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Vacuum Polarization and the Quark-Parton Puzzle

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We suggest a solution to the question of why isolated quark-partons are not observed. It is argued that gauge theories can have the light-cone behavior of quark fields without quark production in deep-inelastic reactions. Our ideas are illustrated in the exact solution of quantum electrodynamics in one space and one time dimension. The theory's crucial physical property is the extreme polarizability of its vacuum. Predictions for deep inelastic final states are presented.

The quark-parton model has provided a simple intuitive guide for predicting and interpreting many features of deep-inelastic scattering processes.[1] Its success, however, underscores an old problem which was born when the quark model was invented: Why are isolated quarks not observed? In the context of deep-inelastic scattering this familiar problem becomes even more puzzling. The reason for this can be found in the fact that the parton model is based on soft (superrenormalizable or cutoff) field theories in which partons experience just soft, finite forces.[2] This suggests that when a parton absorbs a very virtual photon of energy ν, it should propagate essentially freely for lab-frame distances which grow proportionally with ν. Once this distance exceeds the size of a hadron (1 fm, say), one might naively expect that quarks would then be produced. In this Letter, however, we shall argue that this is not necessarily the case. If, in fact, the underlying constituent field theory has a sufficiently polarizable vacuum, quark-partons will never be produced.

Consider first an exactly soluble example of this phenomenon: quantum electrodynamics in one space and one time dimension. We shall argue that the deep-inelastic structure functions of this theory have the scaling laws of underlying free Fermi fields, although only massive bosons appear as asymptotic stable particles. The analog of triality in this simplified model is fermion number. The complete solution of this theory was first given by Schwinger.[3] The exact Green's functions can be obtained through a formal solution of the theory's equations of motion. We shall sketch the derivation here and leave the details and discussion to a lengthier exposition to appear elsewhere.[4] The familiar Lagrangian reads

$$\mathcal{L} = \bar{\psi} i\gamma^{\mu}\partial_{\mu}\psi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\bar{\psi}\gamma^{\mu}\psi A_{\mu}, \tag{1}$$

and the equations of motion are

$$\begin{aligned} &\gamma^{\mu}(i\partial_{\mu} - eA_{\mu})\psi = 0, \\ &j^{\mu} \equiv e\bar{\psi}\gamma^{\mu}\psi = \partial_{\nu}F^{\nu\mu}, \\ &F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}. \end{aligned} \tag{2}$$

A careful definition of the current leads to the equation[3]

$$(\Box + m^2)j^{\mu} = 0, \tag{3}$$

where $m^2 = e^2/\pi$. Thus, the spectrum of the theory contains free massive bosons. It is useful to introduce a scalar function $\varphi(x)$ such that

$$j^{\mu} = \epsilon^{\mu\nu}\partial_{\nu}\varphi, \quad \varphi(z,t) = \int_{-\infty}^{z} j^{0}(z',t)\,dz'; \tag{4}$$

φ will later be interpreted as a dipole density. φ satisfies the equation of motion and canonical commutation relation

$$\begin{aligned} &(\Box + m^2)\varphi = 0, \\ &[\varphi(z,t),\ \dot{\varphi}(z',t)] = im^2\delta(z - z'). \end{aligned} \tag{5}$$

The equation of motion for A_{μ} in the Lorentz gauge follows from Eq. (2),

$$\Box A_{\mu} = j_{\mu}. \tag{6}$$

Therefore, from Eqs. (3) and (6), it is clear that $m^2A_\mu + j_\mu$ satisfies a massless Klein-Gordon equation. Introducing a scalar field $\tilde{\varphi}$ such that

$$m^2A_\mu + j_\mu = \epsilon_{\mu\nu}\,\partial^\nu\tilde{\varphi}, \tag{7}$$

we have

$$\Box\tilde{\varphi} = 0. \tag{8}$$

Furthermore, since the Schwinger term is[3]

$$[j_0(z,t),\ j_1(z',t)] = -im^2\delta'(z-z') \tag{9}$$

and

$$[A_0(z,t),\ A_1(z',t)] = 0, \tag{10}$$

it follows that

$$[\tilde{\varphi}(z,t),\ \dot{\tilde{\varphi}}(z',t)] = -im^2\delta(z-z'). \tag{11}$$

