mor radius ρ being unknown. In the experimental setup there is no independent method for measuring the density, but the temperature measured with an electrostatic probe compares favorably with the value deduced from Fig. 4 ($T \sim 1800°K$).

If these pseudocylindrical waves observed in the laboratory were also detected in space, the measurement of their wavelength at different frequencies would help to reduce the uncertainty which still remains in the evaluation of the temperature of the ionospheric or magnetospheric plasma.

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Normal Fluid Density, Second Sound, and Fourth Sound in an Anisotropic Superfluid

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In a superfluid with an anisotropic quasiparticle energy, the normal-fluid density is a diagonal tensor. The second and fourth sound velocities are distinctly anisotropic, so that heat-pulse or fourth-sound studies with an applied magnetic field $\vec{H}$ may permit identification of the direction of the gap axis with respect to $\vec{H}$. High-frequency sound velocities may display a weaker anisotropy.

I have studied the normal fluid density in an anisotropic superfluid, using Bardeen's[1] and Leggett's[2] generalizations of Landau's arguments for He II.[3] The results have been applied to second sound, fourth sound, and the determination of the energy gap near the transition temperature from a knowledge of the normal-fluid density. These questions are relevant because of the recent flurry of activity, both experimental and theoretical, prompted by the observations of Osheroff, Richardson, and Lee.[4]

It appears that, for $T \lesssim 3$ mK, and under pressure, liquid He^3 undergoes two phase transitions. It has been suggested that these involve superfluid states.[5,6] The higher-temperature phase, because of its unusual NMR resonance shift,[7] appears to be an anisotropic superfluid. The lower-temperature phase shows no such unusual magnetic behavior, but viscosity studies[8] indicate that it is in a superfluid state (presumably isotropic). We will employ the identification[9] of the higher-temperature phase as an AM (Anderson-Morel) state,[10] and the lower-temperature phase as a BW (Balian-Werthamer) state.[11] Recent heat-flow studies support a two-fluid model and have determined a phase diagram.[12]

In the absence of superflow, the momentum density of noninteracting fermion quasiparticles at temperature T is given by[1]

$$\vec{j} = V^{-1}\sum_{\vec{p}}\vec{p}f(\beta(E_p - \vec{p}\cdot\vec{v}_n))$$
$$= -\beta V^{-1}\sum_{\vec{p}}\vec{p}(\vec{p}\,\partial f/\partial x)\cdot\vec{v}_n + O(v_n^2); \tag{1}$$

$$f(x) = (e^x + 1)^{-1}. \tag{2}$$

Here V is the volume of the system, $\vec{p}$ is the momentum of a quasiparticle, E_p is its energy, $\beta = (K_B T)^{-1}$, and $\vec{v}_n$ is the normal-fluid velocity. Using $\vec{j} = \overleftrightarrow{\rho}_n^{(0)}\cdot\vec{v}_n$, one sees that

$$\overleftrightarrow{\rho}_n^{(0)} = -\beta V^{-1}\sum_p \vec{p}\,\vec{p}\,\partial f/\partial x. \tag{3}$$

Since $\overleftrightarrow{\rho}_n^{(0)}$ is real and symmetric, it can always be diagonalized. If the argument of f (i.e., βEp) is isotropic, then $\overleftrightarrow{\rho}_n^{(0)} = \rho_n^{(0)}\overleftrightarrow{1}$. However, if E_p is anisotropic $\overleftrightarrow{\rho}_n^{(0)}$ need not be proportional to the unit tensor $\overleftrightarrow{1}$. To be specific, we consider the model[10,13]

$$E_p = [\epsilon_p^2 + \Delta^2\sin^2\theta_p]^{1/2}, \tag{4}$$
$$\epsilon_p \equiv p^2/2m^* - \epsilon_F.$$

