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[20]In regard to second sound, we note that $C_{2l} \to (\frac{7}{5})^{1/2} \times (\pi/3)K_B T/p_0$ and $C_{2t} \to \frac{2}{3}\Delta/p_0$ as $T \to 0$, for the quasiparticle spectrum given by Eq. (4). These expressions are obtained by employing $S^{\rm qp} = (7\pi^4/60)(K_B/m)(K_B T)^3/\Delta^2\epsilon_F$ and $C_v^{\rm qp} = 3S^{\rm qp}$, where the superscript qp denotes the quasiparticle contribution to S and C_v. In addition, there are much smaller contributions $S^{\rm ph} = (2\pi^4/15)(K_B/m)(K_B T/p_0 u)^3$, $C_v^{\rm ph} = 3S^{\rm ph}$, and $\rho_n^{\rm ph}/\rho = (4\pi^4/15)(\epsilon_F/p_0 u) \times (K_B T/p_0 u)^4$, arising from phonons (high-frequency or zero sound). Here u is the zero-sound velocity and $S^{\rm ph}$, $C_v^{\rm ph}$, $C_v^{\rm ph}$, and $\rho_n^{\rm ph}/\rho$ are taken to be appropriate to the noninteracting gas of phonons (as given, for example, in Ref. 14). In the case of an isotropic gap (or for an anisotropic gap which always remains finite) the phonon contributions dominate as $T \to 0$, in which case $C_2 \to u/3^{1/2}$. However, before the phonon regime is reached, the quasiparticle regime gives, in the case of an isotropic gap, $C_2 \to [3(2\pi)^{1/2}]^{-1/2} K_B T/p_0$ as $T \to 0$. [In deriving this expression, the relation $S^{\rm qp} \approx (K_B T/\Delta)C_v^{\rm qp}$ was employed, and $C_v^{\rm qp}$ was taken from Ref. 14]. As one crosses from the quasiparticle regime (which begins when $S^{\rm qp} \approx S^{\rm ph}$) to the phonon regime (which begins when $\rho_n^{\rm qp} \approx \rho_n^{\rm ph}$), the second-sound velocity rises dramatically.

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Vortex-Ring—Generated Level Differences in Liquid Helium

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Level differences between two baths of liquid helium connected by an orifice are generated by a charged vortex-ring beam incident on a diaphragm containing the orifice. The data are interpreted in light of phase slippage caused by vortex rings which overlap the orifice. The effects of vortex-ring interactions are discussed.

Several years ago Richards and Anderson reported an observation of the analog of the ac Josephson effect in superfluid helium.[1] In this experiment, which was subsequently repeated by others,[2-4] a transducer was placed directly in front of an orifice connecting two helium baths. Activation of the transducer produced quantized level differences between the surfaces of the two baths. These quantized levels were accounted for by slippage of the difference in the phases of the complex order parameters describing the two baths. It was assumed that vortex lines or rings were created synchronously with the transducer frequency. In steady state the phase slippage caused by vortex motion cancels the phase slippage due to a chemical-potential difference between the two baths. This results in a level difference z given by

$$mgz = h\nu, \tag{1}$$

where m is the helium atomic mass, g is the acceleration due to gravity, h is Planck's constant, and ν is the rate at which vortices cross path connecting the surfaces of the two baths. A detailed discussion of this process has been given by Anderson.[5]

An alternative derivation of Eq. (1) has been given by Zimmermann.[6] By considering the reactive thrust of the vortex rings generated at an orifice between two volumes of He II, he shows that a level difference z is produced, given by Eq. (1). Also, using the hydrodynamic equation of motion for a superfluid in terms of a velocity potential, Zimmermann has derived the same equation. More recently, Huggins[7] has considered the role of vortex lines in energy dissipation in the flow of an ideal incompressible fluid, and derived a timewise local relation between chemical potentials and vortex motion, which reduces to Eq. (1) in the steady state.

More recently, careful investigations by other workers[8-10] have shown that level differences generated in similar geometries in their laboratories appear to result from acoustic resonances and do not satisfy Eq. (1). These discrepancies have generated great interest, and suggest that level differences generated by vorticity between two coupled helium baths be subjected to further experimental tests. We report here a novel method of generating level differences which avoids acoustic resonances and obscure mechanisms for the generation and synchronization of vorticity.

