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X-Ray to Visible Wavelength Ratios

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The lattice repeat distance of a nearly perfect single crystal of silicon has been measured in terms of the visible wavelength of a stabilized He-Ne laser. This crystal subsequently has been used to diffract reference x-ray lines (Cu $K\alpha_1$, Mo $K\alpha_1$) thereby establishing their wavelength relative to visible standards. In terms of the x-ray scale in which $\lambda(\text{Cu } K\alpha_1) = 1.537\,400$ kxu, the conversion factor is $\Lambda_{\text{Cu}} = 1.002\,0802$ Å/kxu (1 ppm); if $\lambda(\text{Mo } K\alpha_1) = 0.707\,831$ kxu, $\Lambda_{\text{Mo}} = 1.002\,1017$ Å/kxu (0.6 ppm).

We have measured the wavelengths of two x-ray reference lines, Cu $K\alpha_1$ and Mo $K\alpha_1$, in terms of the 633-nm wavelength of a ^{3}He-^{20}Ne laser stabilized with respect to its Lamb dip. The x-ray measurements were carried out in two steps: In the first of these, we measured the spatial periodicity of a silicon single crystal by means of simultaneous x-ray and optical interferometry of displacements along the (110) crystallographic direction. In the second, we used a closely related sample of this material to diffract Cu $K\alpha_1$ and Mo $K\alpha_1$ x-radiation and obtained the wavelengths via double-crystal spectrometry. This work is a part of a larger measurement program which has been outlined previously[1] and is the first report of results at or exceeding the targeted accuracy.

The principle of the combined x-ray and optical interferometer used to obtain the results reported here is illustrated in Fig. 1. The standing x-ray wave field produced by the two crystals belonging to the stationary assembly, *a*, is intercepted by the third crystal which is part of the movable assembly, *b*. Such a symmetric, Laue-case interferometer produces cosine fringes observed at the x-ray detector, *c*, with translation along the crystal diffraction vector as suggested by the large arrow.[2] The optical cavity indicated has one mirror attached to the stationary assembly and one to the moving assembly. It is a high-finesse Fabry-Perot interferometer whose resonant transmission maxima are detected by the photomultiplier indicated at *d*.

The silicon crystal from which the x-ray interferometer was made was dislocation-free, vacuum float-zoned material.[3] The optical cavity was hemispherical and operated with a mean order number of 10^3 and a working finesse of 10^3 or more. The curved mirror radius was 120 cm so that the Fresnel phase shift,[4] viz., $\pi^{-1}\cos^{-1}[(1 - L/R)^{1/2}]$, gave a correction of 2.50 ppm near midrange. For measurements made over the entire range, $0.8\times10^3\lambda/2 < L < 1.4\times10^3\lambda/2$, this correction would have varied from 2.8 to 2.2 ppm had use been made of the whole range. Laser output was mode matched into the optical cavity by a single lens.[4] Decoupling of the laser from the cavity was effected by a quarter-wave-plate,

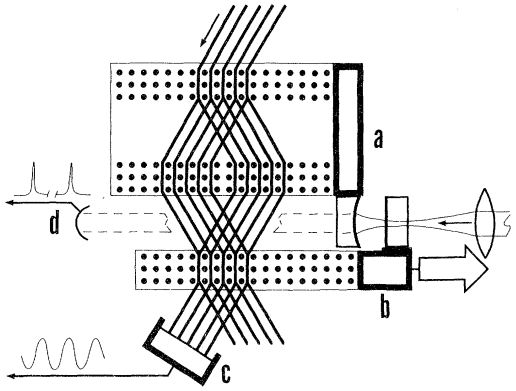

FIG. 1. Principle of simultaneous x-ray and optical interferometry of a common base line. Stationary and moving parts are labeled *a* and *b*, respectively. The x-ray detector is *c* and the optical detector is *d*.

Glan-Thomson–prism combination followed by a neutral attenuator whose transmission was about 10^{-2}.

The laser was calibrated by a beat-frequency measurement against a reference laser stabilized with respect to a saturated absorption peak in $^{129}I_2$, namely, the k peak.[5] This molecularly stabilized laser has been compared in Ref. 5 with the presently defined wavelength standard (the 606-nm line of ^{86}Kr) to an accuracy limited only by the characteristics of the krypton standard. The frequency of this molecularly stabilized laser is also available at the accuracy required in the present work via recent measurements which connect through the ^{86}Kr standard[6] and work in progress which is independent of the krypton standard.[7]

The translation stage and the x-ray/optical interferometer (XROI) were improved relative to their previous state[1] by (1) more accurate grinding of all flexure-defining holes in the translation stage with an intentional reduction of "web" thicknesses; (2) emplacement of a new x-ray interferometer of better quality; (3) use of more closely coupled optical components; (4) careful restraint of the basic structure of the translation stage, so as to compensate for residual distortions during motion; (5) operation in a vacuum. These improvements resulted in a system with a closed-loop noise level of less than 0.01 Å capable of traversing more than 100 optical fringes without any noticeable loss of x-ray fringe contrast.

