in superconductors have long been thought to be overdamped because the electron-lattice interaction readily transfers energy and momentum from the electron-pair fluid to the lattice.[17]

The authors would like to thank Dr. Patrick Lee and Professor Douglas Scalapino for many helpful discussions. They would also like to thank the Office of Naval Research for the helium gas used in these experiments. A grant from the Research Corporation made possible the purchase of the minicomputer used in this work.

†Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1) 1569 and by the Graduate School of the University of Minnesota.

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Dynamic Instability of Vortices in Superconductors*

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(Received 9 August 1973)

We study the nonlinear response of vortices in thin-film superconductors to strong electric fields. In the region of small magnetic fields we find unstable regions of negative differential conductivity. The voltage along the film is therfore predicted to jump from a lower flux-flow value to a higher normal-state value when the transport current is increased beyond the maximum value the flux-flow state can support.

In previous publications[1-4] we have investigated the dissipation rates and changes in structure of vortices in a superconductor when forced into motion by the application of an electric field $\vec{E}$. These investigations were confined to the linear response in weak electric fields. In the present paper we extend our calculations to investigate nonlinear effects.

For the present we confine our attention to films whose thickness d is much less than the penetration depth λ for magnetic fields. For thicker films the magnetic field generated by the transport current becomes important and, even in the absence of an externally applied magnetic field, causes the superconducting structure to break up into an inhomogeneous intermediate-state structure before becoming normal. Thin films, on the other hand, remain homogeneous in this case up to the critical current, where a sudden transition to the normal state occurs with the order parameter jumping discontinuously from a finite value to zero.[5] The main result of our present work is the determination that this discontinuous behavior persists when an external magnetic field B is applied perpendicularly to the film, creating a resistive vortex state, except when B is very near the critical value H_{c2}.

We continue to use the simple time-dependent Ginzburg-Landau equations derived by Gor'kov and Éliashberg[6] for a superconductor containing a high concentration of magnetic impurities sufficient to reduce the critical temperature T_c to a value much less than its original value T_{c0}:

$$\gamma(\partial/\partial t+i2e\psi)\Delta+\xi^{-2}(|\Delta|^2-1)\Delta+(\nabla/i-2e\vec{A})^2\Delta=0, \tag{1}$$

$$\vec{j}=\sigma(-\nabla\psi-\partial\vec{A}/\partial t)+\mathrm{Re}[\Delta^*(\nabla/i2e-\vec{A})\Delta]/4\pi\lambda^2, \tag{2}$$

$$\rho=(\psi-\varphi)/4\pi\lambda_{TF}^2. \tag{3}$$

As before γ is the inverse of the diffusion constant. Δ is the order parameter divided by its value in the absence of fields. ψ, the electrochemical potential divided by the electronic charge e, is practically equal to the scalar potential φ, because the Thomas-Fermi screening length λ_{TF} is much less than the temperature-dependent coherence length ξ and penetration depth λ. σ is the normal-state conductivity. $\vec{j}$ and ρ are the current and charge densities, and $\vec{A}$ is the vector potential. (We set $\hbar=c=1$.) The mean free path for scattering without spin flip is assumed to be much shorter than that with spin flip. Higher-order nonlinear terms in these equations are of order T_c/T_{c0} and may be ignored.

To investigate the character of the phase transition in strong fields we first consider the field region where the order parameter is small everywhere. Expect for corrections of order d/λ the magnetic field B may be taken to be uniform. It is applied in the z direction perpendicular to the film. If the vector potential is chosen as $\vec{A}=B(x-v_xt)\hat{e}_y+\vec{v}\times\hat{e}_z\gamma/4e$ and with $\psi=\psi_0\equiv\vec{v}\cdot\vec{A}$, the ground-state solution to Eq. (1) linearized in Δ is just the undistorted Abrikosov[7] solution translating with a velocity v:

$$\Delta=\sum_n C_n\exp[-eB(x-v_xt)^2+ink(y-v_yt)]. \tag{4}$$

The value of $\vec{v}$ is obtained from $\vec{E}=-\vec{v}\times\vec{B}$, where $\vec{E}$ means the spatial average of $\vec{E}(x,y)$. The second-order phase boundary where $\Delta=0$ is obtained for fields satisfying

$$2eB=2eH_{c2}-(\gamma E/2B)^2, \tag{5}$$

where $H_{c2}=(2e\xi^2)^{-1}$ is the upper critical field when E is small.[8]

The curve of B versus E given by Eq. (5) is illustrated by the solid line in the inset of Fig. 1. The curve has a maximum value of E at $B=\frac{2}{3}H_{c2}$. At first this result appears peculiar since the current flowing in these fields is the normal value, σE, and one would expect the amount of current which can be carried in the superconducting state to increase monotonically as B is lowered below H_{c2}, since the magnetic field tends to destroy superconductivity. The resolution of this puzzle is found by investigating the stability of this solution.

