

nons exist as good normal modes in glasses at temperatures as low as 1.7 K. We conclude that the experimental specific heat must indeed be written in the form of Eq. (2), i.e., the Debye specific heat is simply masked by the specific heat caused by the disorder. From the lower limits of the phonon lifetimes determined in our experiment it follows that thermally excited Debye phonons must contribute to the heat transport in glasses at low temperatures. The questions whether the additional excitations observed in specific heat also carry heat, or whether they only scatter the Debye phonons, or, finally, whether the additional excitations are involved at all in the heat flow, require further studies.

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Diffusivity of Charge Carriers in Semiconductors in Strong Electric Fields*

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Similarities between diffusion of hot charge carriers in semiconductors and in gases are discussed, and formulas for D_{\parallel} and D_{\perp} , the diffusion coefficients parallel and perpendicular to the applied electric field, are given. It is shown that anisotropy of diffusion can be strong even when the carrier distribution function is nearly isotropic in momentum space.

Development of techniques^{1,2} capable of accurate measurement of the diffusivity of charge carriers in a semiconductor parallel and perpendicular to a strong electric field \vec{E} opens the way for a more thorough understanding of the nature of the interaction between carriers and lattice. The parallel and perpendicular diffusion coefficients, D_{\parallel} and D_{\perp} , respectively, have quite a different qualitative dependence¹⁻³ upon the field from that of the mobility coefficient μ , and may therefore yield information not obtainable from mobility data alone. Unfortunately, a satisfactory quantitative theory of hot-carrier diffusion in semiconductors is not available, and so the link between diffusion experiments and charge-carrier-phonon interaction cannot be precisely established. Even qualitative aspects are not properly understood: It is a common misconception that the anisotropy of diffusion in an electric field (i.e., $D_{\parallel} \neq D_{\perp}$) arises *solely* from the asym-

metry (elongation) of the carrier distribution function, $f(\vec{k})$, in momentum space.^{3,4} Further, it has not been recognized that diffusion of charge carriers in a semiconductor is in many ways analogous to diffusion of electrons or ions in a gas of neutral atoms. The theory of the latter problem has received a great deal of attention recently,⁵⁻⁷ and we can use some of these results to provide at least a starting point for a theory appropriate to semiconductors.

In relation to ions diffusing in a gas, Wannier⁸ conjectured the following generalization of the Einstein relation for diffusion parallel to \vec{E} :

$$D_{\parallel} = \frac{\kappa T}{e} \mu \left(1 + \frac{\partial \ln \mu}{\partial \ln E} \right), \quad (1)$$

a formula recently substantiated via nonequilibrium thermodynamics.⁹ It was also shown in Ref. 9 that the perpendicular diffusion coefficient is

given by

$$D_{\perp} = (\kappa T_{\perp}/e)\mu. \quad (2)$$

Here, $\frac{1}{2}\kappa T_{\parallel}$ and $\frac{1}{2}\kappa T_{\perp}$ denote mean random energies of the ions parallel and perpendicular to the field, respectively, κ is Boltzmann's constant, and e is the ionic charge. Because of the generality of the thermodynamic method used in Ref. 9, we might expect (1) and (2) to be true also for charge carriers in a semiconductor. Nougier⁴ has in fact derived these equations specifically for semiconductors. However, in attempting to introduce microscopic concepts into these essentially macroscopic equations, Nougier misinterprets the role played by anisotropy of the carrier momentum distribution function: Under conditions for which $f(\vec{k})$ is *almost* isotropic in \vec{k} space,¹⁰ so that it can be approximated by the first two terms of an expansion in spherical harmonics,

$$f(\vec{k}) = \sum_{l=0}^{\infty} \sum_{m=-l}^l f_m^{(l)}(k) Y_m^{(l)}(\hat{k}) \quad (3)$$

$$\approx f^{(0)} + \vec{F}^{(1)} \cdot \hat{k}, \quad (4)$$

we have an effective equipartitioning of energy in all directions,

$$T_{\parallel} \approx T_{\perp}, \quad (5)$$

and from (1) and (2),

$$D_{\parallel}/D_{\perp} \approx 1 + \partial(\ln\mu)/\partial(\ln E). \quad (6)$$

Nougier's interpretation is effectively the opposite, and he states that (4) implies $D_{\parallel} \approx D_{\perp}$.