Data were obtained by recording the phase of the x-ray signal as could be inferred from the x-ray intensity, first at successive optical fringes, then at intervals of 4, and finally at intervals of 18 to 22 optical fringes. When the ratio of optical to x-ray period is written as an integer (1648) plus a fraction f, the final scan interval of n optical fringes is chosen so that $nf \approx m\pi$. Shorter-interval scans have previously resolved the integer m, so what is measured is the small residual difference, $\varphi = nf - m\pi$; thus $f = (\varphi + m\pi)/n$ is determined with even greater precision. Several systematic problems were encountered near the 0.1-ppm level so that, for the present purposes, we are inclined to treat them collectively in a stochastic fashion. Our results are therefore summarized in a histogram, Fig. 2, where entries are weighted inversely with drift and directly with frequency of occurrence. The curve indicated is a Gaussian obtained by least-squares fitting of the data from approximately forty runs.

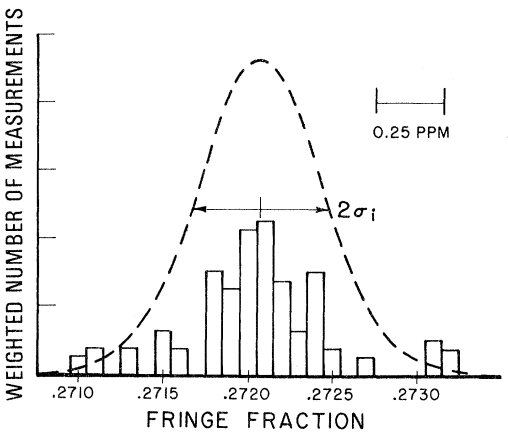

FIG. 2. Distribution of fringe fraction results for XROI measurements. The integer part of the ratio is 1648. The number of measurements is forty, and they are weighted inversely with drift during measurement in forming this distribution. The track mark equals the standard deviation for a single trial.

While the fit is not entirely satisfying as yet, the contribution from this source of error is small compared to those from such uncertainties as temperature and parallelism. Data shown in Fig. 2 were referred to a common temperature, nominally 25°C, using $\alpha = 2.57 \times 10^{-6}\ K^{-1}$.[8] Since all measurements were made within 0.5 K of the nominal temperature, the 1% uncertainty in α is not troublesome. On this basis, the formal result of the data for the mean value of the ratio of the optical to the lattice period is 1648.2721 with a 0.05-ppm value for σ_m.

The total uncertainty σ_T customarily assigned to an experimental result is the square root of the sum of the squares of the statistical uncertainty of the mean, σ_m, and estimates of the uncertainties due to systematic effects which might influence the results. On this basis we obtain a total uncertainty σ_T for the XROI lattice-parameter measurement of 0.30 ppm; this takes into account uncertainties due to possible relative motion of x-ray and optical interferometers, temperature, laser stability and wavelength, cavity size, and σ_m. Thus, the period ratio indicated above, combined with the laser wavelength, 632 991.452 pm (0.01 ppm), gives $d_{220} = 192.017\,15 \pm 0.000\,06$ pm (0.30 ppm) at 25°C

The second part of our measurements used a Si crystal which had been cut from an adjacent section of the same boule as the interferometer used in part one. We have several chains of inference asserting that the grating spacing of the

diffraction specimen used here (taken 1 cm away from the interferometer specimen) is equal to that of the interferometer specimen to 0.1 ppm or better. These inferences are based on (1) experience with other similar boules in regard to lattice parameter variation, (2) density measurements on similar boules, and (3) the existence of essentially "flat" patterns in the static interferograms of both crystals.

Measurements of the diffraction profiles and angles were made using a double crystal spectrometer[9] with a Si(111) sample for the first crystal. Angle measurements were effected by a combined digital angle generator and a two-beam angle interferometer. The former was calibrated with the aid of an optical polygon, and then used to establish the scale (interferometer constant) for the angle interferometer. Output from the interferometer was quantized (digitized) in "quadrants" each amounting to about $\frac{1}{28}$ arc sec. The overall accuracy of these measurements was consistent with this quantization.

The results of the interferometric and goniometric measurements may be combined through the Bragg equation $n\lambda = 2d[1 - (4d^2/n^2)\delta/\lambda^2]\sin\theta$ to yield a value for the (peak) wavelengths of Mo $K\alpha_1$ and Cu $K\alpha_1$. Data were taken using the (110) crystal planes in transmission and the (111) planes in reflection. In symmetric transmission the index of refraction correction vanishes; in reflection, this correction was made using $\delta/\lambda^2 = 3.22\times10^{-6}$ Å^{-2} for silicon.[10] The results of these measurements are summarized in Table I. As indicated in the table, the total uncertainty for the diffraction measurements—taking into account uncertainties of lattice parameter, index of refraction, temperature, angle measurement, alignment, and σ_m—is 0.5 ppm, except for the Cu $K\alpha_1$ transmission case where it is 0.7 ppm.

