[3] A. Heusler, H. L. Harney, and P. von Brentano, Nucl. Phys. A13, 591 (1969).

[4] G. C. Morrison, in *Nuclear Isospin*, edited by J. D. Anderson, S. D. Bloom, J. Cerny, and W. W. True (Academic, New York, 1970), p. 435.

[5] N. Stein, in *Nuclear Isospin*, edited by J. D. Anderson, S. D. Bloom, J. Cerny, and W. W. True (Academic, New York, 1970), p. 451.

[6] H. L. Harney, Nucl. Phys. A119, 591 (1968).

[7] E. Grosse, K. Melchoir, H. Seitz, P. von Brentano, J. P. Wurm, and S. A. A. Zaidi, Nucl. Phys. A142, 345 (1970).

[8] G. Clausnitzer, R. Fleischman, G. Graw, D. Proetel, and J. P. Wurm, Nucl. Phys. A106, 99 (1968).

[9] W. J. Thompson, in *Nuclear Isospin*, edited by J. D. Anderson, S. D. Bloom, J. Cerny, and W. W. True (Academic, New York, 1970), p. 427.

[10] W. J. Thompson, J. L. Adams, and D. Robson, Phys. Rev. 173, 975 (1968).

[11] W. J. Thompson and J. L. Adams, Tandem Accelerator Laboratory, Florida State University, Tallahassee, Fla., Technical Report ANSPEC (unpublished).

[12] F. G. Perey, Phys. Rev. 132, 755 (1963).

[13] P. D. Kunz, unpublished.

[14] C. M. Perey and F. G. Perey, Phys. Rev. 132, 755 (1963).

[15] F. D. Becchetti, Jr., and G. W. Greenlees, in *Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (Univ. of Wisconsin Press, Madison, 1971), p. 682.

[16] R. H. Bassel, Phys. Rev. 149, 791 (1966).

[17] S. Gales, L. Lessard, and J. L. Foster, Jr., Nucl. Phys. A202, 535 (1973).

[18] S. Raman, J. L. Foster, Jr., O. Dietzsch, D. Spalding, L. Bimbot, and H. Wildenthal, Nucl. Phys. A201, 21 (1973).

[19] J. L. Foster, Jr., O. Dietzsch, and D. Spalding, Nucl. Phys. A169, 187 (1969).

Method for Distinguishing between Multiparticle Production Models*

Paul M. Fishbane

Institute of Theoretical Physics, State University of New York, Stony Brook, New York 11790, and Physics Department, University of Virginia, Charlottesville, Virginia 22901†

and

J. S. Trefil

Physics Department, University of Virginia, Charlottesville, Virginia 22901

(Received 9 July 1973)

We discuss differences which differing mechanisms for multiparticle production imply for inclusive nuclear scattering.

The idea that qualitative and quantitative differences between different classes of models for elementary-particle production in single-particle inclusive reactions will emerge from studies of reactions on nuclei has been put forward many times.[1-3] In this note we contrast the results of realistic calculation of production processes on nuclei to show what experiments can differentiate between models of the elementary production process, and to comment on the (sparse) data now available in this area.

In general, current models for multiparticle production can be divided into "coherent-production models" (CPM) (here we might include[4] fragmentation, nova, and diffraction-dissociation models), in which the final state arises from the decay of a long-lived intermediate excitation, and the "incoherent-production models" (IPM) (here we might include[5] multiperipheral and dual-resonance models), in which the particles are produced directly.

If we try to distinguish between these models soley on the basis of data taken from hydrogen targets, we must study correlations in two-body inclusive reactions. While present indications[6] favor the IPM, the experiments are difficult enough, and the interpretation complicated enough, that it would be very useful to have another kind of test available.

