[10]The presence of ^{48}Ti in ^{46}Ti targets used in the (p,t) reaction raises some questions about the reliability of the low-resolution experiment of Ref. 9 for the 1.90-MeV level.

[11]A $|\delta|=1.0$ would give 32-W.u. $E2$ and 3×10^{-3}-W.u. $M1$ components for this transition. The latter is reasonable for $\Delta T=0$ transitions in self-conjugate nuclei.

[12]If the negative parity assignment is correct, then the sequence 3^-, 4^-, 5^- seems novel in this region. In other nuclei, such as ^{40}Ca, the sequence is 3^-, 5^-, 4^-. The latter sequence is improbable in ^{44}Ti in view of the nonobservation of the second level and the strong feeding of the third level in the (p,t) reaction (Ref. 4).

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[15]The two states not included in the band structure are at 3.75 and 3.94 MeV.

Comparison of Vector Analyzing Powers in (d, d) with Those in $(d, {}^3\mathrm{He})$ and (d, t) $l=0$ Transfer Reactions*

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Vector-analyzing-power angular distributions for deuteron elastic scattering and $l=0$ pickup reactions on the same target are compared for $^{32}\mathrm{S}(d,d)^{32}\mathrm{S}$ with $^{32}\mathrm{S}(d,{}^3\mathrm{He})^{31}\mathrm{P}$, $^{30}\mathrm{Si}(d,d)^{30}\mathrm{Si}$ with $^{30}\mathrm{Si}(d,t)^{29}\mathrm{Si}$, and $^{14}\mathrm{N}(d,d)^{14}\mathrm{N}$ with $^{14}\mathrm{N}(d,t)^{13}\mathrm{N}$ at a deuteron lab energy of 15 MeV. The strong similarities between the two angular distributions for each target suggest that an alternative to the distorted-wave Born-approximation description of these direct transfer reactions may be appropriate.

The purpose of this Letter is to present experimental data which show the strong similarity of vector analyzing powers[1] of (d,d) to those of $(d, {}^3\mathrm{He})$ or (d,t) direct reactions in which the transferred nucleon initially has orbital angular momentum $l=0$ (called $l=0$ transfer). In a pickup reaction, such as $(d, {}^3\mathrm{He})$ or (d,t) at sufficiently high energies that direct-reaction mechanisms predominate, if the outgoing particle consists of a deuteron loosely associated with the transferred nucleon, then it may be scattered similarly to an elastically scattered deuteron having the same momentum as the deuteron component of the outgoing particle. If $l=0$, there is no orbital angular momentum transfer, since the nucleon is captured into ^{3}He or ^{3}H with zero orbital angular momentum. The polarization effects in the transfer reaction and in the elastic scattering may then be similar.

The weakly bound projectile (WBP) model[2] has been used in the analysis of polarization effects in $l=0$ (d,p) reactions. One of the first predictions[2] of the model, which agrees quite well with experiment,[2,3] was that the proton polarization produced in an $l=0$ (d,p) stripping reaction initiated by an unpolarized incident deuteron beam should equal the proton polarization in (p,p) for the same scattering angle and outgoing energy. It is straightforward but lengthy to show that, starting from Eq. (18) of Pearson and Coz,[2] the corresponding prediction for the pickup reactions $(d, {}^3\mathrm{He})$ or (d,t), under the same assumptions as in the WBP model, with modifications of only the spins, is that the vector analyzing power for $l=0$ transfer should be equal to that in (d,d) at the same scattering angle and incident deuteron energy.

The above WBP-model predictions follow directly from the analytical formulas. On the other hand, in the distorted-waves Born-approximation model, which is commonly used to describe such transfer reactions, no such parameter-independent predictions relating analyzing powers in $l=0$ transfer reactions to those in elastic scattering are made, and to our knowledge no similarities have been suggested by detailed calculations. There are objections[4] to the formalism of the WBP model originally presented by Pearson and Coz,[2] and an alternative development of the formalism starting from a similar physical viewpoint as in the WBP model has been proposed by Butler and co-workers.[5] The experimental results reported here suggest that some description similar to the WBP model is physically acceptable.

