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Radiation-Reaction and Vacuum-Field Effects in Heisenberg-Picture Quantum Electrodynamics

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It is shown that the radiation-reaction concept, which explains classical radiative effects, cannot be used exclusively to explain atomic radiative effects, and must be supplemented by consideration of the vacuum field.

It has been suggested[1] that the Heisenberg picture might be useful in a search for the resolution of well-known conceptual difficulties of quantum electrodynamics. Since these deal mainly with radiative frequency shifts, it has also been argued[2,3] that the radiation-reaction concept of classical (nonrelativistic) theory, which explains both the damping and frequency shift of classical radiators, be used to explain atomic level shifts and anomalous magnetic moments. The Heisenberg picture presents a suitable vehicle for incorporating this concept.

A Heisenberg-picture analysis[4]—hereafter referred to as I—of the interaction between a special type of atomic system (which includes a two-level system) and a generalized type of radiation field (which includes the electromagnetic field) has been performed, with decay rates and level shifts calculated but not specialized to the electromagnetic field. More recently, a similar analysis was reported for the electromagnetic field in particular,[2] in which the authors conclude that the radiation reaction does indeed explain the radiative level shift, and that the vacuum part of the field, used occasionally to explain spontaneous emission[5] and level shift,[6] "plays essentially no role in determining either the frequency shift Δ or the decay rate A." It is the purpose of the present paper to point out the following:

(1) The above conclusion is not justified when applied to the calculation of the decay rate of a two-level system (TLS).

(2) Interpretation of the anomalous moment of the electron as a radiative frequency shift for a TLS in a nonrelativistic theory yields the wrong sign; a renormalization procedure based on explicit consideration of the effect of the vacuum field will rectify the sign.

(3) The vacuum field indeed plays no role when the atomic system is a harmonic oscillator; thus, antenna theory is conceptually correct not only classically but also quantum mechanically.

We utilize the notation and results of I. The atomic system is described by the (dimensionless) angular momentum variables l_1, l_2, and l_3. The atomic Hamiltonian is $\hbar\omega l_3$, ω being positive. (For a TLS, set $l=\frac{1}{2}$.) The kth field mode is described by the Hamiltonian $\hbar\omega_k(a_k^\dagger a_k + \frac{1}{2})$, and the coupling Hamiltonian between kth mode and atom is $2^{-1/2}\hbar(a_k + a_k^\dagger)\sum_i \gamma_{ik} l_i$. Using $l_\pm = 2^{-1/2}(l_1 \pm i l_2)$, we introduce the "reduced variables" $A_k, A_k^\dagger, L_\pm$, L_3, defined by

$$a_k \equiv A_k \exp(-i\omega_k t), \quad l_\pm \equiv L_\pm e^{\pm i\omega t}, \quad l_3 = L_3,$$

which are constant in the absence of coupling and slowly varying in its presence. With

$$\begin{aligned} \mathcal{A} &= \tfrac{1}{2} i \textstyle\sum_k \gamma_k^* A_k \exp[-i(\omega_k - \omega)t], \\ \mathcal{B} &= \tfrac{1}{2} i \textstyle\sum_k \gamma_k A_k \exp[-i(\omega_k + \omega)t], \\ \mathcal{C} &= (i/\sqrt{2}) \textstyle\sum_k \gamma_{3k} A_k \exp(-i\omega_k t), \end{aligned} \tag{1}$$

where $\gamma_k \equiv \gamma_{1k} + i\gamma_{2k}$, the equations of motion for the atomic system become

$$\dot{L}_+ = \mathcal{A}^\dagger L_3 - L_3 \mathcal{B} + L_+ \mathcal{C} - \mathcal{C}^\dagger L_+, \tag{2a}$$

$$\dot{L}_- = L_3 \mathcal{A} - \mathcal{B}^\dagger L_3 + \mathcal{C}^\dagger L_- - L_- \mathcal{C}, \tag{2b}$$

$$\dot{L}_3 = -(L_+ \mathcal{A} + \mathcal{A}^\dagger L_-) + \mathcal{B}^\dagger L_+ + L_- \mathcal{B}. \tag{2c}$$