Here, m^* is the effective mass of the quasiparticle, and ϵ_F is the Fermi energy, corresponding to the Fermi momentum p_0. θ_p is measured with respect to the gap axis. In this case the nonzero components of $\overleftrightarrow{\rho}_n^{(0)}$ are

$$\rho_n^{(0)xx} = \rho_n^{(0)yy} = -\tfrac{1}{2}\beta V^{-1}\sum_{\vec{p}} p^2 \sin^2\theta_p\, \partial f/\partial x, \tag{5}$$

$$\rho_n^{(0)zz} = -\beta V^{-1}\sum_{\vec{p}} p^2 \cos^2\theta_p\, \partial f/\partial x. \tag{6}$$

Note that more complicated models can yield $\rho_n^{(0)xx} \neq \rho_n^{(0)yy}$. Since $\partial f/\partial x$ is largest for small $\sin\theta_p$, it is clear that $\rho_n^{(0)zz} > \rho_n^{(0)xx}$. Hereafter we write $\rho_{nl}^{(0)}$ and $\rho_{nt}^{(0)}$ for $\rho_n^{(0)zz}$ and $\rho_n^{(0)xx}$, respectively.

The results of an evaluation of Eqs. (5) and (6), using the spectrum given by Eq. (4), are of some interest. At low temperatures, the integrals can be made tractable upon making the approximation

$$[\epsilon_p^2 + \Delta^2 \sin^2\theta_p]^{1/2} \approx |\epsilon_p| + \Delta|\sin\theta_p|.$$

This overestimates the argument of f by no more than a factor of $\sqrt{2} - 1$, at the worst. The results, although somewhat low, should be qualitatively correct. Note that $\sum_{\vec{p}} \to 2V(2\pi\hbar)^{-3}\int d^3p$ includes the summation on spin.

With a bit of algebra and some tricks familar from statistical mechanics, one can show that, for $\beta\Delta \gg 1$,

$$\rho_{nl}^{(0)}/\rho = (\pi^2/2)(m^*/m)(\beta\Delta)^{-2}, \tag{7}$$

$$\rho_{nt}^{(0)}/\rho = (7\pi^4/40)(m^*/m)(\beta\Delta)^{-4}. \tag{8}$$

In Eqs. (7) and (8) m is the bare particle mass. Unfortunately, the AM state seems not to exist in the temperature range for which Eqs. (7) and (8) are valid.[12] These equations, nevertheless, serve as an explicit example of the effects of anisotropy on the normal fluid density. Note that, for fermions with an isotropic gap Δ_0, corresponding to the BW state, Landau and Lifshitz[14] show that

$$\rho_n^{(0)}/\rho = (2\pi\beta\Delta_0)^{1/2}\exp(-\beta\Delta_0)$$

as $\beta\Delta_0 \to \infty$. They neglect Fermi-liquid effects on m^*. This is valid at low temperatures, as Putterman has noted.[15] Therefore, $m^*/m \approx 1$ in Eqs. (7) and (8).

Near T_c, where $\Delta \to 0$, the system becomes isotropic. In order to evaluate $\rho_{nl,t}$ as $\Delta \to 0$, one must incorporate quasiparticle interactions. This has been done by Leggett for an isotropic superfluid.[2] He obtains

$$\frac{\rho_n}{\rho} = \frac{\rho_n^{(0)}/\rho}{1 + (1 - m/m^*)\rho_n^{(0)}/\rho}, \tag{9}$$

where $\rho_n^{(0)}$ is obtained from Eq. (3), using an isotropic gap. Since $\rho_n^{(0)}/\rho = m^*/m$ at T_c, then $\rho_n/\rho = 1$ at T_c. Assuming that Eq. (9) holds for $\rho_{nl,t}$, it is straightforward to include some of the effects of anisotropy. First, as $\beta\Delta \to 0$, one finds that

$$\rho_{nl}^{(0)}/\rho = (m^*/m)_l[1 - 0.4C(\beta\Delta)^2], \tag{10}$$

$$\rho_{nt}^{(0)}/\rho = (m^*/m)_t[1 - 0.8C(\beta\Delta)^2], \tag{11}$$

where

$$C = \int_0^\infty dx\, x^{-1}\, d^2f/dx^2 = (7/4\pi^2)\zeta(3).$$

Here $\zeta(3) = \sum_n n^{-3}$ is the Riemann ζ function. Note that in Eqs. (10) and (11) we employ a generalization of Putterman's phenomenological relationship,[16] defining

$$(m^*/m)_{l,t} = 1 + (\rho_{nl,t}/\rho)F_1/3$$
$$= (m^*/m)_N + (\rho_{nl,t}/\rho - 1)F_1/3. \tag{12}$$