It is well known that (4) holds when the carriers interact primarily with the acoustic branch of lattice vibrations, since there the phonons are of relatively small energy and collisions are quasielastic so that they effectively randomize the momentum of the carriers gained from the field.¹⁰ Similarly, scattering of electrons from heavy gaseous atoms involves only small energy exchange, and here the electron momentum distribution is also nearly isotropic, even at strong fields. These two phenomena have long been known to be kinetically equivalent¹⁰; one can obtain equations for semiconductors directly from the gas equations simply by replacing the atomic mass by the quantity

$$M = \kappa T_L/s^2, \quad (7)$$

where T_L is the lattice temperature and s the sound velocity. In solving the Boltzmann equa-

tion,

$$\left. \frac{\partial f}{\partial t} + \frac{\hbar\vec{k}}{m} \cdot \frac{\partial f}{\partial \vec{r}} + \frac{e\vec{E}}{\hbar} \cdot \frac{\partial f}{\partial \vec{k}} = \frac{\partial f}{\partial t} \right|_{\text{collisions}}, \quad (8)$$

where for the moment, the right-hand side represents the interaction between charge carriers and acoustic phonons only, we can therefore use Eqs. (6), (20), (27), and (28) of Parker and Lowke⁵ to get precise expressions for D_{\parallel} and D_{\perp} . A further modification is that in Ref. 5, the product NQ of neutral gas density and momentum-transfer cross section is to be replaced by the inverse of the mean free path appropriate to semiconductors:

$$l^{-1}(\epsilon) = \frac{\kappa T_L}{8\pi\hbar s \epsilon^2} \int_0^{2(2m\epsilon)^{1/2}/\hbar} dq q^2 V C_q^2, \quad (9)$$

where $\epsilon = \hbar^2 k^2/2m$ is the energy of a carrier (spherical surfaces of constant energy are assumed), m the effective mass, V the volume of the crystal, q the wave number of a phonon, and C_q the matrix element appropriate to the electron-phonon process. For the deformation potential,¹⁰ l is independent of energy, and in this case the Parker-Lowke theory predicts $D_{\parallel}/D_{\perp} \approx 0.5$ in the high-field limit. In other cases where l is a more complicated function of ϵ , D_{\parallel} and D_{\perp} differ quite dramatically (Ref. 6, Fig. 3). Clearly then, diffusion can be strongly anisotropic, even though the carrier distribution function is nearly spherically symmetric.

The solution of (8) becomes much more complex when strong inelastic energy processes (optical-phonon scattering) are included on the right-hand side. Here, since $f(\vec{k})$ is highly asymmetric at strong fields, many terms in (3) have to be included and the simplifications associated with a small phonon energy disappear. The situation can be compared to the solution of the Boltzmann equation appropriate to heavy ions in a neutral gas, where, because of the substantial energy exchanged in ion-atom collisions and the resulting asymmetry of f in momentum space, a method quite different from that used by Parker and Lowke for electrons must be found. Persky and Bartelink³ have adapted the procedure given by Wannier⁸ for ions in a neutral gas, but found it necessary to postulate a particular form (a "double Maxwellian") for the spherically symmetric part of the distribution function, $f^{(0)}$. While this must limit the range of validity of their theory, the assumption implicit in Sect. V of Ref. 3 that $f^{(0)}(k, \vec{r}, t)$ depends upon the number density $n(\vec{r}, t)$

t) of carriers but not upon the gradient $\partial n/\partial \vec{r}$ renders the final expression for D_{\parallel} not even qualitatively correct. To see how this assumption can affect the final result, we take the case where scattering is entirely from acoustic phonons and therefore neglect $f^{(2)}$ in comparison with $f^{(0)}$, as in (4). In this limit, Eqs. (31a) and (31b) of Persky and Bartelink³ give $D_{\parallel} = D_{\perp}$ at all fields, whereas we have just seen that diffusion can be strongly anisotropic in this case, with D_{\parallel} differing appreciably from D_{\perp} .