If we consider a process in which the multiparticle final state is produced on a nucleus, however, a clear difference between these two classes of models emerges. In the IPM case intranuclear cascades can occur, as shown in Fig. 1(a). In CPM, however, only processes such as that shown in Fig. 1(b) are possible. We can envision the intermediate excitation propagating through the nucleus, and when the excitation undergoes an inelastic collision farther on, it can only be excited (or de-excited) to another

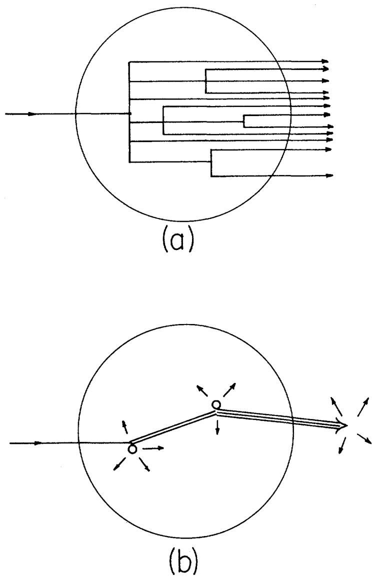

FIG. 1. (a) Schematic picture of the nuclear cascade in incoherent-production models. (b) Schematic picture of the production chain in coherent-production models.

state, with the possible production of slow secondaries through the excitation of the target nucleon. Decay of the projectile excitation takes place only after the excitation has passed through the nucleus.

Without further calculation we can observe some important qualitative features of each model. First, if we assume all quantities in the hydrogen-target reaction are energy independent, then in the CPM case, there is no source of energy dependence in a target of size A. For the IPM case, however, the development of the cascade depends on the multiplicity of the initial collision, which is in turn dependent on $\ln s$. Second, in each case we expect a buildup of the single-particle distribution for slow particles, with an accompanying increase in the slow-particle multiplicity. In the CPM case this is due to the multiple target excitations when the projectile has several inelastic collisions, in the IPM case through obvious properties of the cascade.

To proceed further, we must make more careful calculations. The results of the calculations we shall present are all based upon an extension of the Glauber theory to multistep production processes.[7] The IPM used[2] was a straightforward multiperipheral type, with even distribution of secondaries in rapidity space. The calculation consists of summing all possible cascades with appropriate nuclear weights [see Fig. 1(a)]. The CPM used[3] was a nova model, and the calculation consists of summing all the possible inelastic and elastic collisions in a chain [see Fig. 1(b)]. Further details of the assumptions and numerical methods we used are contained in the indicated references.

In all cases, the nuclear part of the reaction is treated as incoherent, and all final nuclear states are summed over. For comparison with data, *the data must also be summed over all final nuclear states* (see also work by Schaffner and Trefil[8]).

While the IPM calculation is relatively free of assumptions beyond those standard in the Glauber theory, the CPM is not. This follows from the fact that in a cascade, only those elementary-particle interactions occur which can be measured in independent experiments. In the CPM, on the other hand, all inelastic collisions beyond the first involve the collision of the intermediate excitation with downstream nucleons, and some extra assumptions must be made about this process. In what follows, we will assume for convenience that except for kinematic effects, which raise the maximum mass of the fireball beyond that attainable in hydrogen, the mass distributions of all of the excitations in the chain are identical. In addition we assume the scattering cross sections of the projectile excitations are the same as for the projectile (an assumption supported by experiment[9] for low-mass excitations). Our qualitative conclusions are largely independent of these assumptions.

We have found that a number of general qualitative statements can be made which do not depend strongly on the details of the models which were used. Let r be the rapidity, and $dn/dr = (\sigma_{\rm inel}{}^T)^{-1} d\sigma/dr$ be the inclusive number distribution, integrated over transverse momentum, whose integration over r gives $\langle n \rangle$, the average multiplicity of the measured species. Then our results can be summarized as follows:

(i) In both CPM and IPM, the number distribution in the projectile fragmentation region is approximately the same as the distribution on a proton target which was used as input for the calculation.