In this experiment the transducer is replaced by a charged vortex-ring beam which impinges on a thin diaphragm containing an orifice. A schematic diagram of the experiment is shown in Fig. 1. A vortex-ring gun consisting of an ion source (cathode) and an accelerating grid is located in one of the baths. These ions are accelerated through a potential V and form charged vortex rings[11] of energy ϵ_r. The radius R of the vortex rings is given by[12]

$$R = 2m^2\epsilon_r/\rho h^2(\eta - 1.67), \qquad (2)$$

where ρ is the superfluid density, h is Planck's constant, and $\eta = \ln(8R/a_p)$, with a_p the de Broglie wavelength. For isolated vortex rings $\epsilon_r = eV$. Thus, the vortex-ring radius is controlled by adjusting the voltage. At low temperatures the radius is undiminished in drifting across the field-free region to the orifice. In Anderson's interpretation, any vortex ring which cuts a path connecting the surfaces of the two baths once (those which overlap the orifice) causes a phase slippage of 2π. Vortex rings which pass through the orifice either avoid cutting such a path or cut it twice in opposite senses and do not contribute to the phase slippage. Thus we expect to generate a level difference given by Eq. (1), where ν is defined as the number of vortex rings which *overlap* the orifice per second.

We describe here a signal for an idealized model. We assume a uniform areal density of vortex rings from the source. The collected current I is then a measure of the vortex-ring frequency ν. For small accelerating potentials, rings pass through the orifice, and we expect no level difference. Walraven[13] calculates that the critical radius for passage through the orifice, for rings which are coaxial with the orifice, is approximately 0.8 times the radius of the orifice, R_0. For $R = R_0$ we expect $\nu \simeq I\pi R_0^2/eA$, where A is the source area. If we neglect ring expansion near a wall, then for large radii the number of rings which overlap the orifice is equal to all those whose centers fall within a radius R of the orifice. This yields

$$\nu = I\pi R^2/eA. \qquad (3)$$

Thus z/I should vary approximately as V^2 for large V.

Our actual experimental arrangement is shown in Fig. 2. The experimental cell which contains a fixed volume of helium is attached to a ^{3}He refrigerator. ^{3}He temperatures are essential to eliminate vortex-ring attenuation by roton scattering. An inner bath contains a coaxial-capacitor level detector, and has a small volume com-

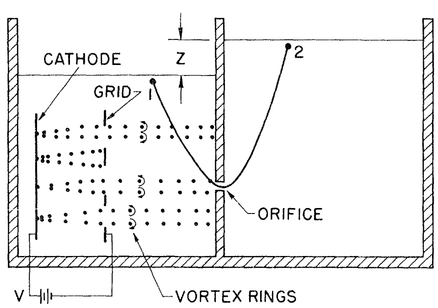

FIG. 1. Schematic diagram of the experiment.

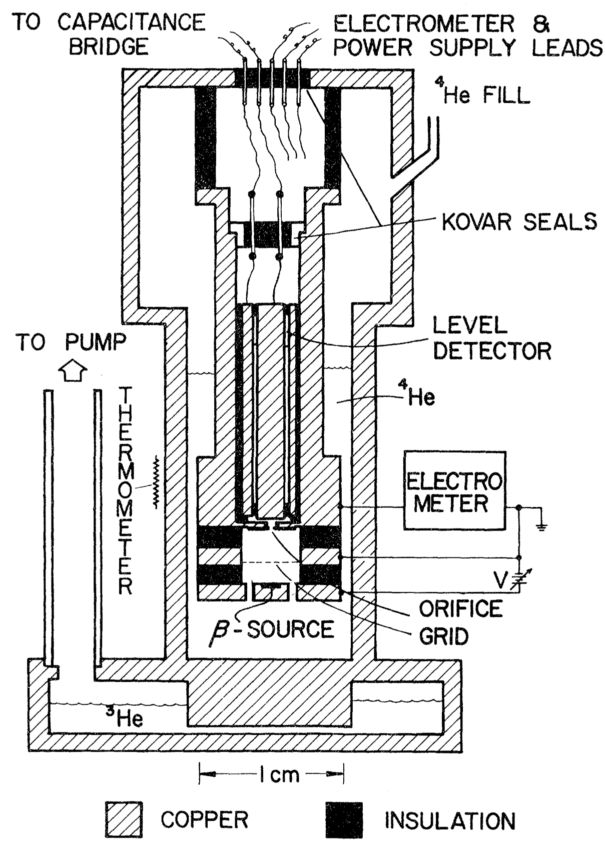

FIG. 2. Experimental apparatus.

