along the chain.[4-6] But the NS equation also includes interactions among similar scales, corresponding to spatial mixing of energy and, hence, cross-linking of the cascade chains. The question of whether such cross-linking is strong enough to limit intermittency buildup needs an answer.[18]

One thing feasible is the exploration of model systems [(5) and (6) are an example] which are more transparent than the NS equation and which may lead to insights. General models of conservative cascade chains, of the forms

$$dy_n/dt = \sum_m A_{nm} y_m \quad (A_{nm} + A_{mn} = 0) \tag{8}$$

and

$$dy_n/dt = \sum_{ml} A_{nml} y_m y_l \quad (A_{nml} = A_{nlm},\ A_{nml} + A_{mln} + A_{lnm} = 0), \tag{9}$$

can be explored simultaneously by analysis and by computer simulation.[19] The following caution must be observed in relating models with limited numbers of y's to the NS equation. Even when there is extreme spatial intermittency at small scales, the univariate distributions of the individual Fourier amplitudes in infinite, homogeneous turbulence with finite correlation scales are *accurately normal*, by the central limit theorem, solely as a consequence of homogeneity.[20] Spatial intermittency is a collective phenomenon in the Fourier representation.

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[1]A. N. Kolmogorov, C. R. Acad. Sci. U.R.S.S. 30, 301, 538 (1941).
[2]A. N. Kolmogorov, J. Fluid Mech. 13, 82 (1962).
[3]A. M. Oboukhov, J. Fluid Mech. 13, 77 (1962).
[4]E. A. Novikov and R. W. Styuart, Izv. Akad. Nauk SSSR, Ser. Geofiz., No. 3, 408 (1964) [Bull. Acad. Sci. USSR, Geophys. Ser., No. 3, 245 (1964)].
[5]A. M. Yaglom, Dokl. Akad. Nauk SSSR, Ser. Geofiz. 166, 49 (1966).
[6]R. H. Kraichnan, in *Statistical Mechanics*, edited by S. A. Rice, K. F. Freed, and J. C. Light (Univ. of Chicago Press, Chicago, 1972), p. 201.
[7]A. S. Gurvich and S. L. Zubkowskii, Izv. Akad. Nauk SSSR, Ser. Geofiz., No. 12, 1856 (1963) [Bull. Acad. Sci. USSR, Geophys. Ser., No. 12, 1130 (1964)]. (1964)].
[8]A. S. Monin, Phys. Fluids 10, S31 (1967).
[9]C. H. Gibson, G. R. Stegun, and S. McConnell, Phys. Fluids 13, 2448 (1970).
[10]H. L. Grant, R. W. Stewart, and A. Moilliet, J. Fluid Mech. 12, 241 (1962).
[11]S. Pond, R. W. Stewart, and R. W. Burling, J. Atmos. Sci. 20, 319 (1963).
[12]P. C. Martin, unpublished.
[13]M. Nelkin, Phys. Rev. Lett. 30, 1029 (1973). In a recent preprint, entitled "Turbulence, Critical Fluctuations, and Intermittency," Nelkin argues that the analogy between μ and critical exponents breaks down because of intermittency effects in the turbulence.
[14]T. D. Lee, Quart. J. Appl. Math. 10, 69 (1952).
[15]G. K. Batchelor, J. Fluid Mech. 5, 113 (1959).
[16]W. J. Cocke, Phys. Fluids 12, 2488 (1969).
[17]S. A. Orszag, Plasma Physics Laboratory, Princeton University, Report No. PPLAF-13, 1966 (unpublished).
[18]This question is discussed in some detail by R. H. Kraichnan, to be published.
[19]E. N. Lorenz, J. Fluid Mech. 55, 545 (1972).
[20]J. Lumley, in *Statistical Models and Turbulence*, edited by M. Rosenblatt and C. Van Atta (Springer, Berlin, 1972), p. 1.

Static Nuclear Magnetism in Extraordinary Liquid ^{3}He†

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Static-nuclear-magnetization measurements show temperature-independent magnetization in the "A" phase of ^{3}He, temperature-dependent magnetization in the "B" phase of ^{3}He, and at the boundary between these phases a discontinuity in magnetization which approaches zero at a polycritical point.

In this Letter we present the first measurements of static nuclear magnetism in an all-liquid sample of ^{3}He. Below the line of second-order transitions[1] at T_c the P-T phase diagram is split into two parts by a line T_{AB} of magnetization discontinuities extending from the melting curve at a temperature probably that of "B"[2] to the T_c line at a pressure of 21.7 bar. The magnetization discontinuity approaches zero as the T_{AB} line approaches the T_c line so the measurements

prove for the first time the existence of a line of first-order transitions which terminates at the line of second-order transitions in what is probably a polycritical point (PCP). A transition in thermal resistance is also observed at the line T_{AB}.[3] Between T_{AB} and T_c the liquid is called ^{3}He-A; otherwise the liquid below T_c is called ^{3}He-B. Our measurements show that in ^{3}He-A the static magnetization is essentially temperature independent, but in ^{3}He-B the magnetization decreases with decreasing temperature. These quantitative results are important in evaluating the state of the liquid.

