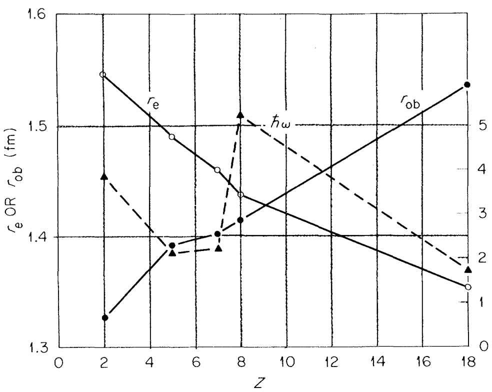

FIG. 2. The set of barrier parameters obtained in the least-squares fit to the total reaction cross sections. The effective radius parameter r_e is defined in terms of the barrier $\bar{E}_0$ by Eq. (21), and the barrier radius parameter r_{0b} is defined by $R_0 = r_{0b}(A_1^{1/3} + A_2^{1/3})$.

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Comparison of Sub-Coulomb Stripping and Analog-Resonance Results near Closed Shells*

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Sub-Coulomb (d,p) reactions have been studied for nuclei near closed shells. Reduced normalizations were extracted from the data and compared with those calculated for the analogs of the parent states using three analog-resonance theories. The R-matrix theory gives the best agreement with the (d,p) data.

In the study of isobaric analog resonances (IAR), spectroscopic factors have been extracted from proton elastic scattering by several different methods. These include the R-matrix approach of Thompson, Adams, and Robson[1] (TAR), and two shell-model methods, that of Mekjian and McDonald[2] (MM) and that of Zaidi and Darmodjo and Harney[3] (ZDH). These theories have been compared and their differences delineated on a theoretical level by Harney and Weidenmüller[4] (HW). Since these differences produce spectroscopic factors which may differ by as much as 50%, it seems desirable to attempt to determine which theory is the most nearly correct by using some experimental means independent of proton elastic scattering.

The method that has been employed in the past is to compare the spectroscopic factors S derived from (p,p_0) scattering via IAR with those found from (d,p) stripping to the low-lying parent states. Even though S is in principle model independent, in the distorted-wave Born-approximation (DWBA) analysis of the (d,p) data, S is strongly dependent on the optical-model parameters used. In many cases, the analysis of the (d,d) elastic-scattering data leads to several equally good families of parameters, which, when applied to the (d,p) reaction, yield spectroscopic factors which may differ by as much as 50%.[5] The dependence of S on the deuteron and proton potential parameters can be strongly reduced by performing the (d,p) experiments at energies in which both entrance and exit channels are below the Coulomb barrier. However, the dependence on the bound-state neutron potential parameters does not decrease appreciably below the Coulomb barrier. Thus, S still cannot be determined uniquely. Therefore, in order to carry out the comparison of IAR theories, a quantity which is parameter independent and can be extracted from both (d,p) and (p,p_0) IAR reactions is desirable.

The quantity which satisfies both these requirements is the reduced normalization Λ, which is essentially the square of the ratio of the transferred-neutron's asymptotic wave function to a spherical Hankel function. Λ was first defined by Rapaport and Kerman,[6] who showed that it is nearly independent of the geometrical parameters used to describe the neutron bound-state well for sub-Coulomb (d,p) stripping. The relationship of Λ to S is given by

$$\Lambda = (N^2/k^3)S,$$

where N is the ratio, outside the nuclear radius, of the DWBA neutron bound-state wave function to a spherical Hankel function, and $k = (2\mu|E_B|)^{1/2}/\hbar$ for a neutron separation energy E_B.

Clarkson, Von Brentano, and Harney[7] have defined this same quantity for (p,p_0) reactions in the context of the three IAR theories. Here, Λ turns out to be insensitive to the optical-model parameters as in the (d,p) case, but is strongly dependent on which theory is used for its extraction.[8] Therefore, it is now possible to use experimental results in an attempt to test the various IAR theories.

The attractive feature of Λ, its parameter independence, is purchased at the price of some basic physical content, in that the value of Λ does not reflect the single-particle character of the neutron wave function as does S.

