New Approach to the Analysis of Nucleon-Deuteron Breakup Data*

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We propose calculations and measurements of $N+d$ breakup cross sections as a function of a new set of kinematic variables. When carried out as a function of these variables, separable-potential calculations predict cross sections which exhibit a very deep destructive interference minimum over a wide range of bombarding energies. We proposed a procedure for isolating the effect of the crucial M_{d2} amplitude on the cross section.

A basic difficulty in attempting to extract new information about the *NN* force from measured three-body observables is that the observables are largely determined by what is already known about the *NN* force from the nucleon-nucleon data. The purpose of this article is to suggest a method of selecting the kinematical situations under which to perform $N+d$ breakup measurements to facilitate the observation of short-range effects. (These we take to include both the off-energy-shell behavior of the *NN* force and explicit three-body forces.) We shall also present estimates of the variation in the cross section which might be caused by these short-range effects.

Since three-body breakup amplitudes depend sensitively on the three final-state *NN* relative energies, we suggest that the relative energies be used to parametrize the kinematics and that comparisons of data and calculations should be done for fixed values of these relative energies. If in addition the direction of one of the final-state nucleon momenta is held fixed, the cross section can depend only on the c.m. rotation angle of a momentum triangle of fixed shape about the direction of the fixed nucleon momentum. This rotation angle, or an equivalent kinematic variable, is the continuous variable against which the cross section is to be measured and calculated in this scheme. The above specification fixes four independent kinematic variables, thus defining a one-dimensional kinematic locus in three-body phase space. We have computed the breakup cross section along a particular one of these constant-relative-energy loci using the *YY* model,[1] where the Watson-Faddeev integral equations are solved with separable spin-dependent s-wave *NN* interactions.[2] The model cross section is given by

$$\frac{d^5\sigma}{d\Omega_3\, d\Omega_4\, dE_3} = F_K\left(\tfrac{2}{3}|M_q|^2 + \tfrac{1}{3}|M_{d1}|^2 + \tfrac{1}{3}|M_{d2}|^2\right), \qquad (1)$$

where F_K is a kinematic factor, M_q is the amplitude for breakup in the quartet state ($S=\frac{3}{2}$), and M_{d1} and M_{d2} are the doublet-state ($S=\frac{1}{2}$) amplitudes in which the two identical nucleons are coupled to spin 1 or 0, respectively. Results from model calculations of elastic N-d scattering phase parameters[3] with different *NN* forces lead us to expect that M_{d2} is much more sensitive to the details of the *NN* force used in the calculations than the other amplitudes. This can be understood because wave-function antisymmetry makes a state contributing to M_{d2} the only one in which there is a significant probability of finding the three nucleons simultaneously close together.[3,4]

The advantage of the above procedure is that the s-wave parts of M_q, M_{d1}, and M_{d2} are all constant along such a locus. If we assume that the deviation between the model-predicted M_{d2} and the true M_{d2} is confined to the s wave, then it is just a single complex number along such a constant-relative-energy locus. The difference between the model M_{d2} and the true M_{d2} we call the doublet defect (Δ).[5] The objective is to determine Δ from comparison of the model calculation with data.

We consider the experiments which involve the detection of two identical nucleons, either ^{2}H(p, $2p$)n or ^{2}H(n, $2n$)p. In this case loci may be chosen such that the s-wave parts of M_q and M_{d1} vanish identically along the entire locus. If the nonidentical particle is labeled 5 and identical particles 3 and 4, this choice of relative energies is the symmetric choice $E_{35}=E_{45}$. In addition to the condition $E_{35}=E_{45}$, these loci are specified to allow symmetric quasifree scattering between the two like nucleons at the point $\Delta\varphi = 180°$. Figure 1 shows such loci for three different values of the nucleon bombarding energy (E_1).