The solution for the Fermi field has a particularly simple form in terms of φ and $\tilde{\varphi}$. From Eq. (2) and the definitions of φ and $\tilde{\varphi}$ one obtains

$$\psi(x) = \exp[-ie\alpha\Phi(x)]\,\chi(x), \quad \alpha = \gamma_0\gamma_1 \tag{12}$$

where χ is a free Fermi field,

$$i\gamma^\mu\partial_\mu\chi(x) = 0, \quad \{\chi(z,t),\ \chi^\dagger(z',t)\} = \delta(z-z'), \tag{13}$$

$$\Phi = \pi(\tilde{\varphi} - \varphi)/e^2. \tag{14}$$

These results lead to a simple derivation of Green's functions which will now be illustrated.

Consider a two-dimensional analog of $e^+e^- \to$ hadrons in which an external "scalar photon" couples weakly into this theory via the operator $s(x) = \overline{\psi}(x)\psi(x)$. The relevant matrix element is

$$S(x) = i\langle 0|\,Ts(x)s(0)\,|0\rangle. \tag{15}$$

Using the solution for ψ from Eq. (12) gives

$$S(x) = i\langle 0|T\overline{\chi}(x)e^{-2ie\alpha\Phi(x)}\chi(x)\,\overline{\chi}(0)e^{-2ie\alpha\Phi(0)}\chi(0)\,|0\rangle. \tag{16}$$

Using Eqs. (5), (8), and (11), this expression is easily simplified to

$$S(x) = i\,\mathrm{Tr}[G_0(x)]^2\langle 0|Te^{2ie\alpha\Phi(x)}e^{-2ie\alpha\Phi(0)}|0\rangle = i(2\pi^2x^2)^{-1}\exp\{-4\pi i[\tilde{\Delta}_F(0,x^2) - \Delta_F(m^2,x^2)]\}, \tag{17}$$

where $G_0(x)$ is the free-fermion propagator, $\tilde{\Delta}_F(m^2,x^2) \equiv \Delta_F(0,x^2) - \Delta_F(0,0) + \Delta_F(m^2,0)$, and $\Delta_F(m^2,x^2)$ is the free-boson propagator. The short-distance and light-cone behavior of $S(x)$ is evidently identical to a free-fermion theory, since as $x^2 \to 0$, $\Delta_F(m^2,x^2) - \tilde{\Delta}_F(0,x^2) \to 0$. To see how fermion singularities are replaced by boson singularities in the intermediate states of $S(x)$, recall that

$$-4\pi i\tilde{\Delta}_F(0,x^2) = \ln(x^2 - i\epsilon) + \text{const.} \tag{18}$$

Thus, the free-fermion factor x^{-2} is precisely canceled by the first exponential in Eq. (17), leaving only an exponential of the massive-boson propagator. This remaining exponential corresponds to a sum of Feynman diagrams in which any number of massive bosons propagate from point 0 to point x.

This calculation illustrates how fermion singularities in matrix elements of currents are canceled in this theory. The mechanism behind this example is, in fact, generally valid. Observe that a matrix element of products $\overline{\psi}\psi$ will consist of loops of fermion propagators times exponentials of the form $\exp\{4\pi i[\Delta_F(m^2,x^2) - \tilde{\Delta}_F(0,x^2)]\}$. The factors involving $\tilde{\Delta}_F(0,x^2)$ will cancel the singularities of the free-fermion loops, leaving only the singularities of the massive bosons. These are obtained by expanding the exponential factors involving $\Delta_F(m^2,x^2)$.