Sub-Coulomb (d,p) stripping has been performed on nuclei near closed shells, and angular distributions have been extracted for ^{92}Mo, ^{140}Ce, ^{142}Nd, and ^{144}Sm. Following the procedure in Ref. 6, we analyzed these angular distributions, together with other sub-Coulomb (d,p) data (near closed shells) extracted from the literature, using the code JULIE.[9] Information on the analog resonances corresponding to the low-lying parent states observed in the (d,p) stripping reactions was obtained from the literature and analyzed following the procedure of Ref. 7 using the code BETTINA.[10] Reduced normalizations were thus obtained from the sub-Coulomb (d,p) reaction and from the (p,p_0) IAR results for each of the three analog-resonance theories, and compared.

Near the $N=50$ closed shell, data were analyzed for sub-Coulomb stripping on ^{90}Zr, ^{92}Zr, and ^{92}Mo. The data from ^{90}Zr(d,p) were from Dally, Nelson, and Smith[11] and the data from ^{92}Zr(d,p) from Kent, Morgan, and Seyler.[12] The DWBA optical parameters used for this region were those of Ref. 12; however, extensive searches over parameter space[13] indicated that the results were fairly insensitive to this choice of parameters, as was expected. The optical parameters for the IAR analysis were those suggested in Ref. 7. The spins of the parent states in this region were taken from the literature with the exception of the 1.69-MeV state in ^{93}Mo, which was taken as $\frac{5}{2}$ based on the recent work of Ball.[14]

For nuclei in the $N=82$ region, ^{138}Ba, ^{140}Ce, ^{142}Nd, and ^{144}Sm, a total of fifteen states were included in this comparison of reduced normalizations. The sub-Coulomb (d,p) work on ^{138}Ba

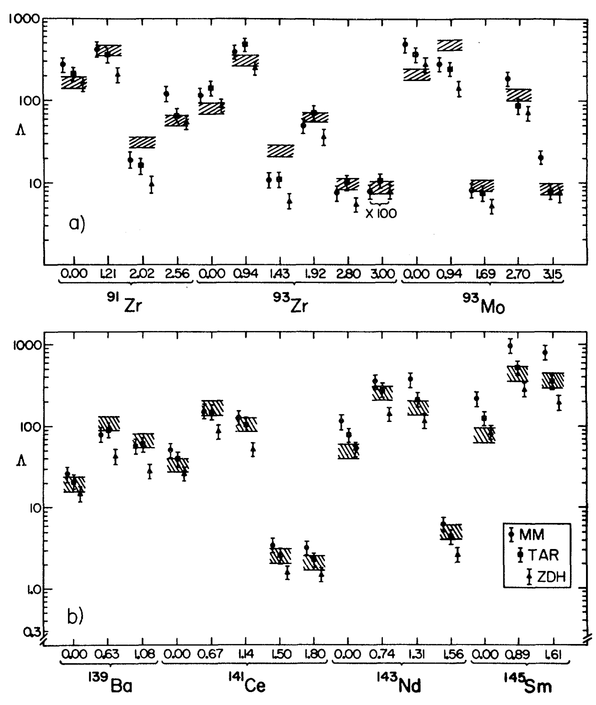

FIG. 1. Comparison of reduced normalization from sub-Coulomb (d,p) stripping and proton elastic IAR results. Cross hatching, (d,p) results with their assigned uncertainties. (a) Nuclei near mass 90. Note that the state at 1.43 MeV is known to be a doublet and has not been included in the comparison. (b) Nuclei near mass 140.

was taken from Ref. 6. Because of the lack of structure in the sub-Coulomb angular distributions, the spins of the various states were obtained from the literature.[15-17] as well as the necessary analog-resonance information required for the (p,p_0) analysis.[18-21] Only states whose spins were well determined were included. Optical-model parameters were obtained from Ref. 6 for the DWBA analysis and from Ref. 15 for the (p,p_0) IAR analysis.

For the $N=126$ region, the sub-Coulomb (d,p) work of Jeans *et al.*[22] on ^{208}Pb has been used for this comparison. The angular distributions they obtained for six states in ^{209}Pb were reanalyzed in order to extract the reduced normalizations for these parent states. The parameters required for the analog-resonance analysis were obtained from Wharton *et al.*[23] and Darmodjo *et al.*[24]

The uncertainties in Λ are as follows. From the sub-Coulomb (d,p) stripping reactions, the uncertainty from the cross-section measurement is $\pm 20\%$, except for the targets of ^{90}Zr, ^{92}Zr, and ^{92}Mo where the uncertainty is $\pm 15\%$. For the proton elastic reactions involving analog resonances, uncertainties of $\pm 20\%$ are assumed from the determination of the proton partial width (since the ratio Γ_p/Γ rather than Γ_p itself is the sensitive parameter in fitting the elastic data). The reduced normalizations for the $N=50$ and $N=82$ regions are plotted with their uncertainties in Fig. 1.

