[1]See, for example, M. E. Fisher, Rep. Progr. Phys. 30, 615 (1967); W. J. Camp and M. E. Fisher, Phys. Rev. 6, 946 (1972), and references given therein; K. Wilson and M. E. Fisher, Phys. Rev. Lett. 28, 240 (1972).

[2]D. Ruelle, *Statistical Mechanics* (Benjamin, New York, 1969); G. Gallavotti and S. Miracle-Sole, Commun. Math. Phys. 12, 269 (1969).

[3]See, for example, B. Simon, to be published.

[4]J. L. Lebowitz and O. Penrose, Commun. Math. Phys. 11, 99 (1968).

[5]D. Ruelle, Phys. Rev. Lett. 26, 303 (1971), and to be published; O. J. Heilmann and E. H. Lieb, Commun. Math. Phys. 25, 190 (1972); L. K. Runnels and J. B. Hubbard, J. Stat. Phys. 6, 1 (1972); H. J. Brascamp and H. Kunz, to be published.

[6]G. Gallavotti and S. Miracle-Sole, Commun. Math. Phys. 7, 274 (1968).

[7]R. B. Griffiths, C. A. Hurst, and S. Sherman, J. Math. Phys. (N.Y.) 11, 790 (1970).

[8]T. Rado, *Subharmonic Functions* (Chelsea, New York, 1949).

[9]B. Kaufmann and L. Onsager, Phys. Rev. 76, 1244 (1949); W. J. Camp and M. E. Fisher, Phys. Rev. B 6, 946 (1972); R. A. Minlos and Ya. G. Sinai, Teor. Mat. Fiz. 2, 230 (1970).

[10]O. Penrose and J. S. N. Elvey, J. Phys. A: Proc. Phys. Soc., London 1, 661 (1968); J. S. N. Elvey, unpublished.

[11]O. Penrose, in *Proceedings of the IUPAP Meeting on Statistical Mechanics and Thermodynamics, Copenhagen, 11-16 July 1966,* edited by T. A. Bak (Benjamin, New York, 1967). See also M. Duneau, B. Souillard, and D. Iagolnitzer, Commun. Math. Phys. 31, 191 (1973), and to be published, for even stronger cluster properties in the low-fugacity region.

Enhanced Microwave Emission Due to the Transverse Energy of a Relativistic Electron Beam

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The role of the transverse energy of a magnetically focused intense relativistic electron beam in the emission of microwaves is investigated experimentally and theoretically.

There has been considerable interest recently in the production of high-power microwaves by the pulsed intense relativistic electron beams which have become available in the last few years.[1,2] Experimental observations of high-power microwaves have been made in a variety of beam configurations, including beams injected into a few hundred milliTorr of neutral gas,[3] and magnetically focused annular beams propagating in vacuum ($<10^{-3}$ Torr) in the presence of special metal boundaries[4,5] and magnetic field perturbations.[6] Theoretical models have been developed, and calculations have been carried out to try to explain the observations,[7-10] but they have been hampered by an incomplete knowledge of the properties of the electron beam. In the present study, we control the transverse energy of beam electrons by varying the static spatial magnetic compression to which the beam, propagating in a straight, metallic drift-tube wave guide, is subjected. We find that if this transverse energy exceeds a certain minimum value, microwave power in excess of that possible by a single-particle mechanism is obtained. Moreover, we find this to be a characteristic of microwave production using the perturbed magnetic-field configuration. A theoretical model based on an interaction between the observed wave-guide mode and the electron beam gives unstable waves which agree well with observations.

The experimental configuration used for the present study is shown schematically in Fig. 1. The electron beam is produced by applying a high-voltage pulse from a 7-Ω, 50-nsec pulse-forming line to a foilless diode.[11] The diode voltage and current are 350–650 kV and 10–25 kA, respectively. The beam propagates in a 4.7-cm-i.d., thin-walled, stainless-steel drift tube immersed in a quasistatic (10-msec risetime) magnetic field applied coaxially to the drift tube by a 22-cm-diam, 1-m-long solenoid. By varying the distance d in Fig. 1 between the cathode and the end of the solenoid from -2 cm (i.e., cathode 2 cm inside the coil) to 18 cm, the magnetic field near the cathode relative to that in the middle of the coil is varied from ~ 0.55 to ~ 0.1. Thus, the distance d controls the magnitude of the radial component of the magnetic field near the cathode. The interaction of the beam electrons with the radial field produces the desired transverse energy. Lucite witness plates obtained in the middle of the solenoid for $d = -2$ and 1 cm are shown on the right-

