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Mode-Coupling Saturation of the Parametric Instability and Electron Heating*

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By solving the hydrodynamic equations for the parametric instability including mode-coupling terms, we predict the saturation level and turbulent spectrum for the oscillating two-stream instability. Coupling the electron distribution function to the spectrum with the quasilinear diffusion coefficient gives the heating rate and high-energy tail formation. These predictions agree with the results of particle simulations.

A problem of considerable interest to laser fusion is the possible formation of high-energy tails on the electron distribution function due to parametric instabilities. These tails have been observed experimentally[1,2] and in simulations.[3] Energetic electrons so formed may lead to serious preheat and decoupling problems in the design of pellets.[4] We report here a one-dimensional analysis of the saturation and heating mechanism for the parametric instability at the critical density (laser frequency equal to the plasma frequency). The analysis is based upon mode-coupling saturation of the linear instability and subsequent electron heating by quasilinear diffusion due to the saturated spectrum.

Previous authors have considered saturation of the decay instability in a plasma with comparable electron-ion temperatures.[5] The dominant nonlinearity for the plasma waves is then their nonlinear Landau damping on the ions, a process which only spreads plasma wave energy to higher phase velocities. (When $T_e \gg T_i$, this process goes over into the resonant coupling of plasma waves by ion acoustic waves.) In these analyses electron-ion collisions play a central role as the process by which energy is coupled from the waves into the particles. In general, however, plasma wave energy spreads to both lower- and higher-phase-velocity waves. Coupling of energy to lower phase velocity can result in the transfer of energy from the waves into the particles by Landau damping. Hence a stationary nonlinear state is possible even when collisions are negligible. Since the energy transfer is via Landau damping, the particle heating will be characterized by the formation of high-energy tails.

The nonlinear evolution of the parametric instabilities driven by an electric field oscillating near the plasma frequency has been extensively investigated in computer simulations. They show that for laser field intensities $\eta^2 \equiv E_0^2/4\pi n T_e \gtrsim 1$, the instability saturation is simply determined by electron trapping in the unstable plasma waves (E_0 is the laser field amplitude; n is the electron density). However, for $\eta^2 \ll 1$, trapping does not occur. In this regime the plasma wave energy saturates at a level several times the pump field energy [see Fig. 1(a)]. The particles heat linearly in time, principally because of generation of suprathermal tails on the distribution function.

In order to discuss a specific instability, let us consider the case in which the laser frequency equals the electron plasma frequency. Then only the oscillating two-stream instability occurs. In this case simulations show that, after saturation, the plasma wave spectrum ($|E_k^2|$) assumes an approximate k^{-2} shape for wave numbers greater than the linearly most unstable mode (Fig. 2). The heated-electron distribution function has a central Maxwellian, an exponential tail, and then a sharp drop (Fig. 3). Our analysis will predict all these basic features observed in the simulations.

Since the fluctuating fields in the saturated state become as large as the pump field, this suggests that mode coupling may be the saturation mechanism. We apply Nishikawa's ordering scheme[6] to the hydrodynamic equations for electrons and ions, but without neglecting the products of pairs of fluctuating quantities. The following equations

