*Work supported in part by the U.S. Atomic Energy Commission under Contract No. AT(11-1)-2130.

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Dispersion and Attenuation of Superthermal "Monochromatic" Phonons in He II

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(Received 25 June 1973)

The propagation of phonons in He II at 0.1 K, in the frequency range (2 to 9)$\times 10^{10}$ Hz, has been studied using superconducting Al-film generators. Such superthermal waves ($\hbar\omega \gg kT$) are shown to be long-lived propagating excitations in the liquid and to have negligible dispersion (group velocity varies by less than 0.5%). These measurements reveal that the current theories of phonon lifetimes in He II are inadequate.

The pressure and temperature dependence of the attenuation of subthermal ($\hbar\omega \ll kT$) ultrasonic waves in He II has been the subject of intensive investigation recently.[1] Maris and Massey[2] and Jäckle and Kehr[3] have successfully explained several unusual features by assuming that the He II excitation spectrum $\omega(k)$ initially curves *upwards*. Experimental support for such anomalous dispersion has been obtained from heat-capacity data[4] and from sound-velocity measurements on helium films.[5] Neutron measurements[6] have so far revealed only normal dispersion, although a region of anomalous dispersion at low k ($\lesssim 0.2$ Å^{-1}) could not be ruled out.

In this Letter we present experimental results on the group velocity ($v_g = \partial\omega/\partial k$) and attenuation (as a function of pressure) for phonons of frequency $\nu \sim 2\times 10^{10}$ to $\sim 9\times 10^{10}$ Hz [$k \sim 0.04$ to 0.22 Å^{-1} at saturated vapor pressure (SVP)] at low temperatures (~ 0.1 K) using "monochromatic" phonon generation and detection and time-of-flight techniques.[7,8] We give strong evidence that such superthermal waves are long-lived propagating excitations in the liquid and have a relative dispersion of less than 0.5% in this low-k region with $v_g = 238$ m/sec with an absolute error of ± 4 m/sec.

The experiments were performed in a large He II cell cooled by means of a dilution refrigerator (see inset of Fig. 1). In the initial exper-

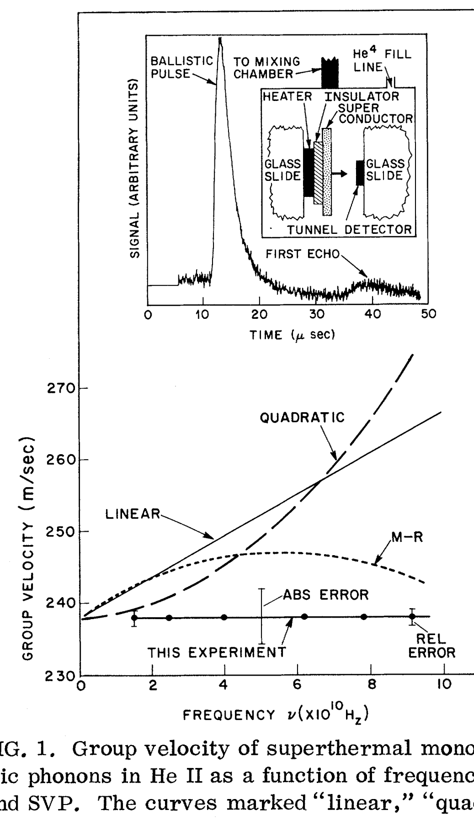

FIG. 1. Group velocity of superthermal monochromatic phonons in He II as a function of frequency at 0.1 K and SVP. The curves marked "linear," "quadratic," and "M-R" were calculated for different forms of the He excitation spectrum (see text). At the top of the figure we show a recorder tracing of a typical ballistic phonon pulse and its first echo. Path length 2.68 mm.

iments the source of phonons was a "pure" Al film (~ 1000 Å thick) pumped by a heat pulse (~ 1 to 5 V amplitude and 0.1 to 0.5 μsec duration) generated by a 50-Ω Constantan film of typical size 3 mm$\times$3 mm. Such an arrangement[8] has been shown to generate primarily a large density of phonons centered about the superconducting energy gap 2Δ. The generated phonons were detected at the other end of the He II sample chamber by means of an Al tunnel junction (2.3 to 2.7 mm away) which served as a quantum detector[7,8] of phonons of energy $\hbar\omega \geq 2\Delta$. Furthermore, their energy was tuned by application of a magnetic field parallel to the films, by use of a superconducting solenoid. A Helmholtz coil was used to cancel out any perpendicular component of field due to misalignment. A PAR 160 boxcar integrator, or a Biomation 8100 transient recorder with 10 nsec resolution, was used to time analyze and signal average the received pulses. By measuring their time of arrival and their amplitude, we measured the group velocity and attenuation of the 2Δ phonons propagating through the liquid. In the presence of anomalous dispersion the group velocity of the detected phonons should have been *greater* than that measured ultrasonically at low frequency.