As an alternative to the purely macroscopic equations (1) and (2) and to avoid a detailed and lengthy discussion of solution of the Boltzmann equation, while bringing out the most important points, we can use the familiar approach of moment equations, in which the Boltzmann equation (8) is multiplied by $\hbar \vec{k}$ and $\epsilon = \hbar^2 k^2/2m$, respectively, and integrated over \vec{k} to give balance equations for momentum and energy. For simplicity, we consider the interaction with acoustic phonons only. The collision terms can be approximated in a number of ways: either by assuming $f(\vec{k})$ to be a displaced Maxwellian appropriate to the temperature T of hot carriers,¹⁰ or, as in Ref. 11, by simply replacing an average over a function of momentum by the same function of the average momentum. In either case, one gets the same equations as are directly obtainable from the equations for electrons in a neutral gas¹¹ by replacing the atomic mass by the quantity M [Eq. (7)]:

$$-\nabla p + ne\vec{E} = n\nu\langle\epsilon\rangle m\langle\vec{V}\rangle, \quad (10)$$

$$0 = n\nu\langle\epsilon\rangle(2m/M)\langle\epsilon\rangle - \frac{3}{2}\kappa T_L - \frac{1}{2}M\langle V\rangle^2. \quad (11)$$

Here, ν is the collision frequency,

$$\nu(\epsilon) \equiv (2\epsilon/m)^{1/2} \nu^{-1}(\epsilon); \quad (12)$$

$\langle\vec{V}\rangle = \hbar\langle\vec{k}\rangle/m$ and $\langle\epsilon\rangle = \hbar^2\langle k^2\rangle/2m$ are the mean velocity and energy, respectively, related to the components of the distribution function by

$$\langle\vec{V}\rangle = (4n\hbar/3nm) \int k^3 \vec{f}^{(1)} dk, \quad (13)$$

$$\langle\epsilon\rangle = (2\pi\hbar^2/nm) \int k^4 f^{(0)} dk; \quad (14)$$

$p \equiv n\kappa T$; and T is the temperature of the hot carriers, related to the mean energy by

$$\langle\epsilon\rangle = \frac{3}{2}\kappa T. \quad (15)$$

In deriving (10) and (11), we have made the assumption that the phonons have an essentially undisturbed Planck distribution at lattice temperature T_L sufficiently high so that equipartitioning

applies.¹⁰ We treat deviations from the stationary, uniform state as perturbations; thus, only the most important of the space and time derivatives, the pressure gradient term, has been retained on the left-hand side of Eqs. (10) and (11). If we write

$$\langle\vec{V}\rangle = \langle\vec{V}_0\rangle + \langle\vec{V}_1\rangle, \quad \langle\epsilon\rangle = \langle\epsilon_0\rangle + \langle\epsilon_1\rangle, \quad (16)$$

where subscripts zero refer to the uniform state and $\langle\vec{V}_1\rangle, \langle\epsilon_1\rangle$ are small perturbations of order ∇n , then, upon linearizing with respect to small quantities, we find the following sets of coupled equations:

$$e\vec{E} = \nu\langle\epsilon_0\rangle m\langle\vec{V}_0\rangle, \quad (17)$$

$$\langle\epsilon_0\rangle = \frac{3}{2}\kappa T_L + \frac{1}{2}M\langle V_0\rangle^2, \quad (18)$$

and

$$-kT_0\nabla(\ln n) = \nu\langle\epsilon_0\rangle m\langle\vec{V}_1\rangle + \nu'\langle\epsilon_0\rangle\langle\epsilon_1\rangle m\langle\vec{V}_0\rangle, \quad (19)$$

$$\langle\epsilon_1\rangle = M\langle\vec{V}_0\rangle \cdot \langle\vec{V}_1\rangle, \quad (20)$$

where

$$T_0 \equiv 2\langle\epsilon_0\rangle/3\kappa. \quad (21)$$

Elimination of $\langle\epsilon_1\rangle$ between (19) and (20) gives

$$\vec{V}_1 = -\vec{D} \cdot \nabla \ln n, \quad (22)$$

where, for a coordinate system in which \vec{E} is parallel to the z axis, the diffusion tensor is of the form

$$\vec{D} = \begin{bmatrix} D_{\perp} & 0 & 0 \\ 0 & D_{\perp} & 0 \\ 0 & 0 & D_{\parallel} \end{bmatrix}.$$

