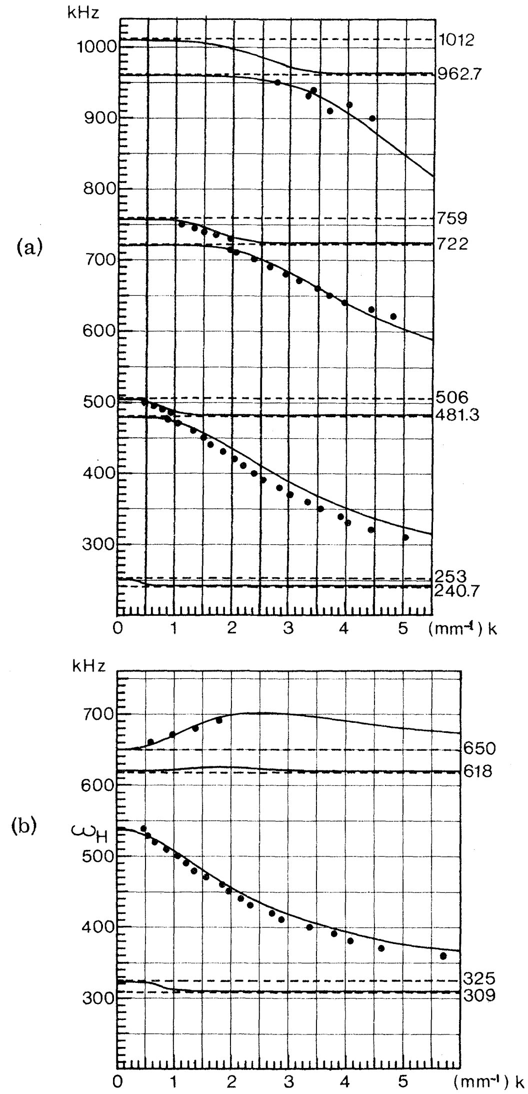


FIG. 3. Dispersion curves as obtained from experimental data. Solid lines, theoretical dispersion curves of pure ion Bernstein waves computed for the same conditions. (a) Same conditions as for Fig. 2(a). (b) $n_0 = 1.5\times10^8$ cm$^{-3}$ and $B_0 = 8310$ G.

This experiment was performed in the $Q$-machine device in the Comissariat à l'Energie Atomique (Fontenay-aux-Roses). The author wishes to thank Dr. Etievant, Dr. Gravier, Dr. Renaud, and Dr. Deschamp for kindly placing this machine at his disposal.

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# Effect of a Limiter on Ohmic Discharges in the Heliotron D

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Ohmic discharges with and without a limiter are being studied in the Heliotron D. Without the limiter, we observe longer containment time and lower impurity concentration than with the limiter, and a stable current with the safety factor $q_{0H} \simeq 0.4$ is obtained. With the limiter, strong current fluctuations are observed near rational values of the rotational transform, i.e., $\iota/2\pi = (\iota_F + \iota_{0H})/2\pi \simeq 0.5$ and 1.0.

The existence of the material limiter is one of the difficult problems to be solved in conventional toroidal systems such as the tokamak. Impurities produced by the interaction between the plasma and the limiter cause fast energy loss. We obtained an Ohmic-heating current column isolated from the wall of the vacuum vessel by the separatrix of the Heliotron magnetic field[1] without using a material limiter in the experiment on the Heliotron B.[2] The separatrix region in the helical Heliotron field[1,3,4] is also expected to act as a natural limiter, since the outermost closed magnetic surface is located inside the helical conductor without crossing the wall of the vacuum vessel. Similar methods have been proposed to remove the limiter recently.[5,6] In this paper experimental results are reported about the effect of a limiter on the Ohmically heated plasma. We also describe the effects of the rotational transform and the shear on the current instability.

Details of the machine have been described in earlier papers.[7,8] An $l = 2$ helical conductor with a major radius of 108.5 cm is hung inside the vacuum chamber. Because of the short pitch (54.5 cm) and the large minor radius (13 cm) of the winding, we can make both the rotational transform and the shear large. The maximum toroidal field strengths due to the toroidal and helical coils are 5 and 3 kG, respectively. An eight-turn air-core transformer for the Ohmic heating is mounted around the vacuum chamber. Disposition of the coil is arranged to minimize stray flux. A capacitor bank of 20 kJ gives a maximum loop voltage $V_L$ of 60 V. The duration of the discharge is 2 msec. A glow discharge is applied[8] in order to clean the vacuum wall and the helical conductor. Base pressure lower than $5\times10^{-8}$ Torr is attained with an oil-free pumping system. Helium gas is used in most cases. By using various preionization methods (such as an electron gun, electron-cyclotron resonance heating, and radio frequency), we can obtain the Ohmic discharge at filling pressure $p_f$ higher than $3\times10^{-5}$ Torr. The plasma current $I_{0H}$ is measured by Rogowskii coils arranged inside the helical coil. The boundary of the plasma is measured by electrostatic probe. The electron density $n_e$ and the temperature $T_e$ are estimated from microwave interferometry and visible light spectroscopy measurements, respectively. The ion temperature $T_i$ is measured by Doppler broadening. Typical parameters are $I_{0H} \lesssim 12$ kA, $n_e = 10^{12}-10^{14}$ cm$^{-3}$, $T_e = 10-100$ eV, $T_i = 5-30$ eV and $\beta = 0.003-0.12$.

In order to have closed magnetic surfaces in the helical Heliotron field, it is necessary to apply an appropriate vertical field. The plasma itself produces an effective vertical field which displaces the magnetic surfaces. In the Ohmic-heating experiments, an additional vertical field is produced by the current ring and the stray