is perhaps necessary to explain the subthermal ultrasonic attenuation and/or (b) that at our high frequencies we are observing a new type of collective excitation[13] such as a zero-sound–like mode. Neutron measurements[14] have shown that such excitations are well defined in liquids. Our measurements would, however, also require them to propagate *macroscopic* distances with little attenuation and essentially zero dispersion in superfluid He. It is clear that numerical calculations of the lifetime of such a zero-sound mode in He II are required to settle the question unambiguously.

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Runaway Electrons in a Plasma*

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The electron runaway rate in a uniform plasma under a uniform electric field is calculated by solving the Fokker-Planck equation numerically. Comparison with other theoretical and experimental results is made.

It is well known that when a uniform electric field E is applied to a uniform plasma, a certain fraction of the electrons will run away; that is, they will gain an energy such that the electric force on them exceeds the drag force and they will accelerate indefinitely. The critical velocity for the latter to happen is $v_c \sim (E_D/E)v_t$, where $v_t = (T/m)^{1/2}$ and $E_D = m\nu v_t/2e$ is the Dreicer field, with $\nu = 4\pi n_e e^4 \ln(\Lambda)/v_t^3 m^2$ the collision frequency and n_e the electron density. If $E \ll E_D$, then $v_c \gg v_t$, so that only an exponentially small fraction of electrons run away in any given time.

The number of such runaway electrons produced per unit collision time has been calculated by a number of authors. One of the first efforts was Dreicer's.[1] Later Kruskal and Bernstein,[2] Gurevitch,[3] and Lebedev[4] also calculated these rates as an asymptotic expansion in E/E_D. Gurevitch's expansion suffered from a singularity which was subsequently removed by Lebedev. Even the latter was less complete than Kruskal and Bernstein, who found five distinct regions of behavior for the distribution function. The latter authors determined the rate up to a multiplicative constant which they have not evaluated yet.

Runaways were first observed experimentally

in the B-1 stellarator,[5] but their number was uncertain. It is now possible to determine the runaway rate more precisely, because in the ST tokamak, x rays produced by runaways in the volume of the plasma are measured.[6] All these results were in disagreement, so it was felt important to carry out a numerical solution. This was done by solving the Fokker-Planck equation for electrons, employing a code first developed by Killeen and Marx[7] but modified extensively by the present authors.

The code is capable of solving the fully nonlinear Fokker-Planck equation for the situation homogeneous in space and axially symmetric in velocity space. However, it was not employed in this form since when electric fields are large enough to produce substantial runaways, there is rapid Ohmic heating and not even a quasisteady state is expected. In the ST tokamak a steady state is reached because the energy of Joule heating is removed to the walls by large thermal-conduction processes.[8] Since the mechanism of this thermal conductivity is not well established as yet, we proceeded differently. We modified the collision integrals in the Fokker-Planck equation, evaluating them for Maxwellian electron and ion distributions. The Joule energy is thus removed by effective collisions with this Maxwellian heat bath, and a steady state is reached. The heat bath is at temperature T.

The Fokker-Planck equation is now linear and solved in this form. In dimensionless variables and in spherical velocity coordinates v, θ, ψ with polar axis along E, it reduces to

$$\frac{\partial f}{\partial t}+E\cos\theta\frac{\partial f}{\partial v}-\frac{E\sin\theta}{v}\frac{\partial f}{\partial\theta}=\frac{1}{v^2}\frac{\partial}{\partial v}v^2\left(\frac{1}{2}\frac{d^2\varphi}{dv^2}\frac{\partial f}{\partial v}-hf\right)+\frac{1}{v\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{1}{2v^2}\frac{d\varphi}{dv}\frac{\partial f}{\partial\theta}, \tag{1}$$

where

$$\varphi=\sum_{j=i,e}\varphi_j=\sum_{j=i,e}Z_j(\alpha_j)^{1/2}\left[x\Phi(x)-\frac{1}{2x}\Phi(x)+\frac{1}{2}\frac{d\Phi}{dx}\right]_{x=v/(\alpha_j)^{1/2}},$$

$$h=\sum_{j=i,e}\alpha_j v^{-2}(d/dv)v^2\varphi_j;\quad \alpha_i=m/M\sqrt{2},\quad \alpha_e=1/\sqrt{2},\quad \Phi(x)=(2/\sqrt{\pi})\int_0^x\exp(-t^2)\,dt.$$

The unit of velocity is v_t, the unit of E is $2E_D$, and the unit of time is v^{-1}. $Z_e=1$, and $Z_i=Z$ is the ion charge number.