The longitudinal and transverse diffusion coefficients are

$$D_{\parallel} = \frac{\kappa T_0}{m\nu\langle\epsilon_0\rangle} \left(1 + M V_0^2 \frac{\nu'\langle\epsilon_0\rangle}{\nu\langle\epsilon_0\rangle} \right)^{-1} \quad (23a)$$

$$\equiv \frac{\kappa T_0}{e} \mu \left(1 + \frac{\partial \ln \mu}{\partial \ln E} \right), \quad (24a)$$

and

$$D_{\perp} = \kappa T_0/m\nu\langle\epsilon_0\rangle \quad (23b)$$

$$= (\kappa T_0/e)\mu, \quad (24b)$$

respectively, where $\mu \equiv \langle V_0\rangle/E$ is the mobility coefficient. It is clear from (24a) and (24b) that this approach gives the same formula connecting D_{\parallel} , D_{\perp} , and μ as does thermodynamics [Eq. (6)]. However, we can now interpret anisotropic dif-

fusion in terms of microscopic concepts: The difference between D_{\parallel} and D_{\perp} is effectively a measure of the gradient of $\nu(\epsilon)$ with respect to energy. For example, when $\nu(\epsilon)$ is an increasing function of energy, collisions occur more frequently as the energy of the carriers is increased, and diffusion parallel to the electric field is reduced in comparison with the perpendicular direction, $D_{\parallel} \leq D_{\perp}$ (equality holds at zero field). For the deformation potential, $l = \text{const}$ and $\nu \propto \epsilon^{1/2}$, and we find from (17), (18), (23), and (24) that $D_{\parallel}/D_{\perp} = 0.5$ (high-field limit) in good agreement with the more exact result of Parker and Lowke.⁵

It is clear that we cannot neglect the dependence of $f^{(0)}$ upon ∇n (as done in Ref. 3), for this implies from (14) and (16) that $\langle \epsilon_1 \rangle = 0$, in contradiction to (20). [If $\langle \epsilon_1 \rangle$ were set equal to zero, then we would find $D_{\parallel} = D_{\perp}$ as given by Eq. (23b).]

The method of moment equations can be extended to the case where optical-phonon scattering is of importance. Here, however, difficulties arise because energy is not equipartitioned and the pressure in (10) becomes a tensor with diagonal entries $n\kappa T_{\parallel, \perp}$. The single equation (11) for energy has to be replaced by two equations, for T_{\parallel} and T_{\perp} . Because of these increasing complexities, it may not be any more advantageous to pursue this approach than to solve the Boltzmann equation directly. In adopting the latter course,

a method recently described by Kumar and Robson⁷ appropriate to ions in a neutral gas may be useful in providing a guide to tackling the problem for semiconductors.

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Mobility Gap and Anomalous Dispersion*

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It is shown that anomalous dispersion of quasiparticles leads to nonpropagating states. Therefore regions of anomalous dispersion define a sort of "mobility gap." Using the coherent-potential approximation, we calculate the conditions for obtaining such a mobility gap in a disordered binary alloy. Just outside the mobility edges located at ω_c the mobility vanishes as $|\omega - \omega_c|$.

We consider the question of the existence of a "mobility gap," of whether in a disordered medium there is a range of allowed energy levels which are all nonpropagating states. This concept, first conjectured by Mott,¹ Cohen, Fritzsche, and Ovshinsky,² and others, is of great physical interest in the electronic band structure of liquids, amorphous semiconductors, and disordered alloys. It has been shown that in the disordered linear chain *all* states are localized,³ so that the "mobility gap"