Figure 2 shows the *YY* model amplitudes and the cross section (solid curves) along these loci

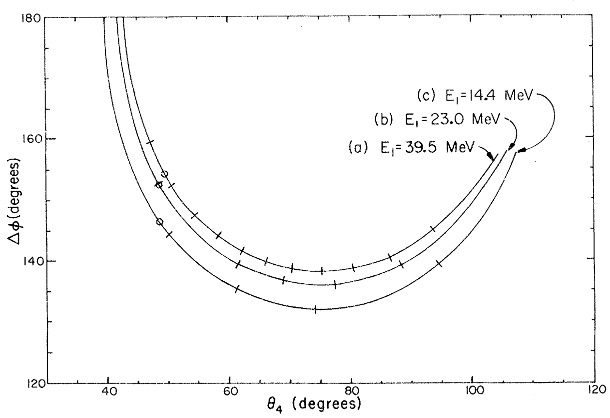

FIG. 1. Constant-relative-energy loci at three different values of the nucleon bombarding energy (E_1). Along each locus the invariant kinematic quantities are the scattering angle (θ_3) and energy (E_3) for one particle and the *NN* relative energies (E_{35}, E_{45}, and E_{34}). For locus *a* $\theta_3 = 43.3°$, $E_3 = 18.64$ MeV, $E_{35} = E_{45} = 9.31$ MeV, and $E_{34} = 17.53$ MeV. For locus *b* $\theta_3 = 41.9°$, $E_3 = 10.39$ MeV, $E_{35} = E_{45} = 5.19$ MeV, $E_{34} = 9.27$ MeV. For locus *c* $\theta_3 = 39.7°$, $E_3 = 6.09$ MeV, $E_{35} = E_{45} = 3.04$ MeV, $E_{34} = 4.97$ MeV. $\Delta\varphi$ is the azimuthal angle difference ($=\varphi_3 - \varphi_4$) between the two nucleon momenta. The E_5 scale is marked along each locus at 1.25-MeV intervals beginning from $E_5 = 0$ at $\Delta\varphi = 180°$. The open circles mark the predicted positions of interference minima.

for the three bombarding energies. The various quantities are plotted as a function of the laboratory energy of the unlike nucleon, a kinematic variable equivalent to θ_4, $\Delta\varphi$, or the c.m. rotation angle mentioned above. Quasifree scattering causes the large peak in the cross section at $E_5 = 0$. The requirement of wave-function antisymmetry makes $M_q = M_{d1} = 0$ at $E_5 = 0$, and M_q and M_{d1} are small compared to M_{d2} throughout most of each E_5 spectrum. In this way we have used antisymmetry to partially isolate the effect of M_{d2} on the cross section. The remarkable feature of each model cross section along the locus is the distinctive interference minimum at small values of E_5. This interference effect was first seen in integrated ^{2}H(p, 2p)n data.[6]

To investigate the dependence of the model cross section on M_{d2} we have computed the cross section along the loci of Fig. 1 for two constant values of Δ. For a reasonable estimate of Δ, we compared Sloan's *YY* calculation[7] with Kloet and Tjon's local potential calculation[8] of elastic n-d scattering at 39.5 MeV. These calculations agree very well except for the $s = \frac{1}{2}$, $l = 0$ state. The doublet-s breakup cross section calculated by Kloet and Tjon is about 15% less than Sloan's value. We have used this model-to-model variation as a rough estimate of the amount by which the square of the magnitude of the s-wave part of our model M_{d2} ($\equiv M_{d2}{}^0$) might differ from its true value.