Here $(m^*/m)_N = 1 + F_1/3$ is appropriate to the normal Fermi liquid.

Combining Eqs. (9)–(12) one finds that, near T_c,

$$\rho_{nl,t}/\rho \approx 1 - A_{l,t}(m/m^*)_{l,t}$$
$$\approx 1 - A_{l,t}(m/m^*)_N, \tag{13}$$

where $\rho_{nl,t}^{(0)}/\rho = (m^*/m)_{l,t}(1 - A_{l,t})$ and Eqs. (10) and (11) define $A_{l,t}$. Since $A_t = 2A_l$ for the model described by Eq. (4), it is clear that a knowledge of the gap axis is required to extract $\Delta(T)$ near T_c from information about the normal-fluid density (such as obtained by the method described in Ref. 15). One must know which normal-fluid density—ρ_{nl} or ρ_{nt}—has been probed in the course of any given experiment. Note that Eq. (13) is dependent on a number of assumptions, so that details of the analysis must be treated with caution.

The tensor nature of $\overleftrightarrow{\rho}_n$ has consequences for second sound in an anisotropic superfluid. Assuming that $\vec{j} \approx 0$ for second sound (so that $\rho \approx$ const and $P \approx$ const) and taking the wave vector to make an angle θ with respect to the gap axis, it is straightforward, using the standard equations of superfluid hydrodynamics,[17] to derive the following expression for the second-sound velocity as a function of θ:

$$C_2^2(\theta) = C_{2l}^2\cos^2\theta + C_{2t}^2\sin^2\theta, \tag{14}$$

where

$$C_{2l,t}^2 = (S^2T/C_v)\rho_{sl,t}/\rho_{nl,t}. \tag{15}$$

Here S is the entropy per unit mass, C_v is the specific heat per unit mass, and $\rho_{sl,t} = \rho - \rho_{nl,t}$. Near T_c, $C_2(\theta) \to 0$ for all θ, as with He II. Since second-sound fluctuations are associated with the backward λ shape of the C_v curve in He II, one is led to inquire about the detailed shape of the C_v curve in He^3. The present data indicate that C_v is discontinuous at T_c,[18] but future work should examine the possibility that a small backward λ shape is also present. We note that, due to quasiparticle collisions with the lattice, second sound does not propagate in superconductors.[19] These materials exhibit a discontinuity, with no indication of a backward λ shape, in C_v at their transition temperatures. The AM state, resembling both He II (superfluidity) and superconductors (pairing states with an energy gap), may exhibit C_v characteristics of both.

Because of the low temperatures involved, implying long collision times, it may be difficult to study experimentally the hydrodynamic regime (and, therefore, second sound) in liquid He^3 for $T \lesssim 3$ mK. However, one should not rule out the possibility that heat pulses, although not very well defined in shape, might show anisotropic propagation characteristics. With an externally applied magnetic field along a known direction, the anisotropy of such pulses would determine the orientation of the gap axis with respect to the magnetic field.[20]

A mode which may be more easily studied experimentally is fourth sound. If the normal fluid is "clamped" by the presence of, e.g., Vycor glass in the sample cavity, so that $\vec{V}_n = \vec{0}$, then it is straightforward to show that

$$C_4^2(\theta) = C_{4l}^2 \cos^2\theta + C_{4t}^2 \sin^2\theta, \tag{16}$$

where

$$C_{4l,t}^2 = \frac{\rho_{sl,t}}{\rho}\left(\frac{\partial P}{\partial \rho}\right)_S \left[1 + \frac{S^2 T C_v^{-1}}{(\partial P/\partial\rho)_S}\right]. \tag{17}$$

In deriving this expression, the thermal expansion coefficient was assumed to be negligible. We remark that pore geometry is rather complex, so that the simple result given by Eqs. (16) and (17) might be in some way averaged out by boundary effects.

