# Generalized Dispersive Photonuclear Sum Rule

W. Weise
*Institute for Theoretical Physics, University of Erlangen-Nürnberg, 8520 Erlangen, Germany*
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The dispersion-relation sum rule of Gell-Mann, Goldberger, and Thirring is generalized to incorporate the hadronic shadowing effects observed in high-energy photon-nucleus interactions. Resulting sum-rule constraints on the high-energy behavior are discussed.

Among the various photonuclear sum rules,[1] the one of Gell-Mann, Goldberger, and Thirring (GGT),[2] being based on dispersion relations, has played an outstanding role. The GGT sum rule states that the total photon-nucleus cross section $\sigma_{\gamma A}(\omega)$, integrated up to the meson production threshold ($\mu = 140$ MeV), is given by[3]

$$\int_0^\mu d\omega\, \sigma_{\gamma A}(\omega) = (NZ/A)S(1+\zeta), \tag{1}$$
$$\zeta \equiv (A/NZ)S^{-1}\int_\mu^\infty d\omega[A\sigma_{\gamma N}(\omega) - \sigma_{\gamma A}(\omega)],$$

where

$$S = 2\pi^2 e^2/M \simeq 60 \text{ MeV mb} \tag{2}$$

is the classical dipole sum, and

$$\sigma_{\gamma N} = (Z/A)\sigma_{\gamma p} + (N/A)\sigma_{\gamma n} \tag{3}$$

denotes the elementary photon-nucleon cross section ($\sigma_{\gamma p}$ and $\sigma_{\gamma n}$ refer to proton and neutron, respectively); $A$ is the nuclear mass number and $Z$ the nuclear charge ($N = A - Z$). The derivation of Eq. (1) requires the introduction of strong assumptions about the properties of the photon-nucleus interaction at asymptotic energies. For the photon forward-scattering amplitudes on the nucleus, $F_{\gamma A}(\omega)$, and the elementary nucleon, $F_{\gamma N}(\omega)$, defined by

$$F_{\gamma N} = (Z/A)F_{\gamma p} + (N/A)F_{\gamma n}, \tag{4}$$

the following relation needs to be valid:

$$F_{\gamma A}(\infty) = AF_{\gamma N}(\infty). \tag{5}$$

The assertion of Eq. (5) is that a photon of extremely large energy interacts with the nucleus as a system of free nucleons, or equivalently, in the language of multiple-collision theory, that the photon-nucleus amplitude is entirely determined by single-scattering events.

A critical re-examination of this point will be the subject of the present paper. Some criticism against the validity of Eq. (5) has already been mentioned briefly by Levinger[4] and by Danos and Fuller,[5] although no systematic analysis has been carried out to date. However, with the advent of a remarkable variety of photonuclear total-cross-section data both below the meson threshold[6] and at high energies (up to about 20 GeV),[7] a new stage of discussion has been reached. In particular, the low-energy data show that the enhancement $\zeta$ of the integrated cross section of Eq. (1) over the classical dipole sum amounts to $0.4 \lesssim \zeta \lesssim 1.2$, depending on the nuclear mass number. The original estimate of GGT was $\zeta \simeq 0.4$ for all nuclei. Furthermore, the high-energy experiments reveal rather strong shadowing effects: For photon energies 2 GeV $\lesssim \omega \lesssim$ 20 GeV, the effective number of nucleons in a photon-nucleus collision, $A_{\text{eff}}$, defined by

$$\sigma_{\gamma A}(\omega) = A_{\text{eff}}(\omega)\sigma_{\gamma N}(\omega), \tag{6}$$

turns out to be nearly independent of $\omega$ and is well approximated by $A_{\text{eff}} \simeq A^{0.91}$. These effects are commonly taken as evidence for the existence of hadronic fluctuations in the photon propagator[8]; they are conventionally treated in terms of the vector dominance model (VDM).[9,10]

Evidently, these facts cast considerable doubt on the validity of Eq. (5). In the following, a sum rule will be developed which formally incorporates the existence of hadronic-shadowing effects and which is free of asymptotic assumptions like Eq. (5).