The experiment used the Triangle Universities Nuclear Laboratory Lamb-shift polarized-ion source[6] to obtain a beam of vector polarized deuterons by using the spin-filter technique[7] with state 3 in a zero-crossover mode.[8] The tensor polarization P was estimated to be small, $0<P_{zz}<0.05$. All data were taken with both spin up and spin down to minimize instrumental asymmetries, as in Hardekopf *et al.*[9]

Scattered deuterons, helium-3 nuclei, and tritons were detected using pairs of silicon surface-barrier detectors in a telescope arrangement, which enabled simultaneous detection of these three types of particles. Two such telescopes were used, placed at equal angles to the left and right of the incident beam. The particle-identification technique of Goulding *et al.*[10] was used. Simultaneous detection of deuterons, helium-3 nuclei, and tritons increased data collection efficiency and eliminated some systematic erros. Detector collimators had a total scattering-angle acceptance of about 6°. Detailed calculations showed that this produced negligible smearing of the analyzing-power angular distributions.

The choice of targets and bombarding energy and the selection of ^{3}He or ^{3}H particles and of states in the final nuclei were made in order to obtain $l=0$ transfers and to maximize yields. The ^{32}S target was natural S as H_2S gas at 1 atm pressure in a 2.5-cm-diam gas cell; the ^{14}N target was natural N_2 at 2 atm pressure in a similar gas cell; and the ^{30}Si target was a self-supporting foil of SiO, prepared from SiO_2 enriched to 96% in ^{30}Si. The energy losses of the 15-MeV incident deuteron beam in the H_2S, N_2, and SiO targets were 70, 200, and 6 keV, respectively. Cross sections for elastic scattering from S and N, measured at a c.m. angle of 90° in the energy range 14.0–15.5 MeV in 80-keV steps, were smooth within the statistical errors of about 3%. This suggests that direct-reaction mechanisms are predominant in this energy region.

Vector analyzing powers A_y in ^{32}S$(d,d)^{32}$S and ^{32}S$(d,{}^3\text{He})^{31}$P $(\frac{1}{2}^+, Q=-3.37$ MeV), in ^{30}Si$(d,d)^{30}$Si and ^{30}Si$(d,t)^{29}$Si $(\frac{1}{2}^+, Q=-4.36$ MeV), and in ^{14}N$(d,d)^{14}$N and ^{14}N$(d,t_1)^{13}$N* $(\frac{1}{2}^+, Q=-6.67$ MeV) in the c.m. angular range 30°–95° were obtained, as for protons,[9] in a standard way[11] from left- and right-detector and spin-up and spin-down yields. Background counts subtacted from the peaks of interest were less than 2% for all peaks, except for ^{14}N$(d,t_1)^{13}$N* for which the backgrounds were approximately 16%.

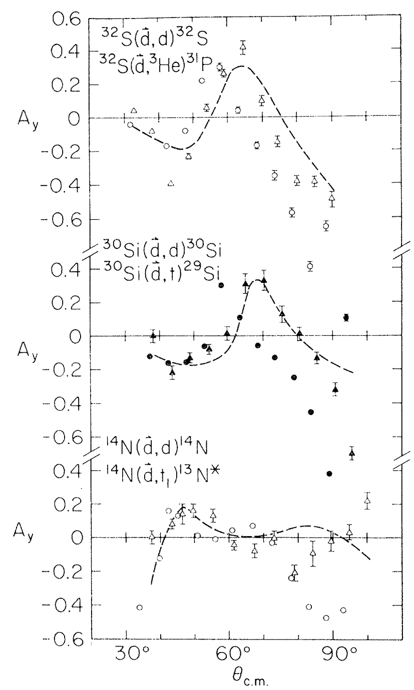

FIG. 1. Vector analyzing powers A_y in the Madison convention (see Ref. 12) *versus* c.m. scattering angle θ for 15-MeV deuterons incident on ^{32}S, ^{30}Si, and ^{14}N. Circles, (d,d); triangles, $(d,{}^3\text{He})$ or (d,t). Error bars shown are statistical only and do not include the additional error which results from the background asymmetries. When not shown the errors are smaller than the symbols. The dashed curves represent the (d,d) analyzing powers at the same deuteron momentum transfer as in the $(d,{}^3\text{He})$ or (d,t) reaction, as discussed in the text. Particularly for the ^{32}S and ^{30}Si targets, the strong similarities between these analyzing-power angular distributions in elastic scattering and in $l=0$ transfer reactions are evident.

