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[8]Note that $L_+(0)|\rangle = \langle|L_-(0) = 0$.

[9]In Eq. (D13) of I, $i\alpha_2$ was neglected in accordance with the approximation used there.

[10]For an electric dipple $\vec{d}$ with the conventional interaction Hamiltonian $-\vec{d}\cdot\vec{A}/c$, we have $|\gamma(\omega_k)|^2 = 16\pi d^2\omega^2/3\hbar\omega_k V$, which yields the results of Ref. 2.

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Interpretation of Radiative Corrections in Spontaneous Emission*†

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Whether the origin of radiative line shifts and widths in spontaneous emission should be attributed to vacuum field fluctuations or to quantum electrodynamic radiation reaction is shown to depend on the ordering of *commuting* atomic and field operators.

A well-known heuristic argument due to Welton[1] shows that vacuum-field fluctuations can be considered a physical basis for atomic level shifts. Very recently, however, Ackerhalt, Knight, and Eberly (AKE)[2] have advanced a fully quantum-electrodynamic treatment of spontaneous emission which attributes the radiative level shift and width to radiation reaction. In their treatment radiative corrections are seen as due entirely to the atom's own source field, and not at all to the field's vacuum fluctuations.

In this Letter an attempt is made to explain these differing perspectives. While the AKE calculation is interesting in its own right, we show below that the AKE results also point to a feature of quantum theory that, we believe, has not been noted before. We show that an apparently central role in the interpretation of quantum-mechanical calculations may be played by the ordering of *commuting* operators.

For simplicity we consider the quantum-electrodynamic radiative corrections to a fictitious atom having only two energy levels. It is then described by R_3, R_+, and R_-, the energy, raising, and lowering operators, which are normalized to satisfy the usual commutation rules, $[R_3,R_\pm] = \pm R_\pm$ and $[R_+,R_-] = 2R_3$. We take the interaction Hamiltonian in the dipole approximation and neglect the A^2 term (we use the notation of Ref. 2).

The Hamiltonian for the illustrative problem reduces to

$$H = \hbar\omega_0 R_3 + i(\omega_0 d/c)[R_+ - R_-]A_d(0) + \sum_\lambda \hbar\omega_\lambda a_\lambda^\dagger a_\lambda, \tag{1}$$

where ω_0 is the unperturbed transition frequency between the two atomic states and d is the magnitude of the electric dipole matrix element between the two unperturbed energy eigenstates. $A_d(0)$ is the component of the vector potential, $\vec{A}(\vec{r})$, along the direction of the dipole moment, evaluated at the center of the atom,

$$A_d(0) = \sum_\lambda \left(\frac{2\pi\hbar c^2}{\omega_\lambda V}\right)^{1/2} \epsilon_{\lambda d}(a_\lambda + a_\lambda^\dagger). \tag{2}$$

We abbreviate below

$$f=\frac{\omega_0 d}{\hbar c} \text{ and } g_{\lambda d}=\left(\frac{2\pi\hbar c^2}{\omega_\lambda V}\right)^{1/2}\epsilon_{\lambda d}.$$

Using this Hamiltonian the Heisenberg equations of motion for the atomic and field operators are

$$\dot{R}_3=f(R_+ +R_-)A_d(0), \tag{3a}$$

$$\dot{R}_+ - i\omega_0 R_+ = -2fR_3A_d(0), \tag{3b}$$

$$\dot{a}_\lambda + i\omega_\lambda a_\lambda = fg_{\lambda d}(R_+ - R_-). \tag{3c}$$

The right-hand sides of these equations represent the perturbations on the atom and field due to their interaction. Denoting the contributions of these small terms by δ we may write Eq. (3) in integrated form:

$$R_3(t)=\tilde{R}_3(t)+\delta R_3(t), \tag{4a}$$

$$R_+(t)=\tilde{R}_+(t)+\delta R_+(t), \tag{4b}$$

$$a_\lambda(t)=\tilde{a}_\lambda(t)+\delta a_\lambda(t), \tag{4c}$$

where the unperturbed solutions are $\tilde{R}_3=R_3(0)$, $\tilde{R}_+=R_+(0)\exp(i\omega_0 t)$, and $\tilde{a}_\lambda=a_\lambda(0)\exp(-i\omega_\lambda t)$. When the interaction is included, in addition to the rapid variation of $R_+(t)$ on a time scale of ω_0^{-1}, there will be a slow variation representing the atomic decay. This slow variation will result in a shift and broadening of the unperturbed transition frequency ω_0.