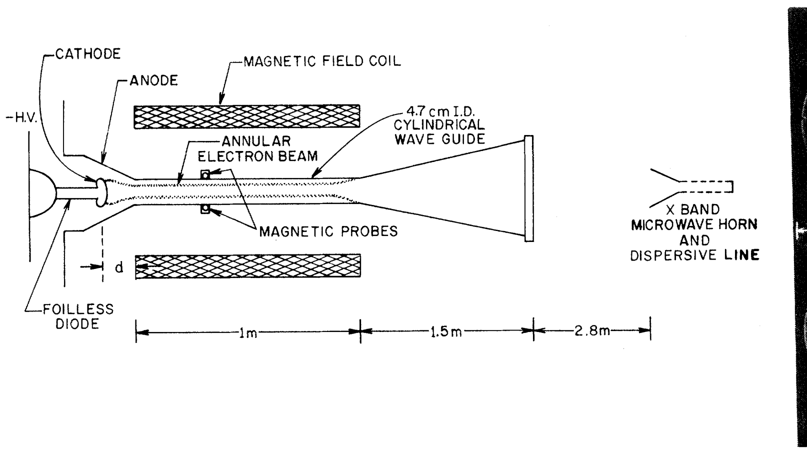


FIG. 1. Schematic of the experiment. Shown on the right-hand side are Lucite witness plates for $d = -2$ cm (upper) and $d = 1$ cm (lower).

hand side of Fig. 1.

The current actually flowing in the drift tube is measured by a magnetic probe located near the middle of the solenoid (~50 cm downstream from the cathode) oriented to measure the beam's self magnetic field B_θ. A similar loop located at the same axial position is oriented to measure the change in the axial magnetic field ΔB_z resulting from the beam's passage. Each loop is connected in a differential mode to eliminate electrostatic effects, and is recessed into the stainless-steel wall to minimize its perturbation of the beam. A standard-gain X-band horn, located 2.8 m from the end of the apparatus, is connected to a crystal detector through 10 or 155 m of X-band wave guide and calibrated attenuators. The detector monitors either the direct microwave signal or the dispersed signal. From the latter, the spectrum and power in the microwave pulse are obtained.[12] Figure 2 shows a data set consisting of diode voltage, B_θ, ΔB_z, and dispersed microwave signal, taken at $d = 1$ cm and an applied field of $B_0 = 5$ kG.

Figure 3 shows the relative microwave power as a function of $\Delta B_z/B_\theta$ at $B_0 = 5$ kG and a diode voltage of 600 kV. Each point is the average of four data shots. Listed for each point is the axial position (d) at which it is obtained, and the average current actually flowing in the drift tube. Thus, although the microwave power at $\Delta B_z/B_\theta = 0.17$, for example, is about the same as at 0.12, the power per unit of current is several times higher. (The main mechanisms for the "lost" current are presumably electrons flowing along field lines connecting the back of the cathode directly to the anode, and the mirroring of those

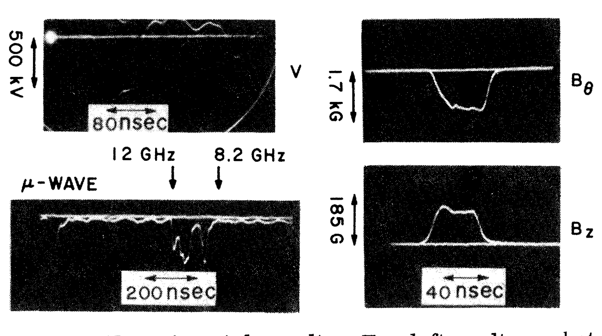
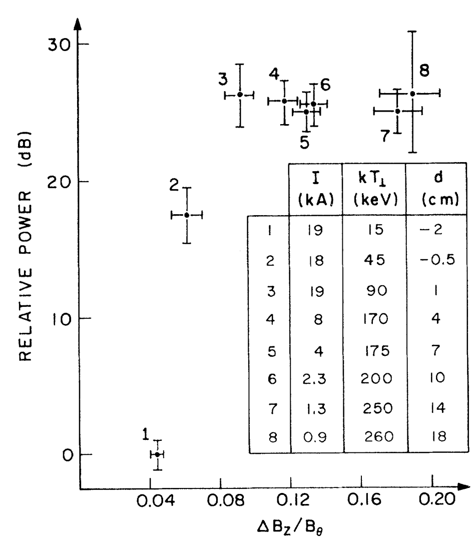


FIG. 2. Experimental results. Top left, voltage; bottom left, dispersed microwave signals; top right, θ component of the magnetic field; bottom right, "diamagnetic" signal of the beam.