Figure 1 shows the experimental analyzing-power angular distributions. Each of the (d,d) angular distributions shows oscillations similar to $(d,{}^3\text{He})$ or (d,t) on the same target. The agreement is comparable to that between (p,p) and (d,p) polarizations for $l=0$ transfers.[2,3] For ^{32}S and ^{30}Si only $l=0$ nucleon transfer is possible in a single-step direct reaction and $2s$ single-particle orbitals are dominant in the target ground-state wave functions.

For ^{14}N the transfer-reaction cross section is on the average a factor of 20 smaller than for the other two targets. Both $l=0$ and $l=2$ transfers are possible since ^{14}N has $J^\pi=1^+$. The probability of $2s$ and $1d$ orbitals admixed into the dominant $1p$ orbitals in ^{14}N may be about 0.1.[13] Therefore, since an $l=1$ transfer is forbidden by parity conservation, this explains why the $^{14}N(d, t_1)^{13}N^*$ cross sections are much smaller than those for the other two transfer reactions investigated here. The transition $^{14}N(d,t)^{13}N$ is therefore very sensitive to contributions from other reaction mechanisms. The predominance of $2s$ over $1d$ is not clearly established,[14] so that the requirement of $l=0$ transfer may be poorly satisfied.

The elastic-scattering data may alternatively be compared to the transfer-reaction data for equal change in deuteron momentum, instead of at equal scattering angles. This takes into account the mass and kinetic energy change in the transfer reaction. At a c.m. scattering angle θ the deuteron momentum change in elastic scattering is

$$q_e(\theta) = 2k \sin(\tfrac{1}{2}\theta), \tag{1}$$

and in the transfer reaction it is

$$q_t(\theta) = [k^2 + (k')^2 - 2kk' \cos\theta]^{1/2}, \tag{2}$$

where $k = (2m_i E/\hbar^2)^{1/2}$ and $k' = \frac{2}{3}[2m_f(E+Q)/\hbar^2]^{1/2}$, with m_i and m_f the reduced masses of the scattering pairs in initial and final states, respectively, and E the c.m. initial kinetic energy. The factor $\frac{2}{3}$ in k' is the fraction of the ^3He or ^3H momentum which is carried by the deuteron. Data can be compared at equal momentum transfers by choosing the elastic deuteron scattering angle θ_e such that

$$q_e(\theta_e) = q_t(\theta). \tag{3}$$

At each angle θ the angle θ_e was determined from Eq. (3). The dashed curves in Fig. 1 were then obtained by quadratic interpolation of A_y between the (d,d) data points in Fig. 1. The agreement between the analyzing powers in elastic scattering and in the transfer reactions is seen to be markedly improved when plotted for the same deuteron momentum change in the elastic scattering as in the transfer reaction.

We have thus shown that for light nuclei at a medium bombarding energy the vector analyzing power for (d,d) elastic scattering is similar to that for $(d,{}^3He)$ and (d,t) $l=0$ transfer reactions when the data are compared at the same momentum transfer. It would be interesting to extend the analyzing-power measurements to other targets and to higher energies than can be attained in our laboratory and to study the analyzing powers as a function of energy. Experimentally, at higher energies the detection of the ^3He and ^3H becomes easier and $l=0$ transfers to higher-excited states which have small yields for 15-MeV deuterons, especially at backward angles, can be measured. This would enable the more general validity of the present observations to be checked.

The present results may stimulate further development of models of direct transfer reactions. In particular, momentum transfer seems to be important in applying such formulations as the WBP model. Therefore such descriptions of direct transfer reactions may be applicable to cases other than (d,p), in particular to $(d,{}^3He)$ and (d,t) reactions.

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Singular-Core-Interaction and One-Pion-Exchange Constraints on the Off-Shell Two-Nucleon T Matrix*

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A method is given for generalizing the Baranger *et al.*–Haftel construction of the off-shell two-nucleon T matrix so as to incorporate the constraints due to singular core interactions (described by the boundary condition model) and the one-pion–exchange tail of the two-nucleon potential.