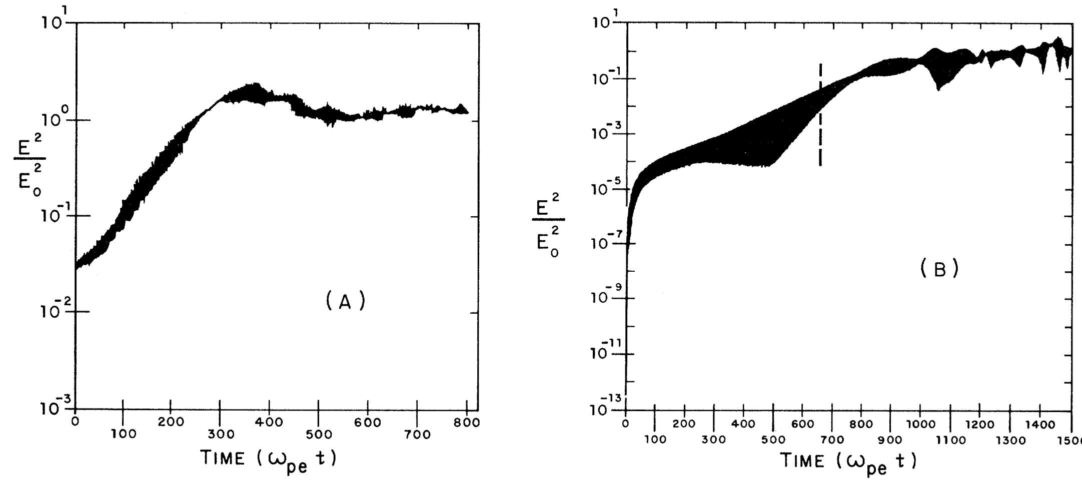

FIG. 1. Field energy as a function of time (a) from a simulation calculation, with driver strength $\eta^2=0.13$, $\omega_0=\omega_{pe}$, and $T_e/T_i=3.33$, and (b) from a hydrodynamic equation calculation with $\eta^2=0.15$, $\omega_0=\omega_{pe}$, and $T_e/T_i=3.00$. The dashed line indicates approximate starting conditions for the simulation. E^2 refers to the sum of the energy in the internal fields.

are obtained for the evolution of the electron and ion density fluctuations:

$$\frac{\partial^2}{\partial t^2} n_{ek} + \nu_{ek}\frac{\partial n_{ek}}{\partial t} + \omega_{ek}^2 n_{ek} = \frac{ike}{m_e} n_{ik} E_0 + \frac{ike}{m_e}\sum_{k'\neq 0} n_{i\,k-k'} E_{k'},$$
$$\frac{\partial^2}{\partial t^2} n_{ik} + \nu_{ik}\frac{\partial n_{ik}}{\partial t} + \Omega_k^2 n_{ik} = -\frac{ike}{m_i} n_{ek} E_0 - \frac{ike}{m_i}\sum_{k'\neq k}\frac{k'}{k-k'} n_{e\,k-k'} E_{k'}. \tag{1}$$

We have followed Nishikawa's notation; ω_{ek} is the Bohm-Gross frequency, Ω_k is the ion acoustic frequency, and ν_k is taken to be the electron or ion Landau damping frequency. We numerically solve these coupled-mode equations to predict the instability saturation.

Simultaneously, the electron distribution function is evolved by solving the quasilinear diffusion equation,[7]

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v} D \frac{\partial f}{\partial v}, \tag{2}$$

where the diffusion coefficient is known in terms of the plasma wave amplitudes determined from the mode-coupling code:

$$D(v) = \frac{e^2}{m^2}\int \frac{\gamma_k E_k^2\, dk}{(\omega_{ek} - kv)^2 + \gamma_k^2}. \tag{3}$$

Here γ_k is the width of the resonance. As long as it is small, its detailed magnitude does not matter. For convenience we have taken γ_k to be the magnitude of the linear growth rate, $0.006\omega_{pe}$, for all modes. From the updated distribution function we compute new values of ν_k and λ_D for Eqs. (1) and (2).

Figures 1(b) and 3 show the results for a typical case ($\eta^2 = 0.13$, $T_e/T_i = 3.33$ initially). As shown in Fig. 1(b) the plasma wave energy exponentiates in agreement with linear theory and then saturates at a level of $\sim 2E_0^2$. The field spec-