The tensor nature of $\overleftrightarrow{\rho}_n$ also has implications for high-frequency sound in the AM state. As mentioned previously, Putterman's determination of the normal-fluid density from data[21] on high-frequency sound in superfluid He^3 assumes that $\overleftrightarrow{\rho}_n$ is a scalar.[15] Because $\rho_n \neq \rho_{nt}$, certain of the equations in that work are ambiguous. Reexamination of the theory of propagation of high-frequency sound in a superfluid, incorporating the effects of anisotropy, therefore appears to be appropriate.

It should be remarked that the experiments on the high-frequency sound velocity have been performed for only one direction of propagation.[21] Studies of propagation in more than one direction might yield an anisotropic high-frequency sound velocity. This would be the case if the quasiparticle interaction $f(\vec{p}, \vec{p}\,')$ has a component depending upon the quasiparticle momenta relative to the gap axis $\hat{n}$. For example, a term in $f(\vec{p}, \vec{p}\,')$ proportional to $\cos\psi = (\vec{p} + \vec{p}\,')\cdot\hat{n}/|\vec{p} + \vec{p}\,'|$ would permit such an anisotropy.

We conclude by noting that de Gennes has come to similar conclusions about the anisotropy of second and fourth sound, using a Ginzburg-Landau approach.[22] This work, which makes a specific assumption about the form of the condensation amplitude, in addition indicates that a rich variety of new phenomena can be associated with the AM state. Such behavior does not appear in our simplified discussion because not all of the hydrodynamic parameters associated with the underlying microscopic structure have been incorporated into the theory. However, we believe that the most important effects relevant to second and fourth sound are included.

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Vortex-Ring—Generated Level Differences in Liquid Helium

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Level differences between two baths of liquid helium connected by an orifice are generated by a charged vortex-ring beam incident on a diaphragm containing the orifice. The data are interpreted in light of phase slippage caused by vortex rings which overlap the orifice. The effects of vortex-ring interactions are discussed.

Several years ago Richards and Anderson reported an observation of the analog of the ac Josephson effect in superfluid helium.[1] In this experiment, which was subsequently repeated by others,[2-4] a transducer was placed directly in front of an orifice connecting two helium baths. Activation of the transducer produced quantized level differences between the surfaces of the two baths. These quantized levels were accounted for by slippage of the difference in the phases of the complex order parameters describing the two baths. It was assumed that vortex lines or rings were created synchronously with the transducer frequency. In steady state the phase slippage caused by vortex motion cancels the phase slippage due to a chemical-potential difference between the two baths. This results in a level difference z given by

$$mgz = h\nu, \tag{1}$$

where m is the helium atomic mass, g is the acceleration due to gravity, h is Planck's constant, and ν is the rate at which vortices cross path connecting the surfaces of the two baths. A detailed discussion of this process has been given by Anderson.[5]

An alternative derivation of Eq. (1) has been given by Zimmermann.[6] By considering the reactive thrust of the vortex rings generated at an orifice between two volumes of He II, he shows that a level difference z is produced, given by Eq. (1). Also, using the hydrodynamic equation of motion for a superfluid in terms of a velocity potential, Zimmermann has derived the same equation. More recently, Huggins[7] has considered the role of vortex lines in energy dissipation in the flow of an ideal incompressible fluid, and derived a timewise local relation between chemical potentials and vortex motion, which reduces to Eq. (1) in the steady state.

More recently, careful investigations by other workers[8-10] have shown that level differences generated in similar geometries in their laboratories appear to result from acoustic resonances