To understand the physics behind these formal results consider a semiclassical treatment of the annihilation reaction "virtual scalar photon" $\to$ anything. The initial state consists of a fermion and an antifermion emanating from one space-time point with momenta $+Q/2$ and $-Q/2$, respectively. This initial state will be represented by a c-number external current consisting of two point charges which travel at (almost) the speed of light in opposite directions:

$$\begin{aligned} &j_0^{\text{ext}} = e\,\delta(z-t), \quad j_1^{\text{ext}} = e\,\delta(z-t), \quad z>0;\\ &j_0^{\text{ext}} = -e\,\delta(z+t), \quad j_1^{\text{ext}} = e\,\delta(z+t), \quad z<0. \end{aligned} \tag{19}$$

In the presence of an external current, Eqs. (3) and (5) become

$$(\Box + m^2)j_\mu = -m^2 j_\mu^{\text{ext}}, \tag{20}$$

$$(\Box + m^2)\varphi = -m^2\varphi^{\text{ext}}, \tag{21}$$

where

$$j_\mu^{\text{ext}} = \epsilon_{\mu\nu}\,\partial^\nu\varphi^{\text{ext}}. \tag{22}$$

From Eqs. (19) and (22) it follows that the external c-number dipole density φ^{ext} is

$$\varphi^{\text{ext}} = -e\,\theta(t+z)\,\theta(t-z), \tag{23}$$

so the induced q-number dipole density satisfies

$$(\Box + m^2)\varphi = m^2 e\,\theta(t+z)\,\theta(t-z). \tag{24}$$

Thus, the radiation field φ is described by a coherent state. The solution of Eq. (24) is

$$\varphi(x) = 2e\int\left(\frac{1}{p^2-m^2} - \frac{1}{p^2}\right)e^{-ip\cdot x}\frac{d^2p}{(2\pi)^2}, \tag{25}$$

with the boundary conditions $\varphi(x) = 0$ unless $t > 0$ and $t^2 > z^2$. In coordinate space,

$$\varphi(x) = e\,\theta(t+z)\,\theta(t-z) - e\,\Delta_R(m^2, x^2), \tag{26}$$

where Δ_R is the retarded commutator. Δ_R is well approximated by $\theta(t+z)\theta(t-z)$ near the light cone ($x^2 < m^{-2}$), and by $\exp(-m|x|)$ for $|x|$ large. Therefore, the resulting amplitude φ vanishes near the light cone and tends to a constant (e) when $|x| \gg m^{-1}$. Finally, since φ is a Lorentz-invariant function, its lines of constant value are the hyperbolas $t^2 - z^2 = \text{const}$.

We can now understand the time development of the final state and the physics of the current flows which neutralize the outgoing fermions. For times $t < m^{-1}$, the dipole density φ is small and nonvanishing only for $|z| < m^{-1}$. For $t \sim m^{-1}$, the dipole density midway between the receding pair has strength e. As t grows the dipole density becomes constant over the entire spatial line between the pair. The polarization charge $j^0 = \partial\varphi/\partial z$ follows the outgoing initial fermions. From Eq. (26) it is easy to see that the polarization charge is confined to an interval of order $(m^2 t)^{-1}$ from the outgoing fermions. Furthermore, the polarization charges are equal and opposite to the charges of the outgoing fermions. The polarization charges can annihilate the outgoing fermions when the distance between the two becomes $\sim m^{-1}$ in the rest frame of the outgoing fermion. This corresponds to a distance $\sim Q^{-1}$ in the c.m. frame of the original pair. This configuration occurs after a time $t \sim Q/m^2$. The formal results of our earlier discussion show that this annihilation always in fact occurs. The proportionality between t and Q is important because it ensures that the outgoing fermions remain free for sufficiently long times to justify the calculations of the naive parton models. In other words, deep-inelastic structure functions can be calculated *as if* the fermions were free, although no fermions are actually produced.