In the inset of Fig. 1 we show a recorder tracing of a typical ballistic pulse and its first echo at 0.1 K and at SVP. This tracing was taken for phonons of frequency 0.91×10^{11} Hz. The leading edge of the ballistic pulse, which corresponds to the *fastest* phonons, was found to have an arrival time of 11.25 μsec. For a propagation length of 2.68 mm this corresponds to a velocity of 238 m/sec, in excellent agreement with neutron and ultrasonic data. At the bottom of Fig. 1 we show the measured velocity as a function of frequency. It is clear from the data that in the region (1.5 to 9) $\times 10^{10}$ Hz the phonon velocity is independent of frequency. Even though the absolute velocity accuracy in our measurements is estimated to be only ± 4 m/sec, our relative accuracy is much better. Despite varying the energy gap of the Al by a large amount, the leading edge of the ballistic pulse was found at the same time within about 0.5%. Our energy resolution does get poorer with increasing field[8] because of gap-edge smearing, but the total absence of any faster phonons is strong evidence that any anomalous dispersion must be extremely small. Numerically, this is illustrated at the bottom of Fig. 1 where we have indicated the expected values of the group velocity, assuming different forms of the excitation spectrum. Taking

$$\omega(k) = ck(1+\alpha k+\gamma k^2+\cdots) \tag{1}$$

we calculated v_g. The curve labeled "linear" assumes $\gamma=0$ and $\alpha = 0.24$ Å as measured by Anderson and Sabisky[5] at 1.38 K for He films. The curve labeled "quadratic" was calculated assuming $\alpha=0$ and $\gamma=0.88$ Å^2 as suggested by Maris[2] to explain the ultrasonic attenuation. The curve "M-R" was calculated from the polynomial expansion given by Molinari and Regge,[9] where, because of higher-order terms, the dispersion is less. The experiments clearly show that if anomalous dispersion exists at these very low temperatures, the relative dispersion over the frequency range (1.5 to 9)$\times 10^{10}$ Hz must be less than 0.5%, and the *maximum* group velocity must be less than 242 m/sec. This is consistent with low-frequency ultrasonic data.[10] Since the ballistic phonons scatter rapidly at elevated temperatures (above ~0.5 K), we are unable to rule out the possibility that the magnitude of the anomalous dispersion may be strongly temperature dependent and/or a film effect.

With increasing pressure the region of anomalous dispersion is expected to move in k space.[3] With $\nu = 0.91\times 10^{10}$ Hz and for pressures up to the solidification point, we find a velocity in agreement with previous ultrasonic data and no evidence for "fast" phonons. At higher frequencies ($>10^{11}$ Hz), however, we begin to see a *slowing* down of the ballistic pulse, consistent with *normal* dispersion. The magnitude of the normal dispersion increases with increasing pressure in agreement with neutron measurements.[6] These higher-frequency measurements, which will be reported elsewhere, were made with Sn generators and detectors, which have a maximum gap of 2.8×10^{11} Hz.

We now show that we have indeed studied the propagation of 10^{11}-Hz phonons and not low-frequency ones. To prove this we replaced the Al "fluorescent" generator with an Al tunnel junction and studied the intensity of the received pulse at the detector as a function of generator bias[7,8] (current). The data are shown in Fig. 2. At 24 bar the received signal was seen to increase[11] linearly with generator current until a current corresponding to a bias of $eV = 4\Delta$ is reached when an abrupt change of slope was noticed. This is because the phonon generation process is a two-step process[7,8] (relaxation of an excited quasiparticle to the gap edge and then recombination) and the detector responds only to phonons

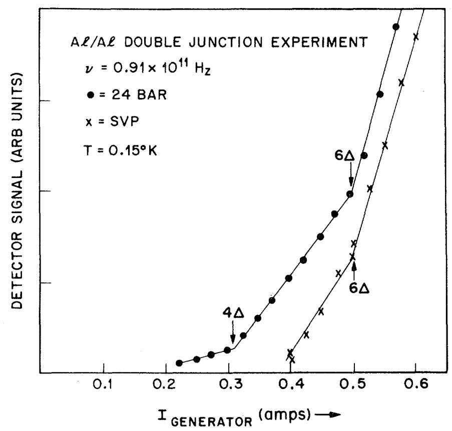

FIG. 2. Amplitude of received ballistic pulse versus generator current. Double-junction experiment, $T = 0.15$ K. The structure at biases of 4Δ and 6Δ are evidence for nonthermal generation and detection and long lifetime of generated phonon in He II (see text).

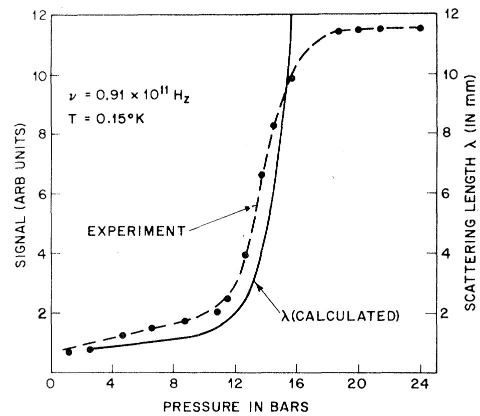

FIG. 3. Pressure dependence of signal strength of 2Δ phonons in He II. The generator and detector biases were kept fixed. Both the actual signal and the calculated scattering length are plotted (see text).

of energy 2Δ or greater. Thus, when a bias equivalent to energy 4Δ is reached in the generator, the detector is sensitive to both recombination and relaxation phonons of energy 2Δ and hence the change in slope. An additional change in slope occurs at a bias of 6Δ because of higher-order processes.[8] At SVP the data (see Fig. 2) were similar, except that the detected signal amplitude for the same generator current was much weaker than at 24 bar because of the increased attenuation of the 2Δ phonons, which is discussed below. At the low pressure we had sensitivity to see clearly only the 6Δ break in slope.