pared to the outer bath. The walls of the inner vessel are made of copper and provided an electrical shield for the level detector. They also provide a thermal short with a calculated time constant of 1 msec for thermal relaxation of the inner bath. An orifice in the bottom of the inner vessel provides the only link between the two baths.

The vortex-ring gun is located beneath the orifice. Ions of both signs are formed by a 10^{-1}-cm-diam low-energy β source which forms the central portion of the cathode. The accelerating grid is kept at ground potential to shield the diaphragm containing the orifice from electrical forces. A nickel diaphragm containing the orifice is soldered to the bottom of the inner vessel and serves as a collector for the vortex-ring current, which is monitored with an electrometer. Both the acceleration and drift spaces are $\simeq 0.2$ cm long. Helium is admitted to the triode via two 0.16-cm-diam holes in the cathode designed to limit the escape of undissipated vorticity into the rest of the outer bath.

Data are taken by measuring the level difference and the current collected on the diaphragm as a function of the voltage V applied to the cathode. Results were obtained both with these measurements taken simultaneously and separately. Ion currents ranged from 1–3 pA. No level differences were detected with a 6-μm-radius orifice. Our experimental results with the 2.25-μm-radius orifice are summarized below. At 1.2 K the level difference is less than 500 Å for all V (≤ 400 V), an expected result since the vortex-ring attenuation length is much less than the drift space. Data from four representative runs at lower temperatures are plotted as z/I versus V in Fig. 3. Here z represents an increase in the level of the inner bath. In runs D, E, and H the voltage was manually chopped between V and zero, and each datum point represents the average of many measurements not taken in sequential order. Run G was taken with the voltage swept at a rate of 0.5 V/sec in the direction shown by the arrows. This curve represents lines drawn through a fine-grained plot of z/I, and is corrected for the instrumental time delay. The error bar shown represents an average uncertainty due to noise, drift, and oscillations. The uncertainty in z is $\simeq 2000$ Å. Uncertainties in z/I are larger for small I (small V). The arrow at 105 V indicates the voltage at which the vortex-ring radius is equal to the orifice radius. The heavy solid curve is the predicted result for the idealized model discussed above.

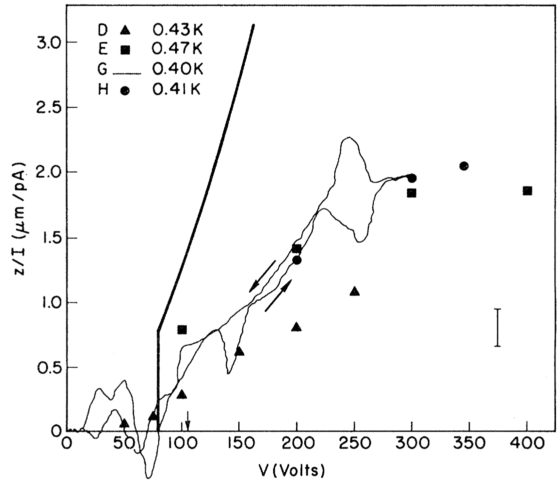

FIG. 3. Ratio of level difference to vortex-ring current versus accelerating voltage for four runs. Heavy solid curve, predicted result for an idealized model.

The results were independent of the sign of V. The results were also independent of whether the accelerating voltage was swept at a rate of 0.5 V/sec or chopped manually between V and zero. The inner level responded to a change in voltage in a time short compared to the instrumental time constant of 10 sec and did not change after attaining equilibrium.