The expressions for $\dot{A}_k$ and $\dot{A}_k^\dagger$ are linear in $L_\pm$, L_3; integration, substitution into Eqs. (1), and approximation based on the fact that the number of modes is large and closely spaced in frequency leads to the results,[4]

$$\begin{aligned} &\mathcal{A} \simeq \mathcal{A}_0 + (\alpha_1 - i\alpha_2) L_-, \\ &\mathcal{B} \simeq \mathcal{B}_0 - i\alpha_3 L_+, \quad \mathrm{C} \simeq \mathrm{C}_0 - i\alpha_4 L_3, \end{aligned} \tag{3}$$

where

$$\begin{aligned} \alpha_1 &= \tfrac{1}{4}\pi |\gamma(\omega)|^2 \rho(\omega), \\ \alpha_2 &= \tfrac{1}{4} \mathrm{P} \int_0^\infty d\omega_k |\gamma(\omega_k)|^2 \rho(\omega_k)(\omega_k - \omega)^{-1}, \\ \alpha_3 &= \tfrac{1}{4} \int_0^\infty d\omega_k |\gamma(\omega_k)|^2 \rho(\omega_k)(\omega_k + \omega)^{-1}, \end{aligned}$$

$\rho(\omega_k)$ being the density of modes at ω_k, and $|\gamma(\omega_k)|^2$ being the average of $|\gamma_k|^2$ over all modes with frequency near ω_k. An explicit expression for α_4 will not be needed. $\mathcal{A}_0$, $\mathcal{B}_0$, and $\mathcal{C}_0$ are obtained from $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ by replacing $A_k(t)$ with $A_k(0)$, and represent the free-field values of $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$. The second term in Eqs. (3) is the source field; it is the part of the field generated by the atomic system and reacts back on the system [by substitution from Eqs. (3) into Eqs. (2)] to produce the "radiation reaction."

Note that each term in Eqs. (2) involves a product of variables which commute. After substitution from Eqs. (3), however, the factors in the individual products no longer commute. The order of the factors in Eqs. (2), as written, is such that the vacuum expectation values of all terms containing the free-field variables vanish after the substitution,[4] the only terms left being the expectation values of the atomic variables. It thus *appears* as though the vacuum field has no effect on the solution for the vacuum expectation values of the atomic variables, and only the source field (or the radiation reaction) is involved. However, the order of the factors in Eqs. (2) has been deliberately chosen to yield normal ordering for the free-field variables. This choice automatically determines the order of the factors $L_\pm, L_3$ in the nonvanishing terms, and affects their expectation values. If we choose the opposite order for the factors in the terms of Eqs. (2), substitution from Eqs. (3) will result in antinormal ordering, the vacuum expectation value of the terms containing the free-field variables will *not* vanish, and the terms containing atomic variables only will not necessarily have the same expectation values.

For purposes of calculating decay rate, or spontaneous emission, we may neglect $\mathcal{B}$ and $\mathcal{C}$.[7] Substitution from Eqs. (3) into (2c) then yields

$$\dot{L}_3 = -(L_+ \mathcal{A}_0 + \mathcal{A}_0^\dagger L_-) - 2\alpha_1 L_+ L_-. \tag{4}$$

An equally valid expression results from inverting the order of the factors in the terms of Eqs. (2), namely

$$\dot{L}_3 = -(\mathcal{A}_0 L_+ + L_- \mathcal{A}_0) - 2\alpha_1 L_- L_+. \tag{5}$$

We calculate, up to second order in perturbation theory, the (spontaneous) emission rate when the atomic system is in its highest-energy state and the field is in its vacuum state. Since α_1 is a second-order constant, we can replace $L_\pm(t)$ by $L_\pm(0)$ in the last term of Eqs. (4) and (5). Equation (4) yields

$$\langle \dot{L}_3 \rangle = -2\alpha_1 \langle L_+(0) L_-(0) \rangle = -2\alpha_1 L_0, \tag{6}$$

where $L_0 = l$, the total angular momentum quantum number. Equation (5), on the other hand, yields[8]

$$\langle \dot{L}_3 \rangle = -\langle \mathcal{A}_0 L_+ + L_- \mathcal{A}_0^\dagger \rangle, \tag{7}$$

which, up to second order, becomes

$$\langle \dot{L}_3 \rangle = -L_0 \int_0^t dt_1 \langle \mathcal{A}_0(t) \mathcal{A}_0^\dagger(t_1) + \mathcal{A}_0(t_1) \mathcal{A}_0^\dagger(t) \rangle. \tag{8}$$

An evaluation by the methods of I gives[9]

$$\int_0^t dt_1 \langle \mathcal{A}_0(t_1) \mathcal{A}_0^\dagger(t) \rangle = \alpha_1 + i\alpha_2. \tag{9}$$