The equation was solved on a uniform mesh in v and θ, $0\le v\le v_{max}$, $0\le\theta\le\pi$. For $v>v_{max}$ it was assumed that collisional diffusion in velocity space was zero, i.e., φ was taken as zero on the right-hand side of Eq. (1). If v_{max} is sufficiently large, this is a good approximation. Thus, for $v>v_{max}$, the equation is first-order hyperbolic and its solution is in principle known. The correct boundary conditions at $v=v_{max}$ can thus be established for this modified problem. For the integrations we chose $v_{max}=10$. Several trial integrations with $v_{max}=5$ and 15 indicated that 10 was a large enough value to produce a determination of the runaway rate to better than 1 – 2%. The mesh was 22 steps in θ and 75 in v. Again, trial integration with other mesh sizes showed that errors due to the finiteness of the mesh were less than 1%.

Integrations of Eq. (1) were carried out with $Z=1$, $E=0.04$, 0.06, 0.08, and 0.10; $Z=2$ for the last three values of E; and $Z=3$ and 10 for the last two. The procedure of integration was to start with $f=f_M=(2\pi)^{-3/2}\exp(-v^2/2)$, and to advance time with a step $\Delta t=0.25$ until f approached a nearly constant function in time which changed only because of the decreasing density produced by the runaway electrons themselves. The integration scheme was a mixture of implicit and

TABLE I. Electron runaway rates γ for various Z as a function of E.

E \ Z	1	2	3	10
0.04	1.914×10^{-6}	...	...	...
0.06	5.411×10^{-5}	2.611×10^{-5}	...	...
0.08	3.177×10^{-4}	1.735×10^{-4}	1.047×10^{-4}	9.0×10^{-6}
0.10	1.004×10^{-3}	5.839×10^{-4}	3.757×10^{-4}	4.49×10^{-5}

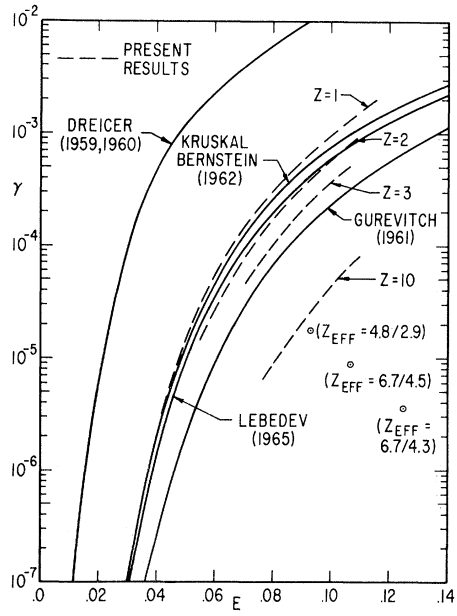

FIG. 1. Comparison of present numerical runaway rates with other theoretical and experimental results. All the theoretical results are given for $Z=1$. Circles, runaway rates deduced from x-ray spectra.

explicit and is described in Ref. 7. The runaway rate $\gamma = -d(\ln n)/dt$ was determined by calculating the flux through a sphere $v = \text{const}$, slightly smaller than v_{max}, and dividing by the density. This value saturated to a constant in a time of order $2/E^2$. It was exponentially extrapolated to its final value.