Still, the general starting point will be the causality, crossing, and unitarity properties of any forward photon scattering amplitude $F(\omega)$ which lead to the Kramers-Kronig subtracted dispersion relation[11]

$$\text{Re}\,F(\omega_0) - \text{Re}F(0) = \frac{\omega_0^2}{2\pi^2}\text{P}\int_0^\infty d\omega \frac{\sigma(\omega)}{\omega^2 - \omega_0^2}. \tag{7}$$

Here $\omega_0$ is some fixed photon energy and

$$\sigma(\omega) = (4\pi/\omega)\,\text{Im}F(\omega). \tag{8}$$

The Kramers-Kronig relation is postulated to be valid for both $F_{\gamma A}(\omega)$ and $F_{\gamma N}(\omega)$; in the latter case, there has been a recent experimental confirmation of Eq. (7) at $\omega_0 = 2$ GeV.[12] The Thomson limits, $\mathrm{Re}F_{\gamma A}(0)$ and $\mathrm{Re}F_{\gamma N}(0)$, are reliably determined by low-energy theorems:

$$\mathrm{Re}F_{\gamma A}(0) = -(Ze)^2/M_A \simeq -Z^2e^2/AM, \quad \mathrm{Re}F_{\gamma N}(0) = -Ze^2/AM, \tag{9}$$

where $M_A$ and $M$ denote the nuclear and nucleon mass, respectively. Introducing the quantity $A_{\mathrm{eff}}(\omega)$ as defined in Eq. (6), the following relations are obtained for $\omega_0 \gg \mu$:

$$\omega_0^2 \mathrm{P}\int_\mu^\infty d\omega \frac{A_{\mathrm{eff}}(\omega)\sigma_{\gamma N}(\omega)}{\omega^2-\omega_0^2} - 2\pi^2\left[\mathrm{Re}F_{\gamma A}(\omega_0) + \frac{Z^2e^2}{AM}\right] = \left[1 + O\left(\frac{\mu^2}{\omega_0^2}\right)\right]\int_0^\mu d\omega\, \sigma_{\gamma A}(\omega), \tag{10a}$$

$$\omega_0^2 \mathrm{P}\int_\mu^\infty d\omega \frac{\sigma_{\gamma N}(\omega)}{\omega_0^2-\omega^2} + 2\pi^2\left[\mathrm{Re}F_{\gamma N}(\omega_0) + \frac{Ze^2}{AM}\right] = 0. \tag{10b}$$

Multiplying Eq. (10b) by $A_{\mathrm{eff}}(\omega_0)$ and adding it to Eq. (10a) yields the general sum rule ($\omega_0 \gg \mu$):

$$\int_0^\mu d\omega\, \sigma_{\gamma A}(\omega) = (NZ/A)S[1+\zeta(A,Z)], \tag{11}$$

$$\zeta(A,Z) = \frac{A}{NZ}\left[\left(\frac{A_{\mathrm{eff}}(\omega_0)}{A} - 1\right)Z + \frac{R(\omega_0)+I(\omega_0)}{S} - O\left(\frac{\mu^2}{\omega_0^2}\right)\right], \tag{11a}$$

$$R(\omega_0) = 2\pi^2[A_{\mathrm{eff}}(\omega_0)\,\mathrm{Re}F_{\gamma N}(\omega_0) - \mathrm{Re}F_{\gamma A}(\omega_0)], \tag{11b}$$

$$I(\omega_0) = \omega_0^2 \int_\mu^\infty d\omega [A_{\mathrm{eff}}(\omega_0) - A_{\mathrm{eff}}(\omega)](\omega_0^2-\omega^2)^{-1}\sigma_{\gamma N}(\omega). \tag{11c}$$

Clearly, this sum rule does not contain any *a priori* assumptions about the asymptotic properties of $A_{\mathrm{eff}}(\omega)$ and $\sigma_{\gamma N}(\omega)$.

The high-energy contribution to the sum rule, i.e., $\zeta(A,Z)$, will evidently be extremely sensitive to the behavior of $A_{\mathrm{eff}}(\omega)$. Since, on the other hand, the value of $\zeta$ is determined by low-energy measurements, it now looks suggestive to use the sum rule as a restrictive condition to study the properties of photon-nucleus interactions at extremely high energies, as represented by the specific form of $A_{\mathrm{eff}}(\omega)$ for large $\omega$. Two limiting cases, corresponding to qualitatively different assumptions about the behavior of photons at asymptotic energies, are of special interest:

(a) The original GGT sum rule follows from Eqs. (11) as a special case for $\omega_0 \to \infty$, $A_{\mathrm{eff}}(\infty) = A$ (i.e., no shadowing effects), and $R = 0$ in Eq. (11b). However, it turns out to be impossible to combine the low- and high-energy data on that basis.

(b) The other extreme situation, $A_{\mathrm{eff}}(\omega_0) = A^{2/3}$ for asymptotic energies $\omega_0$, has been suggested by Gottfried and Yennie[9] within the frame of the VDM. In this case, the sum rule together with the available experimental data implies that $R(\omega_0)$ would have to be extraordinarily large and positive. This could hardly be understood, for example, within the frame of multiple-scattering theory, unless $\mathrm{Re}F_{\gamma N}$ would asymptotically turn to positive values. It is interesting to note that according to recent investigations by Cheng *et al.*[13] such a behavior could be related to an asymptotically rising elementary photon-nucleon cross section.