Substitution from Eq. (9) and its complex conjugate into Eq. (8) produces a result identical to that of Eq. (6). However, while in Eqs. (4) and (6) the spontaneous emission is displayed formally as a consequence of the source field only, in Eqs. (5) and (8) the spontaneous emission is displayed formally as a result of the vacuum field only. One sees that the statements "spontaneous emission is produced by the vacuum field" and "spontaneous emission is produced by reaction of the source-field," when referring to a TLS in its upper state, are merely two sides of the same quantum-mechanical coin, with each statement by itself being an oversimplification motivated by the ordering scheme adopted.

Consider now the frequency shift for a TLS. In the Heisenberg picture, a frequency shift is exhibited by a change in the frequency of oscillation of $l_\pm(t)$. We consider the freely decaying system and look at the vacuum expectation value of $L_+(t)$. A factor $\exp(i\Delta t)$ in the resulting expression yields a frequency $\omega+\Delta$ for the radiating TLS. From I,

$$\Delta = -\tfrac{1}{2}(\alpha_2-\alpha_3) = -\tfrac{1}{8}\mathrm{P}\int_0^\infty d\omega_k\, |\gamma(\omega_k)|^2\rho(\omega_k)f(\omega,\omega_k), \tag{10}$$

where $f(\omega,\omega_k)\equiv(\omega_k-\omega)^{-1}-(\omega_k+\omega)^{-1}$. As in the case of the decay rate, this result can be obtained in two ways. By normal ordering, which suppresses the formal effect of the vacuum field, we obtain[4]

$$\langle\dot{L}_+\rangle \simeq -\tfrac{1}{2}[\alpha_1+i(\alpha_2-\alpha_3)]\langle L_+\rangle, \tag{11}$$

where the expectation-value brackets refer only to the vacuum state of the field. However, by writing Eqs. (2) in antinormal form, substituting from Eqs. (3) as well as from the integrals of Eqs. (2), and ignoring higher-than-second-order effects, we have

$$\langle\dot{L}_+\rangle \simeq \tfrac{1}{2}[\alpha_1+i(\alpha_2-\alpha_3)]\langle L_+\rangle - \langle L_+\rangle\int_0^t dt_1\langle\mathcal{A}_0(t_1)\mathcal{A}_0^\dagger(t)+\mathcal{B}_0(t)\mathcal{B}_0^\dagger(t_1)\rangle. \tag{12}$$

Clearly, the first term comes from the source field and the second term from the vacuum field. Utilizing Eq. (9) and the similarly derived relationship

$$\int_0^t dt_1\langle\mathcal{B}_0(t)\mathcal{B}_0^\dagger(t_1)\rangle = -i\alpha_3, \tag{13}$$

we reproduce the result of Eq. (11). The frequency shift obtained from Eq. (12) can be attributed to the two fields, as follows:

$$\Delta = [\tfrac{1}{2}(\alpha_2-\alpha_3)]_{\mathrm{source}} - [\alpha_2-\alpha_3]_{\mathrm{vacuum}}. \tag{14}$$

An electron in a dc magnetic field H_0 may be considered, if we ignore orbital motion, as a two-level spin system with $\omega=(e/mc)H_0$. The magnetic dipole coupling to the electromagnetic radiation field is given by $|\gamma(\omega_k)|^2 = 8\pi e^2\hbar\omega_k/3m^2c^2V$, where V is the normalization volume.[10] Since $\rho(\omega_k)=V\omega_k^2/\pi^2c^3$, one obtains

$$\tfrac{1}{2}(\alpha_2-\alpha_3) = \frac{2}{3\pi}\omega\frac{e^2\hbar}{m^2c^5}\mathrm{P}\int_0^\Omega d\omega_k\frac{\omega_k^3}{\omega_k^2-\omega^2}, \tag{15}$$

where a cutoff Ω has been introduced. If Ω is chosen to be of the order of magnitude of $mc^2/\hbar$, as is customary in nonrelativistic theories,[11] we can approximate the integrand by ω_k, obtaining

$$\tfrac{1}{2}(\alpha_2-\alpha_3) \sim \frac{2}{3\pi}\frac{e^2}{\hbar c}\frac{e\hbar}{2mc}\frac{H_0}{\hbar}. \tag{16}$$