(ii) In the target fragmentation region, the number distribution exceeds the input distribution. For the IPM, $(dn/dr)_A$ for a nuclear target is given approximately by $A^{\beta(s)} f(s) (dn/dr)_{\rm H}$, where $f(s)$ increases approximately as $\ln s$, and $\beta(s)$ is a complicated function which for $A \gtrsim 10$ is about 0 up to National Accelerator Laboratory energies and becomes significant only at very high energies. For the CPM, on the other hand, $(dn/dr)_A$ is given approximately by $CA^{1/3}(dn/dr)_H$, where C is constant. $A^{1/3}$ here reflects a mean free path effect appropriate for CPM.

(iii) The pionization region interpolates the two fragmentation regions. For IPM this interpolation is calculable and is to a good approximation linear in rapidity r. For CPM, this is dependent on assumption and this region may be particularly model sensitive, since it reflects the large-multiplicity events (i.e., high-mass components) of the projectile excitation. Since we are especially interested in $\langle n\rangle$, for which the sensitivity is not so great, we also use a linear interpolation in r for CPM.

(iv) Both models predict increases of multiplicities for nuclear as opposed to hydrogen targets. For the IPM, the multiplicity is approximately of the form

$$\langle n\rangle = E + F\ln s, \qquad (1)$$

where E and F *are independent of* A for $A \gtrsim 10$ for incident energies into the TeV range. Both E and F have larger numerical values[2] than the corresponding constants for $\langle n\rangle_H$, the multiplicity on hydrogen targets. For the CPM, on the other hand, we have,[3] approximately,

$$\langle n\rangle = \tfrac{1}{2}(CA^{1/3}+1)\langle n\rangle_H, \qquad (2)$$

where in the nova model $C \approx 0.5$.

For quantitative results, we shall give one example, to stress again the differences which the two models can give. For more detailed calculations we refer the reader to Refs. 2 and 3. We show the reaction $p + {}^{12}C \to \pi^\pm + X$. The input reaction is $p + p \to \pi^\pm + X$, which according to the fit of Bali *et al.*[10] is given at 30 GeV/c by

$$dn'(x)/dr = 0.48\exp(-7.5x^2) + 0.38\exp(-12.1x^2).$$

At 1500 GeV/c, intersecting-storage-ring results suggest[11] that this result must be modified for small x,

$$dn(x)/dr = dn'(x)/dr + dn'(5x)/dr. \qquad (3)$$

With the use of Eq. (3) as input, as well as a corresponding[12] result for $\pi + p \to \pi^\pm + X$ (required for the IPM calculation), Fig. 2 shows the output distribution for scattering on ${}^{12}C$ at various energies for the two models referred to here, plotted as a function of rapidity.

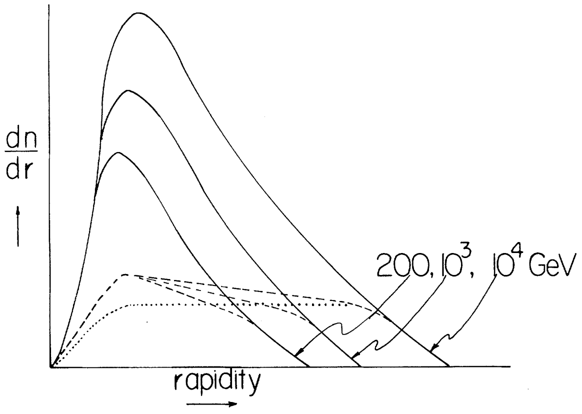

FIG. 2. Inclusive pion number distribution on carbon versus rapidity at various energies. Solid curves, for IPM; dashed curves, for CPM. Dotted curve, corresponding hydrogen distribution at the highest energy.

With regard to this curve, it is rather typical of light nuclei that the "backward boost" effect is smaller in the CPM, because the number of mean free paths in a nucleus is not so very large. Numerically, this also implies the multiplicity is not boosted as far for CPM as for IPM for light nuclei. However, these results can be reversed for larger nuclei at lower energies. For example numerical versions[2] of Eqs. (1) and (2) imply that for ${}^{108}Ag$ the pion multiplicity is greater for CPM than for IPM at low energies, with a crossover near 10^3 GeV. At this crossover, the pion multiplicity on ${}^{108}Ag$ is the same for CPM and IPM.