We use the method of Ackerhalt, Knight, and Eberly to calculate this shift and broadening for a spontaneously radiating atom. Namely, we find an approximate equation of motion for $\langle R_+(t)\rangle$ which is valid for times long compared with ω_0^{-1}. The expectation value $\langle R_+(t)\rangle$ of the operator $R_+(t)$ is calculated in the state which corresponds to the field being in its vacuum state and the atom in some arbitrary state at $t=0$. The equation we seek to approximate is then

$$\langle\dot{R}_+\rangle - i\omega_0\langle R_+\rangle = -2f\langle R_3A_d(0)\rangle = -2f\textstyle\sum_\lambda g_{\lambda d}\langle R_3a_\lambda + R_3a_\lambda^\dagger\rangle. \tag{5}$$

In order to approximate the operators on the right-hand side we must explicitly integrate their equations of motion to find δR_3 and δa_λ:

$$R_3(t)=\tilde{R}_3(t)+f\textstyle\sum_\lambda g_{\lambda d}\int_0^t dt'\,[R_+(t')+R_-(t')][a_\lambda(t')+a_\lambda^\dagger(t')], \tag{6a}$$

$$a_\lambda(t)=\tilde{a}_\lambda(t)+g_{\lambda d}\textstyle\int_0^t dt'\exp[-i\omega_\lambda(t-t')][R_+(t')-R_-(t')]. \tag{6b}$$

The second terms on the right represent the small corrections due to the atom-field interaction.

Since the interaction is small, we can approximate the operators' dynamics by their unperturbed evolution. We replace the operators $R_\pm(t')$ and $a_\lambda(t')$ by $R_\pm(t)\exp[\mp i\omega_0(t-t')]$ and $a_\lambda(t)\exp[i\omega_\lambda(t-t')]$, respectively, and obtain the first-order expressions

$$\delta R_3(t)=f\textstyle\sum_\lambda g_{\lambda d}[R_+(t)a_\lambda(t)I(\omega_\lambda-\omega_0;t)+R_+(t)a_\lambda^\dagger(t)I(-\omega_\lambda-\omega_0;t)+R_-(t)a_\lambda(t)I(\omega_\lambda+\omega_0;t) + R_-(t)a_\lambda^\dagger(t)I(-\omega_\lambda+\omega_0;t)], \tag{7a}$$

$$\delta a_\lambda(t)=fg_{\lambda d}[R_+(t)I(-\omega_\lambda-\omega_0;t)-R_-(t)I(-\omega_\lambda+\omega_0;t)], \tag{7b}$$

where

$$I(x;t)=\textstyle\int_0^t dt'\,e^{ixt'}.$$

Caution is required when using the approximate expressions (7). Since the atom and field operators at equal times commute with each other, their first-order approximations obtained from substitution of (7) into (4a) and (4c) should also commute, at least to first order. Now while the unperturbed portions of R_3 and a_λ commute, δR_3 does not commute with $\tilde{a}_\lambda$, nor does δa_λ commute with $\tilde{R}_3$. However, the noncommuting parts cancel each other, so that the total expressions for a_λ and R_3 do commute to first order. Thus, the net result is the same no matter which ordering we use for products of atomic and field operators in (5). Nevertheless, this lack of detailed commutivity does have a serious effect on the physical interpretation.