	I (kA)	$kT_\perp$ (keV)	d (cm)
1	19	15	-2
2	18	45	-0.5
3	19	90	1
4	8	170	4
5	4	175	7
6	2.3	200	10
7	1.3	250	14
8	0.9	260	18

FIG. 3. Dependence of microwave power on $\Delta B_z/B_\theta$ (a function of beam "temperature").

initially forward moving electrons with the largest transverse energy.) Over the range $\Delta B_z/B_\theta = 0.04$ to 0.09 nearly the same drift-tube current is measured, although the microwave power increases by a factor of 400. Enhanced microwave noise on an electrostatic probe,[13] located at the same axial position as the magnetic probes, accompanies this increase.

To interpret the significance of the $\Delta B_z/B_\theta$ values, we use a "diamagnetic" model including the beam's self-fields for an annular beam equilibrium with only radial variation. This yields (mks)[14]

$$\frac{\Delta B_z(R)}{B_\theta(R)} \cong \frac{2kT_\perp}{eV_zRB_0} + \frac{B_\theta(R)}{2B_0}\left[\frac{(1-f)^2}{\beta^2} - 1\right]. \tag{1}$$

The uniform electron drift velocity is $V_z = \beta c$, $kT_\perp$ is the mean value of transverse energy per electron, R is the radius of the drift tube, f is the fractional charge neutralization, e is the magnitude of the electron charge, and a uniform beam density throughout the annulus has been assumed. The beam self electric field is represented in the $(1-f)^2/\beta^2$ term, but it has been eliminated in favor of B_θ by using the uniform density and velocity assumptions [including $(kT_\perp/m)^{1/2} \ll V_z$]. The values of $kT_\perp$ obtained from Eq. (1), assuming $f=0$,[15] are also given for each of the data points on Fig. 3. The changed magnetic compression alone cannot explain the factor of 6 increase in $kT_\perp$ between $d=-2$ and 1 cm. Note that the microwave power scales with transverse energy to the power of ~3 for fixed current. The power of the observed (X-band) microwave radiation varies with B_0 as well as with d. In addition, the spectrum is a function of d, B_0, and beam energy. These observations are summarized in the following sentences.

The spectrum of the radiation at $B_0 = 5$ kG stretches from between 8 and 9 GHz to 12 or 13 GHz for all values of d except the largest, in which case the highest frequencies are absent. (The cutoff frequency for the dominant mode being observed, TE_{01},[16] is about 7.8 GHz.) The microwave power decreases with increasing magnetic field (e.g., a 10 dB decrease from 5 to 9 kG at $d=1$ cm), and the minimum observed frequency increases both above and below 5 kG, increasing from 8 GHz at 5 kG to 10 GHz at 9 kG for $d=1$ cm. Furthermore, for lower transverse energy cases ($d=-2, -0.5$) the minimum observed frequency increases. Finally, dropping the cathode voltage to ~400 kV produces no significant change in the spectrum except for the higher magnetic fields, at which the observed minimum frequency increases.

To examine the guide-mode interaction mechanism theoretically, the Vlasov-Maxwell equations are employed for small-amplitude waves. The geometry of the model is such that a thin annular beam of average radius a and thickness $\Delta \ll a$ is propagated through a circular wave guide of radius R. A similar calculation in parallel-plate geometry has appeared recently.[17] The beam density is assumed constant within the beam annulus since experimentally $\Delta > r_L$, the Larmor radius. The beam is assumed to be sufficiently weak that its self-field may be neglected. Expanding the azimuthal component of the TE_{01} electric field in an eigenfunction expansion and using the orthogonality properties of Bessel functions, we find that the dispersion relation for electromagnetic waves is given by

$$\frac{\omega^2}{c^2} - k_z^2 - k_{cn}^2 = \frac{8\pi e}{c}\frac{k_\perp^2}{k_{cn}}\frac{\exp[-i(k_z z - \omega t)]}{b_0 R^2 J_2^2(\beta_n)}\int_0^R r\,dr\int d^3p\, f_1(r,z,\vec{p},t)v_\theta J_1(k_{cn}r), \tag{2}$$