Baranger, Giraud, Mukhopadhyay, and Sauer[1] (BGMS) have shown how to construct a partial-wave off-shell two-nucleon T matrix, whose on-shell (phase-shift) behavior is specified, without explicitly using an underlying potential. They showed in particular that the full off-shell T matrix may be generated from an assumed form for the symmetric part $\sigma(p|k)$ of the half-off-shell T matrix $\varphi(k|p)$, where $\varphi(k|k)=(-2k/\pi)\sin\delta(k)$, $\delta(k)$ being the phase shift. This construction should prove very useful in investigations of the sensitivity of the properties of few-nucleon systems to the off-shell two-nucleon T matrix.[2] Sauer[3] has extended the BGMS analysis to the case of tensor interactions and more recently[4] has shown how to incorporate the constraints associated with the one-pion–exchange tail of the two-nucleon potential. Haftel,[5] Amado,[6] and Kowalski *et al.*[7] have modified the BGMS construction so that it may be applied when bound states are present.

The various forms of the BGMS construction just cited are *not* applicable to the case of singular core interactions. Now there are considerable theoretical grounds for assuming that the effective two-nucleon core interaction at low energies is well described by the boundary-condition model[8] which is mathematically equivalent to a singular core interaction.[9] It is thus of some interest to find a generalized BGMS construction in which the underlying two-nucleon potential has a specified singular core interaction. We present such a construction in this paper.

Consider an uncoupled two-nucleon partial-wave eigenchannel with potential $V=V_s+\tilde{V}$, where V_s is a local singular core potential and $\tilde{V}$ is the (nonsingular) potential which gives the interaction for nucleon separations $r>R_c$. The coordinate-space representation of V_s is $\langle r'|V_s|r\rangle=V_s(r)\times\delta(r'-r)/r^2$, where $V_s(r)=V_0\theta(R_c-r)-V_1\delta(r-R_c)$, $V_0\to\infty$, $V_1\to\infty$, $R_c(\sqrt{V_0}-R_cV_1)=F$, so that the radial wave function $\langle r|\psi\rangle$ has the energy-independent boundary condition

$$R_c(d/dr)\langle r|\psi\rangle|_{r=R_c}=F\langle R_c|\psi\rangle,\quad \langle r<R_c|\psi\rangle=0.$$

If there is a bound state with specified energy E_B and wave function $\langle r|\psi_B\rangle$, we assume (following Haftel[5]) that $|\psi_B\rangle$ is an eigenstate of a model Hamiltonian $H_m=K+V_s+\tilde{V}_m$ (K = c.m. kinetic energy), where $\tilde{V}_m$ is the (nonsingular) model potential which gives the interaction for $r>R_c$.[10] For no bound states, $\tilde{V}_m=0$.

Let $|\psi_k^{(0)}\rangle=\exp[-i\delta(k)]|\psi_k^{(+)}\rangle$ and $|\varphi_k^{(0)}\rangle=\exp[-i\delta_m(k)]|\varphi_k^{(+)}\rangle$ be the real scattering eigenstates[11] of the actual Hamiltonian $H=K+V_s+\tilde{V}$ and H_m, respectively, where $|\varphi_k^{(+)}\rangle$ may be calculated[9] by taking the V_0, $V_1\to\infty$ limit, mentioned above, of the solution of the equation

$$|\varphi_k^{(+)}\rangle=|k\rangle+(k^2+i\epsilon-K)^{-1}(V_s+\tilde{V}_m)|\varphi_k^{(+)}\rangle,\qquad \epsilon\to0^+. \tag{1}$$

$\delta(k)$ and $\delta_m(k)$ are, respectively, the actual and model phase shifts. $|\psi_k^{(+)}\rangle$ satisfies the usual integral equation for two-potential scattering[12]:

$$|\psi_k^{(+)}\rangle=|\varphi_k^{(+)}\rangle+(k^2+i\epsilon-H_m)^{-1}(\tilde{V}-\tilde{V}_m)|\psi_k^{(+)}\rangle,\qquad \epsilon\to0^+. \tag{2}$$

From the usual orthogonality relations and the