The distribution of the emitted bosons in "virtual photon" → anything can be calculated from the field φ. Since the field is coherent, the outgoing particles are Poisson distributed on the rapidity axis with a density which will now be determined. From Eq. (25), we have

$$\begin{aligned}\varphi + \varphi^{\text{ext}} &= 2e\int\frac{1}{p^2+m^2}\,e^{-ip\cdot x}\frac{d^2p}{(2\pi)^2}\\ &= 2e\int\frac{i}{2\omega_p}\exp[-i(\omega_p t - p_1 z)]\frac{dp_1}{2\pi},\end{aligned} \tag{27}$$

where $\omega_p = (p_1^2 + m^2)^{1/2}$. This expression should be compared with the second-quantized expression for the field,

$$m\int\frac{1}{(2\omega_p)^{1/2}}a_p^\dagger e^{-ip\cdot x}\frac{dp_1}{(2\pi)^{1/2}} + \text{H.c.}. \tag{28}$$

Thus, we identify the momentum-space number density as

$$\langle a_p^\dagger a_p\rangle = 1/\omega_p. \tag{29}$$

Hence the bosons populate the rapidity axis with a density of unity. This result agrees with an exact calculation based on the Green's functions of two-dimensional quantum electrodynamics.[5] It also agrees with guesses made within the parton model by Berman, Bjorken, and Kogut[6] and Feynman.[2] The physical picture which has been obtained provides a realization of intuitive ideas suggested by Bjorken.[7]

We conclude with a discussion of the appropriate generalization of Schwinger's mechanism to four-dimensional gauge theories of quarks. Schwinger[3] has argued that if the coupling constant of a gauge theory is sufficiently large, the theory's vacuum will become so polarizable that external "charges" will be completely neutralized. If we assume that this is true, then it is not difficult to study the geometry of polarization currents in four dimensions. One finds in the reaction "virtual photon" → anything that a tube of polarized pairs forms between the outgoing fermions and neutralizes them just as in the two-dimensional theory studied here.[4]

Finally, one must construct a theory in which quark triality can be screened in the same way that electric charge is screened in two-dimensional quantum electrodynamics. One way to do this is to introduce additive quantum numbers such that their screening implies the screening of triality. This suggests that three-triplet models, in which the eight SU(3)′ (charm) currents couple to a non-Abelian gauge field, should provide the framework for a Schwinger phenomenon to screen triality.[8] Understanding these quantum number issues is an important problem for the future.

The relevance of these ideas to the real world can be tested by studying the final states of deep-inelastic processes. The produced hadrons should form jets aligned along the direction of the struck parton.[9] Their distribution in momentum space should be given by $dp_{\parallel}/E$ ($p_{\parallel}$ measured along the direction of the struck parton) and their multiplicity should, therefore, grow logarithmically with q^2. Once quark quantum numbers are incorporated into the present framework, many more predictions should result. Our ideas are still in a crude state. However, they possess the promise of providing much insight into many aspects of the problem of hadrons.

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Study of $\pi^- C \to \pi^+\pi^-\pi^- C^*(4.44)$ at 6.0 GeV/c*

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The production of three pions in carbon via $I=0$ exchange has been studied using the process $\pi^- C \to \pi^+\pi^-\pi^- C^*$, where C^* is the $I=0$, $J^P=2^+$ excited state of the carbon nucleus at 4.44 MeV. Preliminary results on the 3π mass distribution, the t distribution, and the spin and parity states within the 3π system are reported.

This paper describes a study of 3π production in carbon. It differs from previous studies in a series of heavy nuclei[1] in that we have selected an excited final state for the carbon nucleus and have analyzed the spin and parity of the 3π system. In two recent experiments[2,3] studying the scattering of pions by carbon, the process $\pi^- C \to \pi^- C^*$ has been identified, where C^* denotes the 4.44-MeV level of carbon with isospin $I=0$ and spin and parity $J^P=2^+$. The process proceeds by $I=0$ exchange in the t channel. The 4.44-MeV state is identified by its γ decay to the ground state of carbon. This technique was applied in the present experiment to the production of three pions in coincidence with a 4.44-MeV γ ray. A secondary beam of 6-GeV/c π^- from the zero-gradient synchrotron at Argonne National Laboratory (ANL) was used with the ANL effective mass spectrometer (EMS) to detect the three outgoing pions, and a NaI detector to detect the 4.44-MeV γ ray.

The ANL effective mass spectrometer has been