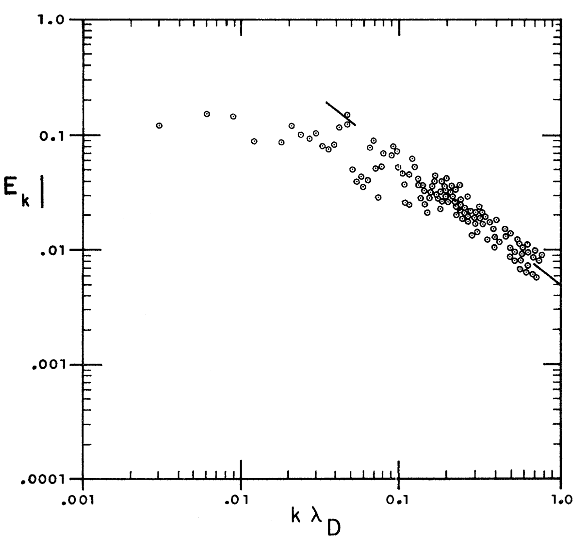

FIG. 2. Saturated turbulent field spectrum from the simulation calculation at time $\omega_p t = 703$. The straight line is for comparison to a k^{-2} spectrum.

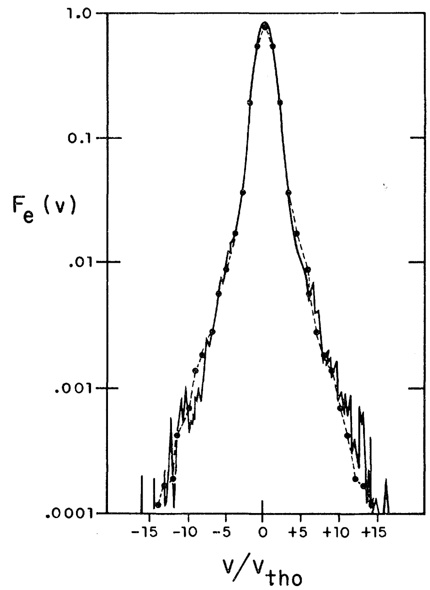

FIG. 3. Electron velocity distribution from the simulation at time $\omega_p t = 326$. Temperature at this time is $2.5T_{e0}$. The dashed line is the distribution calculated from hydrodynamic equations at a temperature of $2.5T_{e0}$.

trum obtained from the mode-coupling equations gives reasonable agreement with the k^{-2} spectrum obtained from the simulations. Figure 3 shows the good agreement between the calculated distribution function and the simulation result. After saturation, the heating rate is constant at about $0.033\omega_{pe}$, whereas the simulation result is about $0.035\omega_{pe}$. This close agreement is typical.

From quasilinear theory, we can show that the spectrum $E_k^2 \sim k^{-2}$ leads to a diffusion coefficient $D(v)$ which is linear in velocity. Since the spectrum depends on temperature (through terms like Ω_k and λ_D), $D(v)$ depends on the time-varying thermal velocity: $D(v) \sim v/v_t(t)$. A simple calculation then gives the correct conclusion that the heating rate is constant and that exponential tails are formed.

It should be noted that the k^{-2} spectrum is not a sensitive function of particle damping but results from the mode-coupling dynamics. This was seen by varying the form of ν_k in the mode-coupling equations and observing that the k^{-2} spectrum persists.

These results show that the fluid equations including mode-coupling terms and the quasilinear diffusion equation predict the basic features of the nonlinear behavior of the oscillating two-stream instability. The fact that the numerical solution of this simple set of equations agrees well with the simulations allows us to abstract the physical mechanisms important in the nonlinear state: mode-coupling of wave energy leading to saturation, and then diffusion of particles leading to formation of a suprathermal tail. We have also investigated cases in which the laser frequency is somewhat higher than the plasma frequency, so that both the oscillating two-stream and ion-acoustic-decay instabilities are operative. The distribution function produced by the turbulence then has fewer high-energy electrons, basically because lower–phase-velocity plasma waves are produced. In addition, we have considered the improvement in the turbulently heated distribution function which is due to the finite spatial extent of the turbulence in pellet applications. This effect can be simply estimated by modifying the diffusion theory to a finite-length system. These results will be reported in detail in an expanded article. Suprathermal distributions found in these analyses have been included in hydrodynamic pellet-design codes[4] in order to determine the sensitivity of the design to high-energy electrons.

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