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Excitation of the Plasmapause at Ultralow Frequencies

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Observational evidence and the theoretical interpretation indicate that magnetic disturbances exterior to the plasmasphere boundary (within the magnetosphere) can excite damped, sinusoidal oscillations, which can be inferred as a magnetohydrodynamic surface eigenmode, at the plasmapause. The wave frequency and damping rate can be used to infer the plasma density inside the plasmapause and the plasma density gradient at the plasmapause.

Earth's magnetosphere represents an excellent plasma laboratory for the study and understanding of naturally occurring ultralow-frequency magnetohydrodynamic (MHD) waves[1] in astrophysical plasmas. Indeed, except for the observations of such waves in the solar wind,[2] the magnetosphere is the only locale where these waves can be examined in detail and their growth and decay properties studied. Sugiura[3] reported the first evidence of elliptically polarized, geophysical hydromagnetic waves from observations made on the ground at College, Alaska. Evidence of ~200-sec waves in the magnetosphere was first reported from data obtained on an early Explorer satellite,[4] and later, satellite-measured waves were associated with similar frequency variations observed at a ground station.[5] Much recent work has been concerned with the use of magnetospheric MHD waves as diagnostic tools for studying magnetospheric properties from the ground,[6] even though the sources of these waves have, until now, not been positively identified.

In order to examine the role of the plasmapause[7] within the magnetosphere as a possible source and sink of wave energy, three digital-recording, three-axis magnetometer stations have been operated in a latitudinal array around the nominal location of the plasmapause.[8] These three stations are located at ~4°W geomagnetic longitude at latitudes corresponding to the intersection with Earth of the $L = 4.4$, 4.0, and 3.2 magnetic shells. A similar station, operated at Siple station in the Antarctic at $L = 4.0$ and at the same geomagnetic longitude as the northern stations, is magnetically conjugate to the northern station at $L = 4.0$. The three orthogonal components (H, D, Z) of the total vector field $\vec{B}$ at each station are sampled and digitized at 2-sec intervals and written in a computer-compatible format on magnetic tape. The noise level of each detector system is ~0.2 γ over the bandwidth considered here ($\gamma = 10^{-5}$ G).[9]

Shown in Fig. 1 are the H-component magnetic records for the three northern stations during the four time intervals 2104–2114 UT 19 December 1972, 1820–1830 UT 21 December 1972,

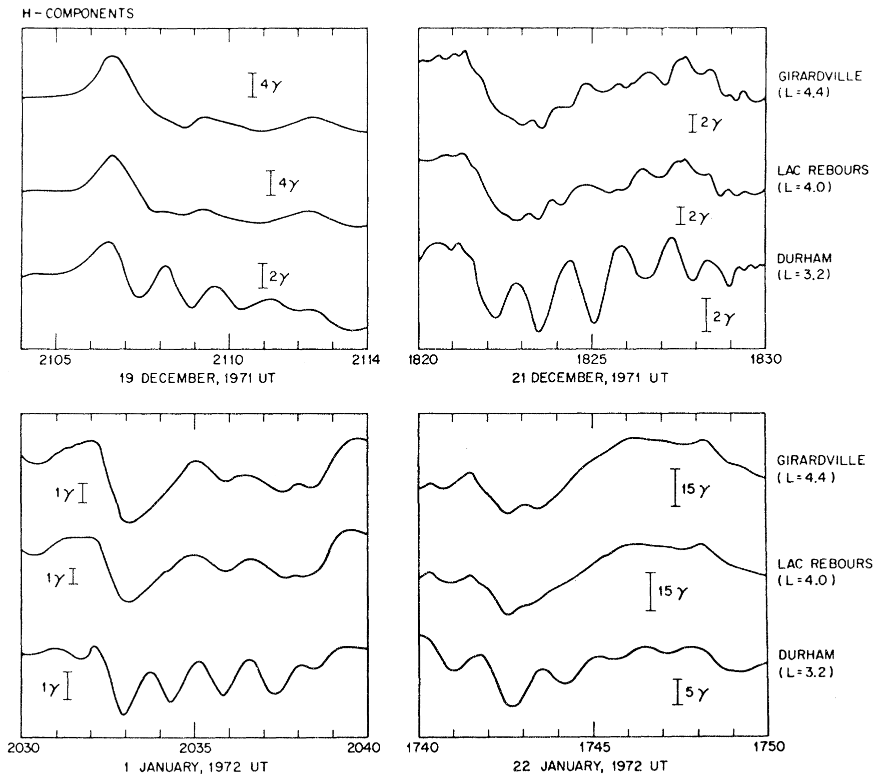

FIG. 1. Latitude dependence of magnetic variations during four time periods when sinusoidal oscillations were observed at the lowest latitude. All three stations are at the same magnetic longitude.