At melting pressure, NMR measurements by Osheroff *et al.*[4] indicated a 40–60% decrease in absorption at "B" and little change between "A" and "B"; this is probably consistent with the present results extrapolated to melting pressure. Also, Halperin, Buhrman, and Richardson[5] reported a 5% change in *cell* magnetization at "B," though this includes an uncertain contribution from solid ^{3}He.

The demagnetization cell used for the present work was fitted above the main cerium-magnesium-nitrate (CMN) refrigerant with two epoxy towers magnetically shielded by niobium tubes. One tower contained the static-magnetization sensing coils. The other tower contained 10 mg of powdered CMN for thermometry.[3] The ^{3}He in the magnetization tower occupied a space 3.0 mm diam and 32 mm long above the main CMN. A 3.0-mm-diam hole 11 mm deep was drilled into the CMN immediately below the tower. Both the magnetization of the 10-mg CMN thermometer and the static magnetization of the ^{3}He were sensed using superconducting devices.[6] Measurements in both towers were possible even with 1000 G on the main CMN. For the measurements presented here a field of about 50 G was trapped in the magnetization tower. Calibration of the ^{3}He magnetization was made over a temperature range from about 0.26 to 1.14 K in a series of condensations by allowing the liquid ^{3}He at vapor pressure to rise in the magnetization tower until it was completely filled. The resulting magnetization changes are proportional to the total ^{3}He magnetic susceptibility $\chi_d+\chi_n$, where χ_d is the diamagnetic susceptibility and χ_n is the nuclear paramagnetic susceptibility C/T^* with $C=(1.362\times10^{-8}\ \mathrm{K})\times(37.0\ \mathrm{cm^3/mole})\times v^{-1}$, v being the molar volume. We used the values of T^* given by Ramm *et al.*[7] both in the calibration and for computation of the low-temperature normal-state paramagnetic susceptibility. During calibration we plotted magnetometer output versus $1/T^*$, obtaining from the slope the calibration of the instrument output in terms of paramagnetic susceptibility and from the intercept the diamagnetic susceptibility. We found $\chi_d v=-(2.14\pm0.11)\times10^{-6}$ cm^3/mole, which can be compared with $-(2.02\pm0.08)\times10^{-6}$ cm^3/mole measured by Barter, Meisenheimer, and Stevenson[8] and with a calculated value[9] of -1.99×10^{-6} cm^3/mole. This comparison is important since it reflects the accuracy and precision of our measurements. Our accuracy is based on the ±1% accuracy of the T^* from Ref. 7. The precision of the calibration of instrumental output in terms of χ_n is estimated to be ±5%.

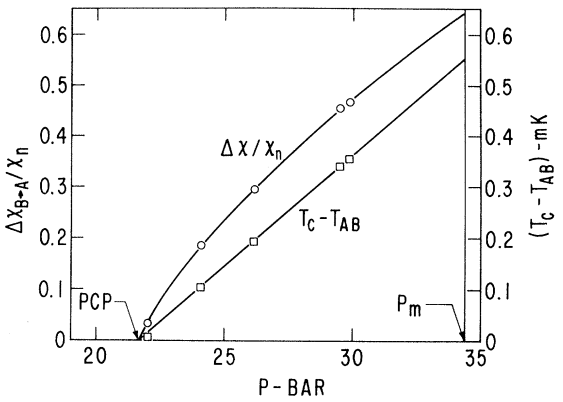

FIG. 1. Increase of nuclear susceptibility relative to that in the normal state when ^{3}He-B transforms into ^{3}He-A at T_{AB} (circles) and the difference between T_c and T_{AB} (squares) as functions of pressure. P_m is melting pressure and PCP denotes the polycritical point.

Measured properties along the T_{AB} line as a function of pressure are shown in Fig. 1. The quantity $\Delta\chi/\chi_n$ is the increase of static susceptibility in the transition from ^{3}He-B to ^{3}He-A referred to the full nuclear paramagnetic susceptibility in the normal state χ_n calculated as indicated above for each pressure. Higher pressures were precluded because of supercooling. Each $\Delta\chi$ point is the mean of typically ten observations obtained by shifting the temperature back and forth through T_{AB} using an external field from the magnetizing solenoid on the main CMN. Scatter in $\Delta\chi$ was typically a few tenths of a percent of χ_n. The magnetic temperatures T_{AB}^* and T_c^*, referred to the main CMN magnetic temperature scale, were obtained by letting the cell drift through those temperatures under residual heat leak. T_{AB}^* was noted at the sudden increase in magnetization (both the $B\to A$ and the