where $k_\perp^2 = \omega^2/c^2 - k_z^2$, $k_{cn} = \beta_n/R$, b_0 is the amplitude of the longitudinal magnetic field of the TE_{01} mode, and β_n is the nth zero of J_1. The perturbed distribution function f_1 is obtained by integrating $(\vec{E} + \vec{v}\times\vec{B}/c)\cdot\partial f_0/\partial\vec{p}$ along unperturbed particle orbits with the assumption that $\vec{E}$ and $\vec{B}$ are wave-guide fields for a TE_{01} mode. Since r_L is assumed much smaller than a, the zero-order relativistic trajectories may be approximated by keeping terms up to order r_L/a. In this approximation, knowledge of cyclotron harmonics above the fundamental is lost. Choosing the anistropic velocity-space distribution function (normalized to 1) as $f_0 = \delta(p_\perp - p_{0\perp})\,\delta(p_z - p_{0z})/2\pi p_\perp$ within the annular beam, and zero elsewhere, gives the dispersion relation for the beam-guide system,

$$\frac{\omega^2}{c^2} - k_z^2 - k_{cn}^2 = \frac{\alpha_n^2}{\gamma_0}\left[\frac{2(\omega - v_{0z}k_{0z})}{\omega - k_z v_{0z} - \Omega/\gamma_0} - \frac{(v_{0\perp}^2/c^2)(\omega^2 - c^2k_z^2)}{(\omega - k_z v_{0z} - \Omega/\gamma_0)^2}\right], \tag{3}$$

$$\alpha_n^2 = \frac{\omega_b^2}{c^2}\frac{1}{\pi a}\frac{a^2}{R^2}\frac{v_{0\perp}}{\Omega}\frac{J_1^2(k_{cn}a)}{J_2^2(\beta_n)}, \quad \omega_b = \left(\frac{4\pi e^2 N}{m}\right)^{1/2},$$

$$v_{0z} = p_{0z}/\gamma_0 m, \quad v_{0\perp} = p_{0\perp}/\gamma_0 m, \quad \gamma_0 = [1 + (p_{0\perp}^2 + p_{0z}^2)/m^2c^2]^{1/2}.$$

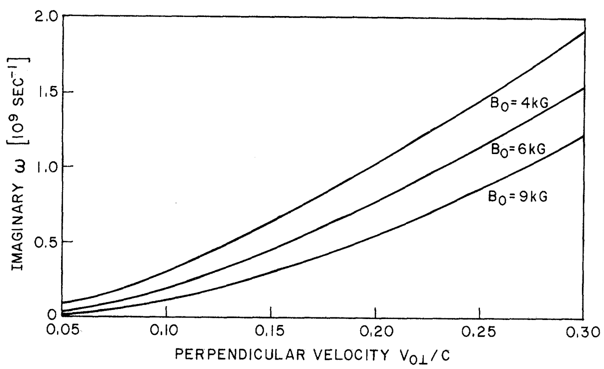


FIG. 4. Theoretical maximum growth rate as a function of transverse velocity and magnetic field.

The last term on the right-hand side of Eq. (3), proportional to $v_{0\perp}^2/c^2$, is the term which gives rise to an instability. The source of free energy is the anisotropy of the particle distribution.

The results of numerically solving Eq. (3) for imaginary ω in terms of real k_z for $n=1$, $\omega_b/c = 1.5\ \text{cm}^{-1}$ (beam density $\approx 10^{12}\ \text{cm}^{-3}$), $v_{0z}=0.866c$ ($\gamma_0 \gtrsim 2$), $R=2.35$ cm, and $a=1.2$ cm is shown as a function of $v_{0\perp}/c$ in Fig. 4. For $v_{0\perp}/c \lesssim 0.05$, corresponding to $\lesssim 5$ keV, growth times are an appreciable fraction of the beam pulse. However, for the larger values of $v_{0\perp}/c$, growth times can be $\lesssim 1$ nsec. Figure 4 also shows lower growth rates for higher B_0, in agreement with experiment. Furthermore, the lowest frequency (Reω) for growing waves is found to vary with B_0, total electron energy, and electron transverse energy, in good quantitative agreement with experimental observations. This frequency is very close to the frequency corresponding to maximum growth.