2023–2040 UT 1 January 1973, and 1740–1750 UT 22 January 1972. All of these records show that a magnetic disturbance (a "step-function-like" or an "impulselike" magnetic change) at higher latitudes is observed at the lowest latitude ($L=3.2$) as a damped, sinusoidal wavelike variation with a frequency in the range ~0.009 to ~0.0125 Hz.

The damped, sinusoidal magnetic-field variations are predominantly elliptically or linearly polarized waves, as is shown in Fig. 2 for the wave component lying in the horizontal (H-D) plane. For three of the four waves, the polarization plot is divided into two parts, corresponding to successive time intervals, to show better the polarization changes as the wave damps. The waves on 19 December and 1 January are right-handed polarized (when looking in the direction of the ambient magnetic field), while on the other two days the waves are predominantly linear.

The type of waves measured at $L=3.2$ as shown in Fig. 1 is frequently observed on the dayside of Earth. It is clear from a thorough examination of the data that the waves are not the same as the irregular waves ($Pi\,2$) that are usually associated with magnetic substorms and that are observed at all latitudes on the nightside of Earth.[10] The waves are not associated with identified sudden commencements or sudden impulses produced at the magnetosphere boundary by a change in the solar-wind pressure. Although the source of the high-latitude impulse is unknown, it is clear that such an impulse excites a damped oscillation deep within the magnetosphere.

An attempt was made, using satellite data, to determine the location of the plasmapause at the time of the wave events. No exact correspondence (in time) of a satellite measurement of the plasmapause occurred in the vicinity of the stations' longitude when an event occurred. The events shown in Fig. 1 occurred on days when the floating-probe d.c. electric-field instrument[11] on the S^3 satellite indicated that the plasmapause in the midnight sector was at $L<4$ during the approximately 12-h interval preceding the ground measurement. From previous satellite-based studies,[12] it can be reasonably assumed that the midnight L value of the plasmapause corresponds well to the plasmapause location on the dayside some 12 h later, as the plasmasphere co-rotates with Earth. Thus, the damped oscillations are observed inside or close to the boundary of the

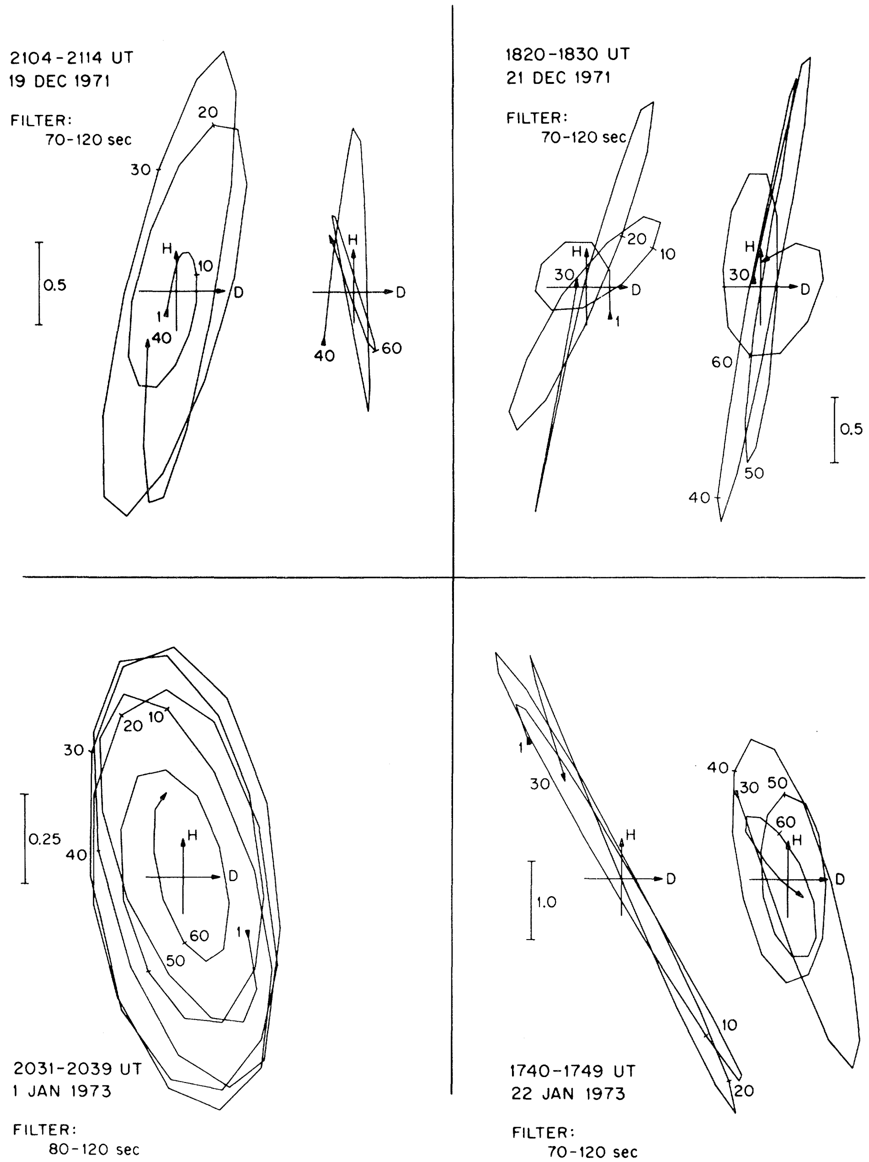

FIG. 2. Polarization diagram in the horizontal plane of the magnetic variations shown in Fig. 1.