An example of a more quantitative estimate will be presented to elucidate how the sum rule of Eq. (11) might be used to introduce strong constraints on the high-energy properties of the photon-nucleus interaction. For that purpose, let $\omega_0$ be chosen somewhere at the upper end of the energy region covered by experiments (for example, $\omega_0 = 15$ GeV). For the evaluation of $I(\omega_0)$ of Eq. (11c), $A_{\mathrm{eff}}$ and $\sigma_{\gamma N}$ need to be specified for all $\omega > \mu$. $\sigma_{\gamma N}(\omega)$ is known up to about $\omega \simeq 20$ GeV[14]; a smooth (though perhaps questionable) extrapolation to infinity may be taken from Damashek and Gilman.[15] As mentioned before, $A_{\mathrm{eff}}(\omega)$ is measured in the range between 2 and 20 GeV, and it may be reproduced satisfactorily within the frame of a simple model,[16] which combines the existence of hadronic components in the photon propagator with Glauber's multiple-scattering theory. The same model can be used to obtain a reliable estimate of $R(\omega_0)$ of Eq. (11b), the result being shown in Fig. 1.

Practically no experimental information exists about $A_{\mathrm{eff}}(\omega)$ in the resonance region $\mu \leq \omega \leq 2$ GeV. We therefore introduce an average shadowing parameter $\alpha$ in that region, defined by

$$\int_\mu^{2\,\mathrm{GeV}} d\omega\, A_{\mathrm{eff}}(\omega)\sigma_{\gamma N}(\omega) = A^\alpha \int_\mu^{2\,\mathrm{GeV}} d\omega\, \sigma_{\gamma N}(\omega). \tag{12}$$

The integrated resonance cross section on the

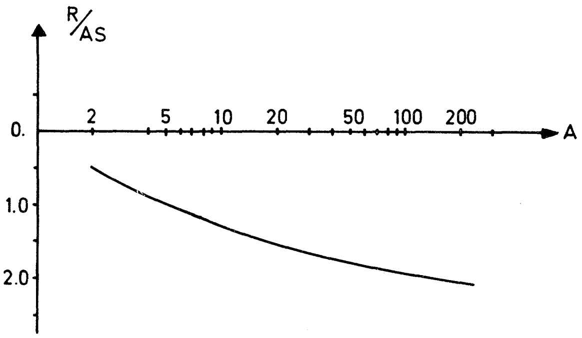


FIG. 1. Estimate of the quantity $R(\omega_0)$ in units of $AS$ [see Eq. (11b)] at $\omega_0 = 15$ GeV as a function of the nuclear mass number, using the multiple-scattering model described in Ref. 16.

right-hand side of Eq. (12) is well determined by experiment.

The asymptotic region ($\omega > 20$ GeV) remains open to speculation. However, the sum rule integrally combines the resonance and asymptotic regions, so that there will be a strong relationship between the resonance-shadowing parameter $\alpha$ of Eq. (12) (which is measurable in principle) and the results of any model describing asymptotic properties. As an example, let us study the consequences of the (purely hypothetical) assumption that the difference between $A_{\mathrm{eff}}(\omega)$ and $A_{\mathrm{eff}}(\omega_0)$ is extremely small for $\omega > 20$ GeV, so that the asymptotic part of the integral $I(\omega_0)$ can be neglected. Then the resonance-shadowing parameter is fixed by the sum-rule requirement of consistency between high- and low-energy data and turns out to be $\alpha = 0.79-0.80$, as shown in Fig. 2. If, on the other hand, $A_{\mathrm{eff}}(\omega)$ would be slowly and continuously rising (falling) beyond $\omega \simeq 20$ GeV, then $\alpha$ would have to be larger (smaller) than 0.8. Actual numerical examples show that $\alpha = 0.72-0.73$ for $A_{\mathrm{eff}} \to A^{0.90}$ and $\alpha = 0.85-0.87$ for $A_{\mathrm{eff}} \to A^{0.92}$. In other words, slight modifications of the asymptotic behavior may stipulate rather strong and observable changes in the resonance region. An extension of the experimental studies along the lines of Ref. 6 up to about 2 GeV would thus be highly desirable.

Finally, since the asymptotic behavior of $A_{\mathrm{eff}}$ is expected to be closely related to the asymptotic properties of the photon-nucleon cross section, the sum rule of Eqs. (11) might also contribute to the discussion of asymptotic constraints for elementary cross sections.