The Mo $K\alpha_1$ measurements give essentially the same wavelength in transmission as reflection, whereas for Cu $K\alpha_1$ these two types of measurements differ by ~1 ppm. In view of the problems encountered at longer wavelengths[11] the average value is still probably the best choice with somewhat increased total uncertainty. Taking the average of the reflection and transmission values, these yield λ(Mo $K\alpha_1$) = 0.709 3187 Å (0.6 ppm) and λ(Cu $K\alpha_1$) = 1.540 5981 Å (1 ppm). If Mo $K\alpha_1$ is taken as the basis of an x-ray wavelength scale where λ(Mo $K\alpha_1$) = 0.707 831 kxu,[12] then the conversion factor Λ_{Mo} = 1.002 1017 Å/kxu (0.6 ppm). If, however, one prefers the x-ray scale in which Cu $K\alpha_1$ has the "defined" value of 1.537 400 kxu, then the conversion factor is Λ_{Cu} = 1.002 0802 Å/kxu (1 ppm). Finally, if we follow Bearden's proposal[13] that the definition be λ(W $K\alpha_1$) = 0.209 0100 Å* and use the ratio for λ(Mo $K\alpha_1$)/λ(W $K\alpha_1$), being 3.393 620 (1.7 ppm),[14] then Λ^* = 1.000 0256 Å/Å* (1.8 ppm).

TABLE I. Summary of wavelength measurements. The columns indicate successively the crystal planes used and whether the crystal was in transmission (T) or reflection (R); average Bragg angle in degrees; resultant wavelength in angstroms, where the uncertainty is 1 standard deviation; N, the number of measurements involved.

Planes	Bragg angle	Wavelength	N
		Mo $K\alpha_1$	
(440) T	21.678 850	0.709 3186 (0.5 ppm)	86
(444) R	26.899 637	0.709 3188 (0.5 ppm)	40
		Cu $K\alpha_1$	
(220) T	23.650 829	1.540 5990 (0.7 ppm)	20
(333) R	47.475 712	1.540 5973 (0.5 ppm)	48

The uncertainty contributed to Λ by the XROI measurement is evidently smaller than that contributed by the goniometric measurement. This is not a defect in our procedures but rather follows from large intrinsic widths (~300 ppm) of x-ray lines, the shot-noise–limited conditions under which they are normally observed, and the lack of a theoretical model for line shape in this region. Our diffraction angle measurements give results at approximately the limiting level of accuracy permitted by the x-ray lines themselves. On the other hand, the higher lattice-parameter accuracy available may be useful with regard to γ-ray wavelengths, should values of such be required to much better than 1 ppm. The lattice parameter accuracy available will also be more fully exploited in a determination of Avogadro's constant, which is currently underway.

We may now turn to a comparison of these results with previous measurements, and, finally, consider comparison with indirect evidence.

In dealing with Λ we need only consider direct ruled-grating measurements and calculated hydrogenlike spectra. The latter are of inferior precision and accuracy so we consider only direct ruled-grating measurements. The best and most recent of these is of Henins.[15] He reports Λ_{Cu} = 1.002 0655 Å/kxu (9.8 ppm). Our new result is seen to be within his error estimates, but is about a factor of 10 better.

By way of indirect evidence, the most current

readjustment of the constants by Cohen and Taylor[16] uses the $N_A\Lambda^3$ results of Henins and Bearden,[10] and of Bearden,[17] together with the two most recent μ_p/μ_n measurements (which can be arranged to yield Avogadro's constant, N_A) to obtain a value of Λ. In terms of Cu $K\alpha_1$ = 1.537 400 kxu, their result is 1.002 0772 Å/kxu (5.3 ppm) which differs by 3.0 ppm from our direct measurement.

The authors wish to thank Dr. William C. Sauder and Dr. Ernest G. Kessler, Jr., for helpful discussions.

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[3]This sample was produced several years ago by Dow Chemical and is of the type which was marketed under the trade name Perfex.

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Observation of the $g-2$ Resonance of a Stored Electron Gas Using a Bolometric Technique*†

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A bolometric technique is used to determine the g-factor anomaly a of the free electron from measurements made on 78°K electrons contained in a Penning-style quadrupole ion trap. The value obtained is $a = 0.001\,159\,667(24)$, where the electron g factor is given by $g = 2(1+a)$.

Precision measurements of the g-factor anomaly a [where $g = 2(1+a)$] of the free electron have provided the most sensitive tests of QED up to the present time. In this paper a new method of measuring a is described which differs in every detail from the method used to obtain the most precise, currently accepted experimental value of a.[1]

We have applied a bolometric technique described earlier[2,3] to the measurement of the cyclotron and the $g-2$ resonances of an electron gas stored in a quadrupole ion trap. The value of a for the free electron is then determined using corrections derived from direct measurements of the perturbations caused by the trapping fields and the stored electrons.