$A \rightarrow B$ transitions took place in less than a second) on warming through the line of first-order transitions and $T_c{}^*$ was obtained as in Ref. 1 from the change in cell warming rate at the line of second-order transitions. We plot $T_c - T_{AB}$, the corresponding difference in *absolute* temperatures on the tentative absolute scale established in our sound experiment,[10] where the P_c-$T_c{}^*$ curve is used as intermediary. The extrapolations of these curves suggest a pressure near 21.7 bar for the PCP and, at the melting curve, values of $(\Delta\chi/\chi_n)_m = 0.6_4$ and $(T_c - T_{AB})_m = 0.5_6$ mK. The NMR absorption results of Ref. 4 are probably consistent with $(\Delta\chi/\chi_n)_m$, and the value of $(T_c - T_{AB})_m$ is reasonable in comparison with the value of 0.5_9 mK expected from our melting-curve thermometry work[11] for the temperature difference between the "A" and "B" features.

If we use the measurements of Ref. 4 for the effect of magnetic field H on the pressure of the B' feature together with the zero-field slope[11] of the melting curve to estimate the slope $(\partial T_{AB}/\partial H)_p \simeq -(\chi_A - \chi_B)H/(s_A - s_B)$, we can use our value for $(\Delta\chi)_m$ to evaluate the entropy change per unit volume, $s_A - s_B$, at the line AB at melting pressure. This procedure has some accuracy since the field effect on B' is much greater than the field effect on the melting pressure.[12] We find $(s_A - s_B)_m = 0.6_1$ erg/cm^3 mK or, assuming $T_{B'} = 2.0_4$ mK, a latent heat per unit volume at melting pressure of 1.0_2 erg/cm^3. Further, using this value of $s_A - s_B$ and an approximate slope $(\partial P_{AB}/\partial T_{AB})_{H=0}$ of -30 bar/mk for the AB line near the melting curve from Ref. 3, we find $(v_A - v_B)/v \simeq -2\times10^{-8}$. We cannot detect volume changes of this size. We did search for a latent heat at the AB transition in Ref. 3, but its existence could not have been proven if the transition in the main cell were spread out in temperature because of thermal inhomogeneity to the same extent as the transition at T_c.

Temperature dependence of magnetization was measured both by rapidly (20 sec) applying a magnetic field (~40 G) to the main CMN to effect the change from $\chi(T)$ to χ_n and, starting with $T > T_c$, by rapidly decreasing this field to effect the change from χ_n to $\chi(T)$. The temperature, measured in the other tower, was taken to be the equilibrium value just before application of field for field-*on* data, and after removal of the field for field-*off* data. The two methods gave consistent results. Temperatures from the 10-mg CMN thermometer were converted to those in the main CMN in a separate drift experiment where both were measured (recall that T_c is determined using the main CMN). Susceptibility changes in ^{3}He-A are less than 2 to 3% of χ_n, so one may say to this accuracy that the static nuclear magnetization in ^{3}He-A is temperature independent and equal to that for the normal Fermi liquid. Measurements of the temperature-dependent static nuclear susceptibility in ^{3}He-B at 20.8 bar, just below the PCP, and at 29.9 bar below T_{AB} are shown in Fig. 2. For the 20.8-bar data T_c was taken to be that temperature (2.3_7 mK) for which χ/χ_n extrapolated to 1. This is slightly lower than the thermal T_c (0.05 mK) and probably reflects thermal inhomogeneities in our cell. For the 29.9-bar data, T_c was taken to be that for the thermal transition at this pressure, even though it may not be appropriate for ^{3}He-B at this pressure.

The above results are helpful in understanding the nature of the ^{3}He-A and ^{3}He-B liquids. The temperature independence of χ in ^{3}He-A is characteristic of the equal-spin-pairing odd-L state of Anderson and Morel.[13] We have compared the temperature-dependent susceptibility in ^{3}He-B with Leggett's formula[14]

$$\frac{\chi}{\chi_n} = \frac{(1+\frac{1}{4}Z_0)\chi_w/\chi_n}{1+\frac{1}{4}Z_0\chi_w/\chi_n}, \qquad (1)$$

where $Z_0 = -2.9_4$ for the data on Fig. 2 and χ_w is the susceptibility calculated without taking into account Fermi-liquid effects. We compare our data with Eq. (1) both using χ_w calculated by Anderson and Morel[13] for a particular linear combination of D states[15] and using χ_w given by Balian and Werthamer[16] (BW) for a particular

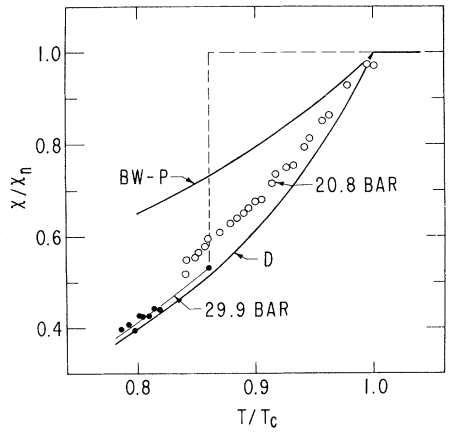

FIG. 2. Nuclear susceptibility relative to that in the normal state as a function of temperature relative to T_c for two pressures. The smooth curves are based on theory and described in the text.