Note that $\frac{1}{2}\hbar\Delta/H_0$ is a change in the effective moment. Since the anomalous moment is approximately $(2\pi)^{-1}(e^2/\hbar c)(e\hbar/2mc)$,[12] the above order-of-magnitude calculation is surprisingly good as far as the absolute value is concerned. However, it yields the wrong sign.[13] The anomalous moment has the same sign as the "bare" moment, producing an increase in ω, while Δ of Eq. (10) is negative. If the anomalous moment is to be interpretable as a radiative frequency shift for a TLS in nonrelativistic quantum electrodynamics, the way out of this dilemma appears to be a renormalization procedure which absorbs the vacuum-field contribution in Eq. (14). Then the physical anomalous moment is given by the source-field contribution only in a calculation where the vacuum-field effect must be explicitly subtracted.

How is the significance of the vacuum field to be reconciled with classical electrodynamics, in which radiative frequency shifts are due to the source field only? The answer goes beyond a statement that quantum effects become negligibly small in the classical domain. It is easily seen that for harmonic (or linear) oscillators, the pertinent effects of the vacuum field vanish even in the quantum-mechanical domain. If the harmonic oscillator is described by the Hamiltonian $\hbar\omega\times(a^\dagger a+\frac{1}{2})$, with coupling to the kth mode given by $\frac{1}{2}\hbar(a_k+a_k^\dagger)\gamma_k(a+a^\dagger)$, then, instead of Eq. (2), we obtain for the reduced variable A

$$\dot{A} = -\mathcal{A}+\mathcal{B}^\dagger, \tag{17}$$

and, in place of Eqs. (3), we have

$$\mathcal{A}=\mathcal{A}_0+(\alpha_1-i\alpha_2)A,\quad \mathcal{B}=\mathcal{B}_0-i\alpha_3A^\dagger. \tag{18}$$

The linearity of Eq. (17) obviates questions of ordering, and, after substitution from Eqs. (18) into Eq. (17), the vacuum expectation value eliminates unambiguously vacuum field effects. Thus, linear oscillators, such as antennas, cannot, in principle, experience the effect of the vacuum field.

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Interpretation of Radiative Corrections in Spontaneous Emission*†

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Whether the origin of radiative line shifts and widths in spontaneous emission should be attributed to vacuum field fluctuations or to quantum electrodynamic radiation reaction is shown to depend on the ordering of *commuting* atomic and field operators.

A well-known heuristic argument due to Welton[1] shows that vacuum-field fluctuations can be considered a physical basis for atomic level shifts. Very recently, however, Ackerhalt, Knight, and Eberly (AKE)[2] have advanced a fully quantum-electrodynamic treatment of spontaneous emission which attributes the radiative level shift and width to radiation reaction. In their treatment radiative corrections are seen as due entirely to the atom's own source field, and not at all to the field's vacuum fluctuations.

In this Letter an attempt is made to explain these differing perspectives. While the AKE calculation is interesting in its own right, we show below that the AKE results also point to a feature of quantum theory that, we believe, has not been noted before. We show that an apparently central role in the interpretation of quantum-mechanical calculations may be played by the ordering of *commuting* operators.

For simplicity we consider the quantum-electrodynamic radiative corrections to a fictitious atom having only two energy levels. It is then described by R_3, R_+, and R_-, the energy, raising, and lowering operators, which are normalized to satisfy the usual commutation rules, $[R_3,R_\pm] = \pm R_\pm$ and $[R_+,R_-] = 2R_3$. We take the interaction Hamiltonian in the dipole approximation and neglect the A^2 term (we use the notation of Ref. 2).

The Hamiltonian for the illustrative problem reduces to

$$H = \hbar\omega_0 R_3 + i(\omega_0 d/c)[R_+ - R_-]A_d(0) + \sum_\lambda \hbar\omega_\lambda a_\lambda^\dagger a_\lambda, \tag{1}$$

where ω_0 is the unperturbed transition frequency between the two atomic states and d is the magnitude of the electric dipole matrix element between the two unperturbed energy eigenstates. $A_d(0)$ is the component of the vector potential, $\vec{A}(\vec{r})$, along the direction of the dipole moment, evaluated at the center of the atom,

$$A_d(0) = \sum_\lambda \left(\frac{2\pi\hbar c^2}{\omega_\lambda V}\right)^{1/2} \epsilon_{\lambda d}(a_\lambda + a_\lambda^\dagger). \tag{2}$$