Preliminary observations comparing microwave properties of electron beams in the rippled magnetic field[6] and present configurations indicate that the basic property of a minimum of transverse energy being required for high-power microwave production is the same. A sharp cutoff in transverse energy production by the rippled magnetic field above the critical magnetic field[6,18] is observed and has been explained by Siambis.[19] A detailed study of similarities and differences between the ripple field and the compressed beam microwave properties is presently underway.[16]

We would like to acknowledge helpful discussions with J. A. Nation and many of our colleagues at the Naval Research Laboratory, particularly L. S. Levine and V. L. Granatstein.

[1]J. C. Martin, U.S. Patent No. 3,344,298 (1967), and private communications.

[2]See, for example, L. S. Levine and I. M. Vitkovitsky, IEEE Trans. Nucl. Sci. 18, 255 (1971), and references therein.

[3]J. A. Nation and W. L. Gardner, Nucl. Fusion 11, 5 (1971).

[4]J. A. Nation, Appl. Phys. Lett. 17, 491 (1970).

[5]M. Friedman and D. A. Hammer, Appl. Phys. Lett. 21, 174 (1972).

[6]M. Friedman and M. Herndon, Phys. Rev. Lett. 28, 210 (1972).

[7]B. J. Eastlund, Nucl. Fusion 11, 15 (1971).

[8]J. D. Gibbon, J. Phys. A: Proc. Phys. Soc., London 5, L85 (1972).

[9]W. M. Manheimer and E. Ott, to be published.

[10]W. B. Thompson, private communication.

[11]M. Friedman and M. Ury, Rev. Sci. Instrum. 41, 1334 (1970).

[12]J. A. Nation, Rev. Sci. Instrum. 41, 1097 (1970).

[13]C. A. Kapetanakos and A. W. Trivelpiece, J. Appl. Phys. 42, 4841 (1971).

[14]C. L. Longmire, *Elementary Plasma Physics* (Interscience, New York, 1963), Chap. 1; D. A. Hammer, to be published.

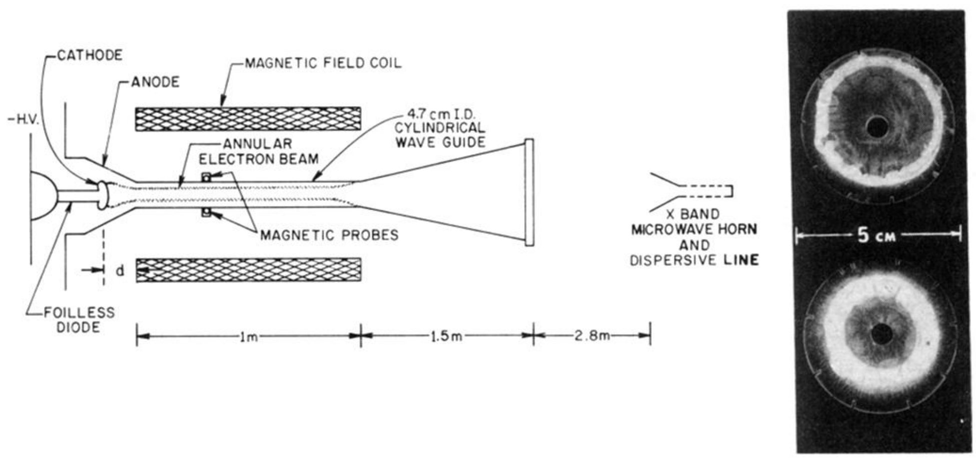
[15]This is justifiable since the drift tube pressure was $< 5\times 10^{-5}$ Torr air.

[16]V. L. Granatstein, M. Herndon, and R. K. Parker, private communication.

[17]E. Ott and W. Manheimer, Bull. Amer. Phys. Soc. 18, 585 (1973).

[18]M. Friedman and M. Herndon, to be published.

[19]J. Siambis, private communication.



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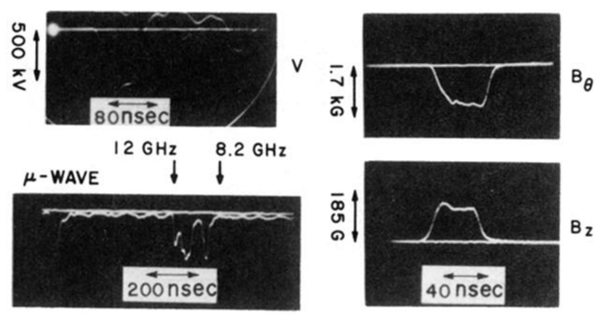

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