In order to determine which of the analog-resonance theories agrees best with the (d,p) reduced normalizations, a "goodness-of-fit parameter" I is defined,[13]

$$I=\sum_{\text{states}}\frac{(\Lambda_{dp}-\Lambda_{pp})^2}{(\Delta\Lambda_{dp})^2+(\Delta\Lambda_{pp})^2},$$

where the sum is over states of the same spin and parity, and Δ is equal to 0.2, representing the 20% uncertainty in Λ_{dp} and Λ_{pp} (except for the 15% uncertainty case mentioned above where Δ is equal to 0.15). Table I lists the various values of I for all the states used in this comparison. There is a total of 29 states, with ten different l and j values. For the $d_{5/2}$, $f_{7/2}$, and $g_{9/2}$ states, the ZDH theory gives the best agreement to the (d,p) results, and for all other states, it is the TAR method that yields the best agreement. Also, the TAR method produces the lowest total value of I.

The reason why the TAR R-matrix approach yields the best overall agreement to the sub-Coulomb (d,p) stripping results is not clear. In their extensive comparison of these three theories, HW pointed out that true, substantive differences exist among the theories. In particular both of the shell-model approaches utilize statistical assumptions in the construction of the analog states. These assumptions ignore second-order effects in the imaginary optical potential W, an assumption which HW show is violated even for very small values of W. Also HW show that application of an R-matrix theory to analog resonances appears to violate the R-matrix assumptions of no internal mixing and no external polarizing potential. It may be that the better results obtained with the TAR method is an indication of which of

TABLE I. Summary of comparison results using the goodness-of-fit parameter.

State	No.	I MM	I TAR	I ZDH	Targets used
$s_{1/2}$	7	30.29	17.69	30.27	^{90}Zr, ^{92}Zr, Mo, Pb
$p_{1/2}$	4	12.73	0.94	18.21	Ba, Ce, Nd, Sm
$p_{3/2}$	4	8.38	1.11	18.84	Ba, Ce, Nd, Sm
$d_{3/2}$	4	30.14	10.92	24.71	^{90}Zr, ^{92}Zr, Mo, Pb
$d_{5/2}$	5	22.98	11.16	8.29	^{90}Zr, ^{92}Zr, Mo, Pb
$f_{5/2}$	3	3.46	0.40	8.70	Ce, Nd
$f_{7/2}$	4	18.58	5.30	1.74	Ba, Ce, Nd, Sm
$g_{7/2}$	2	10.71	0.60	3.46	^{92}Zr, Pb
$g_{9/2}$	1	9.43	1.87	0.05	Pb
$i_{11/2}$	1	1.35	0.77	3.46	Pb
Total	35	148.05	50.76	117.73	
Total/state [a]		4.23	1.45	3.36	

[a]In order for Λ_{dp} and Λ_{pp} to be within 1 standard deviation, I/state must be less than 0.71.

these violations has a stronger effect on the calculation. It should be pointed out that the R-matrix approach does have an adjustable parameter, the channel radius, outside of which there exists no nuclear potential for the channel in question. In every case this parameter was set at the first maximum in the single-particle proton width outside the nuclear radius. This was done automatically by the code BETTINA.[10]

It should also be noted that for the six states in ^{209}Pb, the ZDH approach agrees best with the (d,p) results for four states, while the TAR method yields the best agreement for the other two states. Also, the ZDH theory produces the lowest total value of I (8.54) for the ^{208}Pb target, while the TAR method gives an I equal to 12.96 and the MM method produces a value of I of 63.43. The TAR method gives the best overall agreements for the mass-90 and -140 regions; the ZDH approach produces the best overall agreement for the parent states in ^{209}Pb.

The details concerning the sub-Coulomb (d,p) angular distributions, optical-model parameters, and the analog-resonance data used will be published for the mass-90 region by Morgan, Kent, and Seyler[13] and for the mass-140 region by Norton *et al.*[25]

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