The results for γ are given in Table I and Fig. 1. γ is the fraction of electrons which run away in a "collision time" v^{-1}, and E is the electric field in units of $2E_D$. (Note the factor Z does not occur in these units.) Also included in Fig. 1 are the theoretical results. The best fit to our results is the Kruskal-Bernstein theory in which the constant has been chosen to make the results agree at $E = 0.04$:

$$\gamma_{K\text{-}B} = 0.35E^{-3/8}\exp\{-[(2/E)^{1/2} + 1/4E]\}.$$

The slight discrepancy with Lebedev occurs because he omitted region II of velocity space in Kruskal *et al.*'s notation. It is seen that Dreicer's result is too large by over an order of magnitude. This can probably be traced to his assumption of a simple angular dependence of f. All authors treat the modified Fokker-Planck equation (1), i.e., Maxwellian collisions. We also include some experimental runaway rates inferred from x-ray measurements on the ST tokamak. The x-ray spectrum reached a steady state whose profile corresponded to that obtained from a freely accelerating high-energy electron tail with a cutoff in the MeV region. Thus the electron distribution and the associated runaway rate could be calculated from an assumed effective value of Z in the discharge. This Z was derived from a measurement of the resistivity. The comparison with our results is uncertain for three reasons: (a) The effective charge number Z is not known. The charge number to achieve agreement seems unreasonably large (>10). (b) Our Eq. (1) does not represent the loss rates very well. (c) Instabilities produce collisions enhanced above the Coulomb value. By solving the full Fokker-Planck equation with various assumptions with respect to the loss rate we hope to explore point (b) and see if agreement can possibly be achieved without invoking (c).

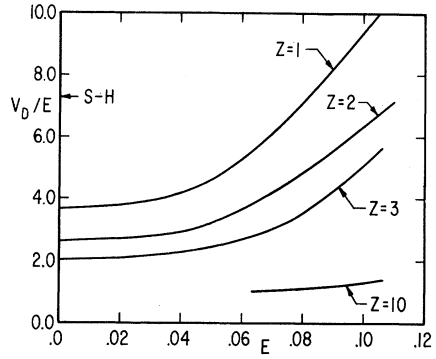

FIG. 2. Conductivities v_D/E as a function of E.

We also computed the mean velocity v_D and electron temperature T' of the distribution f, which differs from T because of Ohmic heating. These are given in Table II. The conductivity for our model, v_D/E in our units, is plotted in Fig. 2. It is seen that the conductivity increases with E. The zero-electric-field value was also computed using a "linearized" form of our code. This value does not represent the Spitzer-Härm conductivity since we did not perturb the collision integrals. The Spitzer-Härm conductivity in our units is indicated by S-H. It is seen that the S-H value is roughly a factor of 2 larger. This is probably because our assumption of Maxwellian collisions corresponds physically to neglect of the reaction on one electron in an electron-electron encounter. However, it is noteworthy that the conductivity increases so rapidly with E, although some calculations with self-consistent collisions indicate that this increase is not real, but

TABLE II. Mean velocity v_D and electron temperature T', shown in parentheses, for various Z as a function of E.

E \ Z	1	2	3	10
0.04	0.1664 (1.074)	...	...	...
0.06	0.3143 (1.310)	0.2186 (1.200)	...	...
0.08	0.5698 (1.872)	0.3904 (1.606)	0.2822 (1.411)	0.090 (1.124)
0.10	0.9345 (2.757)	0.6279 (2.196)	0.4597 (1.875)	0.1289 (1.218)

is a product of our model. Finally, the fact that the temperature is of order 1 indicates that our model gives sufficiently effective cooling of the electrons to lead to a steady state consistent with observations, even if the cooling mechanism is not the true physical one.

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Transmission of High-Frequency Phonons through a Solid-Liquid-Helium Interface

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Heat-pulse experiments at 1.3 K show that the conversion of ballistic high-frequency phonons in a solid into second sound in superfluid helium can be used to study the transmission coefficient of the phonons through the interface. The transmission coefficients for longitudinal and transverse phonons in silicon are found to be equal for heater temperatures between 10 and 20 K.

During the last few years the experimental techniques for investigating the transmission and reflection coefficient of phonons at an interface between a solid and liquid helium have been considerably improved. A direct measurement of the transmission coefficient is to be preferred over a determination of the reflection coefficient of phonons at the interface.[1] As a result of the acoustic mismatch between a solid and liquid helium, the reflection coefficient is large, and small changes are not so easily detected. Furthermore, because of diffuse reflection caused