Thus we see that the characteristics of the IPM are (i) a large increase in multiplicity which is independent of A for $A \geq 10$ at accelerator energies and increases more quickly with energy than the multiplicity on hydrogen, (ii) a large increase in the number of low-rapidity particles, and (iii) an increase of the height of the backward peak with energy. The CPM, on the other hand, is characterized by (i) a relatively modest increase in multiplicity which is $\sim A^{1/3}$, (ii) a relatively small increase in the number of low-rapidity particles for light nuclei, and (iii) an unchanging height of the backward peak with energy.

The data available at the present time are not very extensive. The existence of increased multiplicities and the skewing toward low rapidity in cosmic-ray data have been noted,[13] although it is not really possible to distinguish between the IPM and CPM data there because of poor statistics.

Although it is not possible on the basis of the preliminary data of the emulsion exposures at the National Accelerator Laboratory[14] to rule out the CPM, these data do indicate a multiplicity increase on nuclear targets which is consistent with the IPM picture. We can only hope at this point that more data will become available soon.

One of us (P.M.F.) would like to thank Professor C. N. Yang and the members of the Institute of Theoretical Physics at Stony Brook for their gracious hospitality and many interesting discussions.

---

*Research supported by the National Science Foundation under Grants No. GP-32998X and No. GP-39179, and by a Cottrell Research Grant, The Research Corp.

†Permanent address.

[1] L. W. Jones, in *Proceedings of the International Conference on Expectations for Particle Reactions at the New Accelerators, Madison, Wisconsin, April 1970* (Univ. of Wisconsin, Madison, Wis., 1970); P. M. Fishbane and J. S. Trefil, Phys. Rev. D 3, 238 (1971); P. M. Fishbane, J. L. Newmeyer, and J. S. Trefil, Phys. Rev. Lett. 29, 685 (1972), and Phys. Rev. D 7, 3324 (1973); A. Dar and J. Vary, Phys. Rev. D 6, 2412 (1972); A. Subramanian, Tata Institute Report No. TIFR-BC-72-12, 1972 (to be published); A. S. Goldhaber, in Proceedings of the Fifth International Conference on High Energy Physics and Nuclear Structure, Uppsala, Sweden, 1973 (to be published).

[2] P. M. Fishbane and J. S. Trefil, Phys. Rev. D 8, 1467 (1973), and to be published. These works deal with IPM.

[3] P. M. Fishbane and J. S. Trefil, Phys. Rev. D, to be published. This work deals with CPM.

[4] J. Beneke *et al.*, Phys. Rev. 188, 2159 (1969); M. Jacob and R. Slansky, Phys. Rev. D 5, 1847 (1972); R. Hwa, Phys. Rev. Lett. 26, 1143 (1971).

[5] D. Amati *et al.*, Nuovo Cimento 26, 896 (1962); D. Sivers and J. Yellin, Rev. Mod. Phys. 43, 125 (1971).

[6] C. Quigg, in Proceedings of the Vanderbilt Conference on Multiparticle Production, Nashville, Tennessee, 1973 (to be published).

[7] J. S. Trefil and F. von Hippel, Phys. Rev. D 7, 2000 (1973).

[8] J. Schaffner and J. S. Trefil, unpublished.

[9] E.g., C. Bemporad *et al.*, in *Proceedings of the International Conference on Elementary Particles Amsterdam, 1971,* edited by A. G. Tenner (North-Holland, Amsterdam, 1971).

[10] N. F. Bali *et al.*, Phys. Rev. Lett. 25, 557 (1970).

[11] L. G. Ratner *et al.*, Phys. Rev. Lett. 27, 68 (1971); A. Bertin *et al.*, Phys. Lett. 38B, 260 (1972).

[12] E.g., W. D. Shepard *et al.*, Phys. Rev. Lett. 27, 1164 (1971).

[13] Subramanian, Ref. 1; Goldhaber, Ref. 1.

[14] J. J. Lord, private communication.