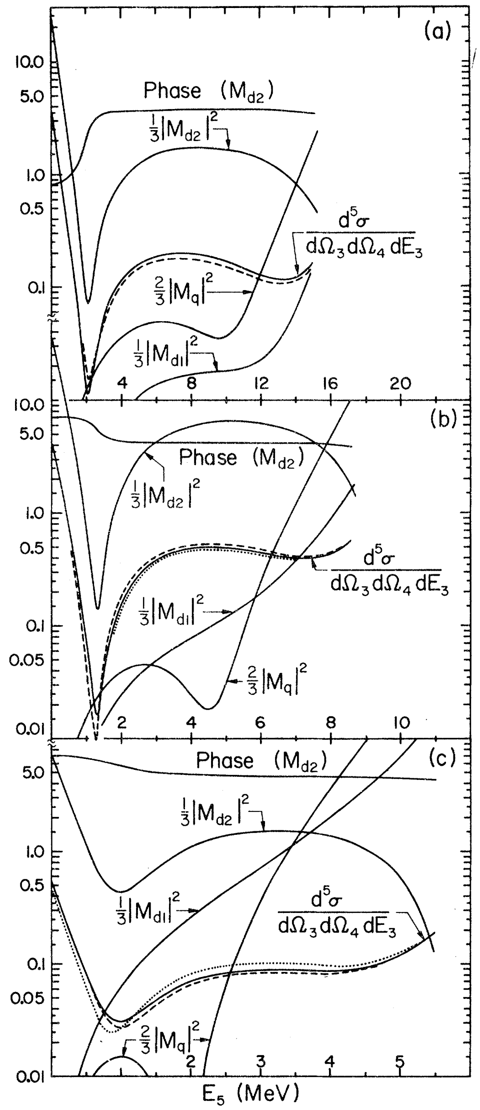

FIG. 2. (a)–(c) Results of the separable potential (*YY* model) calculation along the respective loci *a*–*c* of Fig. 1. The curves marked $\frac{2}{3}|M_q|^2$, $\frac{1}{3}|M_{d1}|^2$, and $\frac{1}{3}|M_{d2}|^2$ show the relative contributions of the three scattering states on an arbitrary scale. The solid curves labeled $d^5\sigma/d\Omega_4 d\Omega_3 dE_3$ are the model prediction for the differential cross section as given by Eq. (1) in units of mb/sr^2 MeV. The dotted and dashed curves are the cross sections which result when the defects Δ_1 and Δ_2 are added, respectively. Δ_1 changes the magnitude of $M_{d2}{}^0$, and Δ_2 changes its phase. The phase of M_{d2} is plotted in radians. Where the dotted and dashed curves are not plotted they are nearly identical to the solid curve.

The dotted curves in Fig. 2 show the cross sections which result when the *magnitude* of $M_{d2}{}^0$ is decreased by 7.5% by adding a constant Δ_1 (corresponding to a decrease of 15% in $|M_{d2}{}^0|^2$). The dashed curves in Fig. 2 show the cross sections which result when the *phase* of $M_{d2}{}^0$ is changed by -15% by adding Δ_2. (The Kloet and Tjon calculation predicts a value for ${}^2\delta_0$ which is 15° less than the prediction of the *YY* model; we have taken this difference in the elastic phase shift as a rough estimate of the expected defect in the phase of $M_{d2}{}^0$.)

Except in the vicinity of the minimum, the changes in the cross section produced by these "reasonable" choices of Δ are in the range ± 0–12%. These variations are of the same order as the typical differences between model calculations and data.[1,9] Therefore it is possible that the discrepancy between data and model calculations can be largely explained as a defect in the $l=0$ part of the M_{d2} amplitude. To test this hypothesis, data should be acquired along loci such as those of Fig. 1 and compared with calculations which include a variation of Δ to obtain the best fit. ^{2}H$(n, 2n)p$ data at lower bombarding energies would be especially valuable, because the separable s-wave approximation to the *NN* force is more realistic in this case and the model cross section depends more sensitively on the s-wave part of M_{d2} at the lower energies. It is expected that the comparison of data with calculations in the vicinity of the interference minimum such as those of Fig. 2 will provide a sensitive test of three-body calculations.

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Test of Backbending Models Using Odd-*A* Nuclei*

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We have studied the properties of decoupled bands in particular odd-*A* nuclei, and the results provide information on the origin of backbending in even-even nuclei. Our data are in agreement with the rotation-alignment model and in apparent disagreement with the pairing-collapse model. This proposed test also provides a means to determine which particles are involved in the two-quasiparticle band that intersects the ground band in the rotation-alignment picutre of backbending.

A process known as "backbending" has recently been discovered[1] to occur at high spins in the ground-state rotational bands of some even-even rare-earth nuclei. The name refers to the fact that a plot of moment of inertia $\mathcal{I}$ versus the square of the rotational frequency, $(\hbar\omega)^2$, for the various spin states of these nuclei has an s-shaped form. That is, $\hbar\omega$ becomes temporarily smaller around $I \approx 16$, while $\mathcal{I}$ increases rather sharply with I. Since $\hbar\omega$ is very nearly half the rotational transition energy, the above shape results from several transition energies around the critical spin value being lower than those for spins just below or above this value. It is by now quite clear that this occurs for many rare-earth nuclei, but it does not occur (at least in the same