Consider the right-hand side of (5) in normal order, so that $a_\lambda^\dagger$ appears to the left of R_3 and a_λ to the right of R_3. Upon substitution of (4c) and (7b) into (5), we find that the entire contribution to the right-hand side is due to the terms $\delta a_\lambda^\dagger R_3 + R_3\delta a_\lambda$. However, if we choose to use the symmetric ordering $\frac{1}{2}[R_3(a_\lambda+a_\lambda^\dagger)+(a_\lambda+a_\lambda^\dagger)R_3]$ in the right-hand side of (5), we find that the entire contribution is from the term $\delta R_3\tilde{a}_\lambda^\dagger + \tilde{a}_\lambda\delta R_3$. In either case (5) reduces to Eq. (8) of AKE,

$$(d/dt - i\omega_0)\langle R_+\rangle = [-i\Delta - \tfrac{1}{2}A]\langle R_+\rangle + \text{H.c.}, \tag{8}$$

where A is the Einstein spontaneous decay coefficient and Δ is the frequency shift.[3] In obtaining

(8) we have gone to the continuum limit $V \to \infty$ and used the result[4] $I(x;t) \to \delta(x) + i\mathrm{P}(1/x)$ for $t \gg \omega_0^{-1}$.

When normal ordering is employed we see that the source part of the field, δa_λ, couples with the atomic energy operator R_3 in producing the frequency shift term. Thus the radiative correction to the transition frequency is obtained through the coupling of the atom's source field with the atom itself, so we can interpret the radiative correction as entirely a radiation reaction effect.

In the case of symmetric order, however, the "vacuum" part of the field, $\tilde{a}_\lambda$, couples to the perturbation δR_3 in the atomic energy operator. After recognizing that to lowest order in f, δR_3 consists of a "vacuum" field operator multiplied by either R_+ or R_-, we clearly see that the radiative correction in this case is due to the "vacuum" field. We can therefore interpret the same correction as an effect entirely of the vacuum field fluctuations. A strong argument in favor of this interpretation can be obtained by repeating these calculations with the initial condition that the field is in some highly excited state. In this case the vacuum fluctuation term is enhanced in proportion to the strength of the field, since it contains products of the field operators, $\tilde{a}_\lambda \tilde{a}_\lambda^\dagger$ and $\tilde{a}_\lambda^\dagger \tilde{a}_\lambda$.

By choosing a suitable linear combination of these orderings, the frequency shift can even be apportioned between the radiation reaction and vacuum fluctuation interpretations according to taste. For example, if we use the ordering $R_3 a_\lambda^\dagger + a_\lambda R_3$, the vacuum fluctuation term contributes $-\Delta$ to the shift, whereas the radiation reaction term contributes 2Δ. For other possible choices neither the radiation reaction nor vacuum fluctuation term produces the entire shift; rather, each contributes a portion complementary to the other.

The equation of motion for the average of the atomic energy operator $\langle R_3 \rangle$ is

$$\langle \dot{R}_3 \rangle = -\tfrac{1}{2}A - A\langle R_3 \rangle . \qquad (9)$$

If in obtaining Eq. (9) we used normal ordering, then the decay could be considered entirely a radiation reaction effect. However, there is no ordering which would attribute the decay entirely to a vacuum fluctuation effect. The fact that atomic fluctuations are necessary for spontaneous emission has been emphasized by Fain and Khanin,[5] who used an ordering for which atomic and field fluctuations contributed equally to the natural linewidth. They point out that a ground-state atom does not undergo spontaneous absorption due to the vacuum field fluctuations because in this state they are exactly canceled by the atomic fluctuations. This fact shows the importance of having both types of fluctuations contributing to the overall physical process.

We have shown that different orderings of commuting operators in the Heisenberg picture can lead to an ambiguity in the interpretation of a physical process. In the case of spontaneous emission we found that the two interpretations, radiation reaction and vacuum fluctuations, could be almost intertwined at will. Since this arbitrariness is equivalent to the ordering of commuting operators, it must imply that they are closely related interpretations. They are, however, not fully equivalent since we cannot describe the net process exclusively in terms of vacuum fluctuations. We are, as AKE were, able to attribute spontaneous emission entirely to radiation reaction. In orderings other than normal ordering the radiative corrections must be attributed to the combined effect of both radiation reaction and field fluctuations.

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