We believe that the discrepancy between experimental data and the idealized model is in part due to vortex-ring interactions which result in reduced ring radii and beam spreading. An isolated vortex ring expands when its energy is increased. On the other hand, some of the increase in energy of a system of densely packed vortex rings goes into the interaction energy $U_{\rm int}$. We write the total energy, $\int \frac{1}{2}\rho[\vec{v}(\vec{r})]^2 d^3r$, as

$$E = \sum_i \int \tfrac{1}{2}\rho v_i^2 \, d^3r + \sum_{i\neq j} \int \tfrac{1}{2}\rho \vec{v}_i \cdot \vec{v}_j \, d^3r, \tag{4}$$

where $\vec{v}(\vec{r})$ is the vector sum of the velocity fields $\vec{v}_i$ of individual vortex rings. For a system of N identical vortex rings which have been accelerated through a voltage V, we rewrite Eq. (4) as

$$NeV = N\epsilon_r + U_{\rm int}. \tag{5}$$

The ring radius, given by Eq. (2), is smaller for a given V than for the case of isolated vortex rings. Hence, fewer rings overlap the orifice resulting in a smaller level difference.

We estimate the radius of vortex rings in a strongly interacting system with the use of a magnetic analog. Formal analogs exist between the magnetic field $\vec{B}$ generated by a current I and the velocity field $\vec{v}$, resulting from a circulation $\kappa = h/m$. In cgs units they are

$$\vec{B} \leftrightarrow \vec{v}, \quad 4\pi I/c \leftrightarrow \kappa. \tag{6}$$

We consider a long cylinder composed of a lattice of vortex rings of density n and R. Its velocity field is analogous to the magnetic field of a long cylindrical bar magnet composed of small current loops. The macroscopic velocity field inside the cylinder is $v_m = n\kappa\pi R^2$, and the energy density of the system is approximately

$$neV = \tfrac{1}{2}\rho v_m^2. \tag{7}$$

The macroscopic flow accounts for an appreciable portion of the total energy for $nR^3 \geq 1$. In this experiment nR^3 is greater than 10^{-1} for $V \gtrsim 80$ V. Our estimated value of z/I for large V obtained by including vortex-ring interactions is a factor of 5 larger than the experimental values. We estimate the maximum ring radius to be $\simeq 6$ μm which accounts for the absence of an effect with a 6-μm-radius orifice.

We believe that the discrepancy between our model and experiment is due to beam spreading near the orifice. Boundary conditions require that streamlines for the macroscopic fluid flow at the diaphragm be tangent to it. Rings are swept radially from the center of the beam, and the areal density of rings is reduced. This effect is important for $v_m/v_{\text{ring}} \gtrsim 1$, which condition is satisfied for $V \geq 320$ V. Beam spreading is a rapidly increasing function of V.

At all times 1-Hz U-tube oscillations between the two baths are present. The resultant flow through the orifice perturbs rings near it, and may quantitatively affect our results.

Despite the quantitative disagreement of the data with our predictions at large V, the following experimental facts lead us to believe that we have observed level differences generated by a vortex-ring beam: (1) the absence of an effect at 1.2 K, (2) an onset in level difference at a ring radius which is 50–70% of the orifice radius, and (3) quantitative agreement with theory to within a factor of 5 despite vortex-ring interaction and beam spreading effects which are difficult to estimate. The alternative of using a beam of lower density has so far proved impractical because of signal-to-noise problems.

Alternative explanations due to heating and electrostrictive effects associated with turning on the vortex-ring beam can be excluded since these occur in the outer bath and would decrease rather than increase the inner level. Further, neither of these effects can explain the onset at 70 V.

We have not discussed the implications of Huggins's calculation[14] which shows that, in an appropriate geometry, pressure is exerted on the orifice while the vortex rings are geing accelerated. However, the absence of an effect for a 6-μm-radius orifice suggests that this contribution is unimportant in our experiment.

In conclusion, we have employed a vortex-ring beam to create a level difference between two baths of helium connected by a small orifice. This experiment is free of acoustic reasonances which have plagued similar experiments using transducers. In addition, we suggest that vortex-ring interactions must be taken into account in a dense beam.

We wish to thank Elisha Huggins for suggesting the bar-magnet analog, and S. Srikantia for technical assistance.

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