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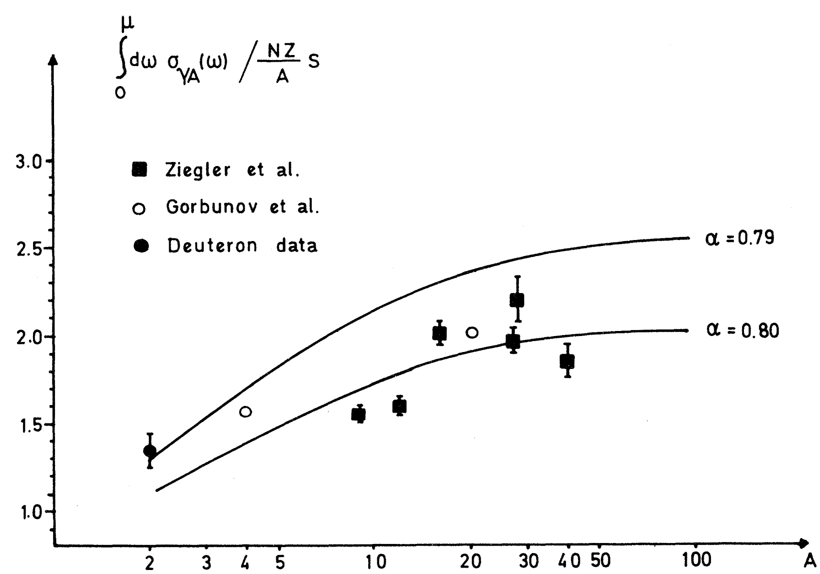


FIG. 2. Example of a sum-rule prediction assuming that the asymptotic part of Eq. (11c) can be neglected (see text). $\alpha$ is the average shadowing exponent in the resonance region, as defined by Eq. (12). The experimental data are taken from Refs. 6 and 1.

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# Pseudo Magnetic Moments of $^1$H and $^{51}$V Measured by a New Method

A. Abragam, G. L. Bacchella, H. Glättli, P. Meriel, M. Pinot, and J. Piesvaux
*Service de Physique du Solide et de Résonance Magnétique, Centre d'Etudes Nucléaires de Saclay, 91190 Gif-sur-Yvette, France*

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We describe a new method to measure pseudo magnetic moments of nuclei, based on the two-coil beam device of Ramsey. As a test, pseudo magnetic moments of the proton and the vanadium nucleus were measured and found equal to their known values within experimental accuracy.

We review first the definition of a pseudo magnetic nuclear moment and of a pseudo magnetic field. The formal analogy between the magnetic scattering of a slow neutron by a pointlike magnetic moment and the spin-dependent part of its *nuclear* scattering by a nucleus of spin $I$ has led to the concept of the nuclear pseudo magnetic moment[1-3] which is defined in the following.

The nuclear scattering length of a slow neutron by a nucleus can be written as

$$A_N = a_0 + a_N \vec{I} \cdot \vec{s}_n, \qquad (1)$$

where $\vec{s}_n$ is the spin of the neutron, $\vec{I}$ is that of the nucleus, and $a_N = (a_+ - a_-)/(I+\frac{1}{2})$, $a_+$ and $a_-$ being the scattering lengths for the two spin channels $J = I \pm \frac{1}{2}$ of the compound nucleus. We assign to every nucleus with $I \neq 0$ a nuclear pseudo magnetic moment $\mu^*$ by the formula[1-3]

$$\mu^* = -(\mu_B I / g_n r_0) a_N. \qquad (2)$$

where $\mu_B$ is a Bohr magneton; $g_n = -1.91$, the value of the neutron magnetic moment expressed in *nuclear* Bohr magnetons; and $r_0 = e^2/mc^2$, the classical radius of the electron. The definition (2) of $\mu^*$ is that of a hypothetical magnetic moment that would scatter a neutron magnetically with an amplitude equal to the actual *nuclear* scattering length $a_N$ (for a scattering plane perpendicular to $\mu^*$).[1-3]

From this magnetic analogy it is plausible to expect that a neutron going through a polarized nuclear target will "see" a pseudo magnetic field,

$$H^* = 4\pi M^* = 4\pi N \mu^* P, \qquad (3)$$

where $N$ is the number of nuclei per unit volume and $P$ is the nuclear polarization. That this is indeed so is easily proved[3] using the pseudopotential introduced by Fermi[4] to describe the thermal scattering of neutrons by bound nuclei. The existence of $H^*$ as evidenced by a change $\gamma_n H^*$ in the Larmor frequency of the neutron was in fact demonstrated theoretically much earlier[5] using the somewhat different approach of the refractive index of neutron optics rather than the magnetic analogy above. Some nuclei have large pseudo magnetic moments. The largest, that of the proton, calculated from (2), is equal to 5.4 *electronic* Bohr magnetons,[6] and in a polarized proton target $H^*$ can be quite large as in lanthanum magnesium double nitrate (LMN), where