The current flowing in the film can be calculated from Eq. (2). It consists of three parts. One is the current circulating around the vortex cores just as in the static case but now translating with velocity $\vec{v}$. This circulating current is calculated from the last term in Eq. (2), using only the first term in our expression for $\vec{A}$. The remaining terms are $\sigma\vec{E}(x,y)-|\Delta|^2\vec{v}\times\hat{e}_z\gamma/8\pi e\lambda^2$, whose space average is the transport current $\vec{j}_t$. The difference between these terms and $\vec{j}_t$ is called backflow.

By applying the continuity equation to $\vec{j}$ we can find $\psi-\psi_0$, and it has the same spatial depen-

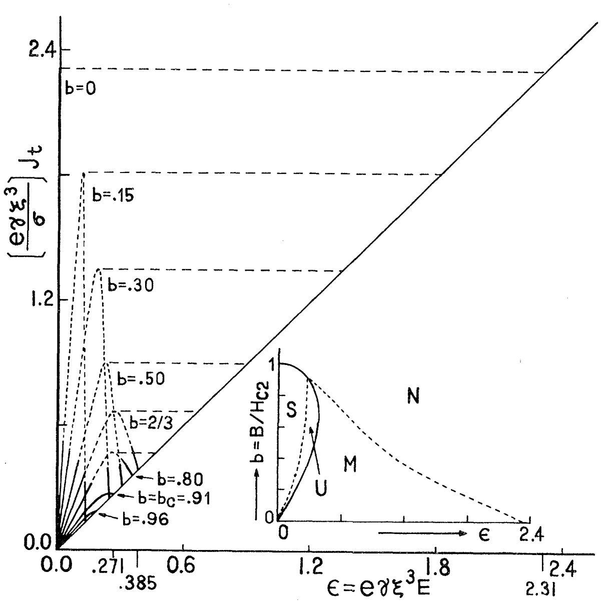

FIG. 1. The transport current j_t of a thin-film superconductor in the vortex state plotted as a function of the electric field strength E along the film for different values of the magnetic field B (normalized to H_{c2}) applied perpendicular to it. Inset: diagram of the four phases obtained for different values of E and B. S is for stable superconductivity; U, unstable superconductivity; M, metastable normal state; and N, stable normal state.

dence as found in Ref. 1, which is an odd function with respect to any vortex core. Then using Abrikosov's procedure,[7] the average value of $|\Delta|^2$ for fields near the second-order phase boundary is independent of $\psi - \psi_0$ and is found to be

$$\langle|\Delta|^2\rangle = [1-(\gamma\xi E/2B)^2 - B/H_{c2}]\beta_A^{-1}.$$

Expressing $\vec{v}$ in terms of $\vec{E}$ and $\vec{B}$ we obtain

$$\vec{j}_t = \sigma\vec{E} + \frac{[1-(\gamma\xi E/2B)^2 - B/H_{c2}]\gamma\vec{E}}{16\pi e\lambda^2\beta_A B}. \tag{6}$$

The differential conductivity $(\partial j_t/\partial E)_B \geq 0$ at $\Delta = 0$ only if $B/H_{c2} \geq \xi^2/(\xi^2 + \beta_A\zeta^2) = 0.912$, where $\zeta^2 = 4\pi\lambda^2\sigma/\gamma = \xi^2/12$. Only for this high field can this calculated curve of j_t versus E at constant B be continuous and monotonically increasing. If $B < 0.912H_{c2}$, equal values of j_t can be found in a certain range for different values of E but the same B. We think the solution with the highest conductivity, corresponding to the smallest E for fixed j_t, is the thermodynamically preferred stable one.[9] In the range where E is multivalued the magnitude of j_t is higher than that calculated for a continuous transition to the normal state from Eq. (5). When j_t reaches the maximum value for any superconducting state, E must jump discontinuously for the film to reach the normal state.

This predicted behavior is illustrated in Fig. 1. The solid line with a slope of 45° is the normal state with constant conductivity σ. The solid lines leaving this curve are calculated from Eq. (6) and represent superconducting states having higher conductivities than the normal state. The solid lines starting from the origin are calculated from our previous theory of the linear flux-flow conductivity,[1,2] with a smooth interpolation being made between our high- and low-B values. The short-dashed lines are extrapolations of these two limiting curves. The long-dashed lines, drawn from the maximum values of j_t obtained in a superconducting state across to the normal state, represent the discontinuity predicted for E when j_t exceeds this peak value. The maximum value of j_t for a superconducting state is now observed to increase continuously as B is lowered below H_{c2} to the maximum value at $B=0$, which is calculated rigorously for the state with constant Δ.