P state since the identification of this state with ^{3}He-B has been suggested, for example, by Anderson and Brinkman.[17] On the other hand, Soda and Yamazaki[18] recently suggested F-state pairing for ^{3}He-A and D-state pairing for ^{3}He-B. The D- and BW P-state weak-coupling theoretical curves[19] are given on Fig. 2. Even at $T = 0$ the BW-state susceptibility is 0.35 of χ_n, so it appears, using weak-coupling theory, that ^{3}He-B is not a pure BW state. The experimental data are rather close to but apparently not coincident with the theoretical curve for D-state pairing. We note in this connection that Eq. (1) was derived for S-state pairing so that it may not be accurately applied to the present case although reasonable agreement is expected.[14] Further, the specific-heat ratio[1] at T_c is greater than that expected[20] for weak-coupling theories and is a weak function of pressure. Hence we might also expect a more complicated behavior for the susceptibility, rather than a universal, pressure-independent behavior scaling with T_c.

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[1]R. A. Webb, T. J. Greytak, R. T. Johnson, and J. C. Wheatley, Phys. Rev. Lett. 30, 210 (1973).

[2]D. D. Osheroff, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. 28, 885 (1972).

[3]T. J. Greytak, R. T. Johnson, D. N. Paulson, and J. C. Wheatley, Phys. Rev. Lett. 31, 452 (1973).

[4]D. D. Osheroff, W. J. Gully, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. 29, 920 (1972).

[5]W. P. Halperin, R. A. Buhrman, and R. C. Richardson, Bull Amer. Phys. Soc. 18, 642 (1973).

[6]R. P. Giffard, R. A. Webb, and J. C. Wheatley, J. Low Temp. Phys. 6, 533 (1972).

[7]H. Ramm, P. Pedroni, J. R. Thompson, and H. Meyer, J. Low Temp. Phys. 2, 539 (1970), Tables III and IV.

[8]C. Barter, R. G. Meisenheimer, and D. P. Stevenson, J. Chem. Phys. 64, 1312 (1960).

[9]Ya. G. Dorfman, *Diamagnetism and the Chemical Bond* (American Elsevier Publishing Co., New York, 1965), p. 33. We note that the experimental values of χ_d for both Ne and Ar are also high with respect to their calculated values.

[10]D. N. Paulson, R. T. Johnson, and J. C. Wheatley, Phys. Rev. Lett. 30, 829 (1973).

[11]R. T. Johnson, D. N. Paulson, C. B. Pierce, and J. C. Wheatley, Phys. Rev. Lett. 30, 207 (1973).

[12]W. J. Gully, D. D. Osheroff, D. T. Lawson, R. C. Richardson, and D. M. Lee, to be published.

[13]P. W. Anderson and P. Morel, Phys. Rev. 123, 1911 (1961).

[14]A. J. Leggett, Phys. Rev. 140, A1869 (1965).

[15]An earlier calculation for a particular D state was given by L. H. Nosanow and R. Vasudevan, Phys. Rev. Lett. 6, 1 (1961).

[16]R. Balian and N. R. Werthamer, Phys. Rev. 131, 1553 (1963).

[17]P. W. Anderson and W. F. Brinkman, Phys. Rev. Lett. 30, 1108 (1973).

[18]T. Soda and K. Yamazaki, to be published.

[19]For the BW P state we used the curve given by A. J. Leggett, Phys. Rev. Lett. 14, 536 (1965), with Z_0 changed to -2.94.

[20]G. Barton and M. S. Moore, to be published.

Decay of Correlations

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For lattice systems with a symmetric transfer matrix, the correlation functions decay exponentially with distance if the fugacity z lies in a region of the complex plane that contains the origin and is free of zeros of the grand partition function. When these interactions decay slower than exponentially the correlations do not decay exponentially for small z and, for Ising ferromagnets with pair interactions, for all values of the magnetic field.

The correlations between widely separated regions of a thermodynamic (infinite) system are of great interest. They play a central role in the theory of critical phenomena[1] and help us to understand the microscopic structure of thermodynamic systems.[2] (They also play an important role in recent work in field theory.[3]) We are interested here in how the asymptotic decay of the correlation functions at large distances is related to the analyticity properties of the free energy