The data shown in Fig. 2 may be summarized as follows. The discrete structure observed at the detector as a function of generator bias proves that the generated phonons do indeed propagate at frequency 2Δ through the liquid helium. This excludes the possibility of strong small-angle scattering at these frequencies.

We now turn to the pressure dependence of the attenuation of the 2Δ phonons. The data are shown in Fig. 3 where we have plotted the amplitude of the ballistic pulse as a function of pressure at $T = 0.15$ K for $\nu = 0.91 \times 10^{10}$ Hz. The data were taken with a fixed generator current (~0.45 A) and a fixed detector bias. It is clear from this figure that the attenuation changes markedly in the pressure range 10 to 16 bar. Taking the scattering length λ at high pressures to be large on the scale of our sample length L, we have calculated the pressure dependence of the attenuation by assuming an exponential damping of the signal, i.e.,

$$S(P) = S_\infty \exp[-L/\lambda(P)], \qquad (2)$$

where $S(P)$ is the observed intensity at pressure P (< 14 bar), S_∞ the saturation signal at high pressures, and $\lambda(P)$ the scattering length. The limiting low-pressure value of λ for $\nu = 0.91 \times 10^{11}$ Hz is ~0.8 mm, about $\frac{1}{3}$ of our sample length. The solid curve in Fig. 3 shows the variation of λ with P as calculated from (2).

This large increase in phonon lifetime with increasing pressure at 0.1 K is to be contrasted with the large decrease observed in neutron measurements[6] at high temperatures (~1.2 K). This latter result, we believe, is due to the presence of thermal rotons at high temperatures. The phonon lifetime is then limited by phonon-roton scattering and the roton number density increases with increasing pressure. At our frequency and temperature, however, the phonon lifetime is limited by scattering with other phonons via the three-phonon process. This scattering rate would be expected to decrease[12] with increasing pressure as the normal phonon dispersion increases markedly with pressure.

In summary, we have studied the propagation of superthermal "monochromatic" phonons in liquid helium. At frequencies as high as 0.91 $\times 10^{11}$ Hz these phonons have been shown to propagate through the liquid with a group velocity which is the same as the low-frequency ultrasonic velocity. This lack of anomalous dispersion is qualitatively consistent with the long lifetimes observed in our experiments. Our data, therefore, suggest (a) that a theoretical re-evaluation is perhaps necessary to explain the subthermal ultrasonic attenuation and/or (b) that at our high frequencies we are observing a new type of collective excitation[13] such as a zero-sound-like mode. Neutron measurements[14] have shown that such excitations are well defined in liquids. Our measurements would, however, also require them to propagate *macroscopic* distances with little attenuation and essentially zero dispersion in superfluid He. It is clear that numerical calculations of the lifetime of such a zero-sound mode in He II are required to settle the question unambiguously.

We would like to thank M. A. Chin, S. Darack, and J. P. Garno for technical assistance; S. Geschwind, C. C. Grimes, and J. M. Rowell for their support; W. L. McMillan for helpful discussions; and C. H. Anderson for helpful correspondence.

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Runaway Electrons in a Plasma*

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The electron runaway rate in a uniform plasma under a uniform electric field is calculated by solving the Fokker-Planck equation numerically. Comparison with other theoretical and experimental results is made.

It is well known that when a uniform electric field E is applied to a uniform plasma, a certain fraction of the electrons will run away; that is, they will gain an energy such that the electric force on them exceeds the drag force and they will accelerate indefinitely. The critical velocity for the latter to happen is $v_c \sim (E_D/E)v_t$, where $v_t = (T/m)^{1/2}$ and $E_D = m\nu v_t/2e$ is the Dreicer field, with $\nu = 4\pi n_e e^4 \ln(\Lambda)/v_t^3 m^2$ the collision frequency and n_e the electron density. If $E \ll E_D$, then $v_c \gg v_t$, so that only an exponentially small fraction of electrons run away in any given time.

The number of such runaway electrons produced per unit collision time has been calculated by a number of authors. One of the first efforts was Dreicer's.[1] Later Kruskal and Bernstein,[2] Gurevitch,[3] and Lebedev[4] also calculated these rates as an asymptotic expansion in E/E_D. Gurevitch's expansion suffered from a singularity which was subsequently removed by Lebedev. Even the latter was less complete than Kruskal and Bernstein, who found five distinct regions of behavior for the distribution function. The latter authors determined the rate up to a multiplicative constant which they have not evaluated yet.

Runaways were first observed experimentally