The four regions on these curves are indicated in the phase diagram in the inset to Fig. 1. For small E the state is stable and superconducting if $B < H_{c2}$, or normal if $B > H_{c2}$. If $H_{c2} > B > 0.912H_{c2}$ a continuous transition occurs at the solid line. If $B < 0.912H_{c2}$, the current reaches a peak for a stable superconducting state. The locations of such peaks are indicated by a dashed line. Beyond the value of E at this peak the superconducting state is unstable, with a negative differential conductivity. If this unstable curve could be followed as E is increased through the unstable superconducting region, the solid curve would be reached, leading to the normal state. However, we think the part of the normal-state curve first reached is only metastable, since, although the differential conductivity is positive, the stable superconducting state can carry the same current with a higher conductivity. If the voltage along the film is held constant instead of the current, the film can enter into an intermediate state with alternating stripes of the superconducting state at the maximum current and the normal state at the same current, again increasing the conductivity above the normal-state value. Only when the electric field is increased so the normal current exceeds the highest current possible for any superconducting state, as indicated by the second dashed line, is the normal state absolutely stable.

Unfortunately, in the above discussion we have not calculated the complete curves including the regions near the current maxima. Therefore, it is interesting to notice that we can calculate the complete curves for a special case considered previously in the linear-response region.[2] For this special case, $\kappa = \lambda/\xi = 1/\sqrt{2}$, and two slightly unphysical assumptions are made. First, ζ is set equal to $\lambda = \xi/\sqrt{2}$, although its physical value is $\xi/\sqrt{12}$. This change does not alter the basic structure of the equations or solutions. Secondly, the boundary conditions on the fields at the sample surfaces are ignored. This has the effect, among others, of replacing the denominator of the slope of $\langle|\Delta|^2\rangle$ versus B near H_{c2}, which is $2\kappa^2\beta_A$, by $(2\kappa^2-1)\beta_A+1$, i.e., replacing β_A by 1, implying a 16% error. Then using the method of Ref. 2 we find the exact solution:

$$\Delta = s\Delta_0((\vec{r}-\vec{v}t)s/\xi),$$
$$B = H_{c2}(s^2 - |\Delta|^2) = s^2 B_0((\vec{r}-\vec{v}t)s/\xi), \tag{7}$$
$$j_t = s^2\sigma E H_{c2}/B,$$

where $s = [1-(\frac{1}{2}\gamma\xi v)^2]^{1/2} = [1-(2\pi\sigma\xi E/B)^2]^{1/2}$ is a scaling factor for distances, and Δ_0 and B_0 are the static solutions. This solution indeed shows the behavior indicated in Fig. 1. The only difference is that the maximum values of j_t are all the same $[=(\frac{1}{3})^{3/2}2e\pi\xi^3]$ for $B \leq \frac{2}{3}H_{c2}$, with continuous transitions for larger B.

Although our calculation has been carried out in a particularly simple regime, we expect that the general behavior we predict should be observed in more general cases. Thus, further experimental work characterizing resistive transitions in thin-film superconductors should be interesting, particularly observations of when continuous transitions to the normal state are obtained as the electric field strength is increased, and when discontinuous transitions occur. Some curves showing discontinuous transitions in thin films have been obtained by Ōgushi, Takayama, and Shibuya.[10] Unfortunately, discontinuous transitions can also occur if the thermal conductivity of the substrate is too poor to remove readily the heat generated so that macroscopic hot normal regions occur, which we have not considered. Considerable care must be taken to separate the two types of effects experimentally. The thin-film regime is favorable since the total current is reduced for a constant current density. The required current density can be further reduced by working close to the reduced T_c, although fluctuations become important and smear out the transitions in a region too close to T_c.[11]

*Work supported by the National Science Foundation.

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Ultrasonic Anomalies in Simple Metals due to Backward Scattering of the Conduction Electrons

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The acoustic attenuation for a one-band free-electron metal has been derived using a generalized scattering function. If the latter has an appreciable backward lobe, the attenuation exhibits an anomalous peak when the sound wavelength is comparable to the electron mean free path. Similar effects might be observable in the alkalis as a result of umklapp scattering processes. If so, further study could enhance our understanding of the electron-phonon interaction in these metals.

The free-electron model of a metal has the merit that Fermi-surface disturbances due to applied fields can be expressed in terms of spherical harmonics. It is well known[1] that if the scattering function $W(\vec{k},\vec{k}')$ depends only on the angle θ between $\vec{k}$ and $\vec{k}'$, then each spherical harmonic Y_{LM} relaxes towards zero with time constant τ_L given by

$$1/\tau_L = \int[1 - P_L(\cos\theta)]W(\theta)\,d\Omega, \tag{1}$$

where P_L is a Legendre function, $W(\theta)$ is the differential scattering probability, and $d\Omega$ is the element of solid angle. Associated with each τ_L is an effective mean free path defined by

$$l_L = v_0\tau_L, \tag{2}$$

where v_0 is the Fermi velocity of the electrons. A special case is that of isotropic scattering, where $W(\theta)$ is independent of θ so that all the τ_L