plasmasphere and are excited by the disturbance external to the plasmasphere.

If we assume that an observed oscillation is a consequence of the external impulse, the oscillation must be a localized eigenmode of the system. It is tempting to attribute the oscillation to an excitation of the local field line arising from the standing shear Alfvén wave. However, such a wave mode cannot be an eigenmode of a system with a continuously varying plasma density[13]; the shear-Alfvén-wave resonance contributes only a part of the continuous spectrum.

Hence, we must attribute the observed oscillations to an excitation of the MHD surface wave at the plasmapause. For simplicity we assume a straight magnetic field $B(y)$ (taken in the z direction), but nonuniform in the y direction. The ideal MHD equation then gives, for the fluid displacement vector $\vec{\xi}$ ($\vec{v}=\partial\vec{\xi}/\partial t$),

$$\mu_0\rho_m\ddot{\vec{\xi}} - (\vec{B}\cdot\nabla)^2\vec{\xi} = -\mu_0\nabla\rho' - \vec{B}(\vec{B}\cdot\nabla)\nabla\cdot\vec{\xi}, \qquad (1)$$

where $\rho'=\rho+\vec{B}\cdot\vec{b}/\mu_0$ is the total pressure, $\vec{b}$ the wave magnetic field, and $\rho_m(y)$ the plasma mass density. Assuming incompressibility[14] ($\nabla\cdot\vec{\xi}=0$), and applying Fourier (in space) and Laplace (in time) transformations, we can eliminate ξ_x from

(1) and obtain the wave equation for ξ_y:

$$\frac{d^2\xi_y}{dy^2}+\frac{d\ln\epsilon(y)}{dy}\frac{d\xi_y}{dy}-(k_z^2+k_x^2)\xi_y=S(y,\omega). \tag{2}$$

Here S is the source function and ϵ is the effective dielectric constant given by

$$\epsilon(y)=\omega^2\mu_0\rho_m(y)-k_z^2B^2(y)$$
$$\equiv B^2(y)[\omega^2/v_A^2(y)-k_z^2], \tag{3}$$

where v_A is the Alfvén velocity. The undamped frequency of the surface eigenmode can be easily obtained by assuming a sharp boundary ($d\epsilon/dy=0$ except at the boundary, $y=0$). The solution of the homogeneous equation of (2) is then

$$\xi_y \sim c\exp[i(k_xx+k_zz-\omega t)-(k_z^2+k_x^2)^{1/2}|y|].$$

The dispersion relation is obtained by matching the solution across the boundary using the continuation of ξ_y and the pressure $p'\propto\epsilon\, d\xi_y/dy$, as

$$1/\epsilon_{\rm I}+1/\epsilon_{\rm II}=0, \tag{4}$$

where $\epsilon_{\rm I}$ and $\epsilon_{\rm II}$ are the dielectric constants at $y<0$ and $y>0$, respectively. If the density ratio in region I to that in region II is significantly small, the eigenfrequency given by Eq. (4) is

$$\omega_0=\sqrt{2}\,k_zv_{\rm A\,II}, \tag{5}$$

where $v_{\rm A\,II}$ is the Alfvén speed in the region with the *larger plasma density*. In application to the magnetosphere, $k_zv_{\rm A\,II}$ is taken as representative of the appropriate dipole field line.

If the density jump from region I to region II is not sharp, there is a local point in y ($=y_0$) at which the shear Alfvén wave meets the resonant condition, i.e.,

$$k_zv_{\rm A}(y_0)=\sqrt{2}\,k_zv_{\rm A\,II}.$$

At this point where the eigenmode faces phase mixing, wave damping results. The imaginary part of the eigenfrequency associated with this phase mixing can be calculated by solving Eq. (2) if a specific value of $\epsilon(y)$ is given. When $\epsilon(y)$ has a linear profile given by

$$\epsilon(y)=\tfrac{1}{2}[(\epsilon_{\rm II}-\epsilon_{\rm I})y/a+\epsilon_{\rm I}+\epsilon_{\rm II}],\quad |y|\le a, \tag{6}$$

the eigenmode of this equation is given by Sedláček.[15] For $|ak_x|\ll 1$ the dispersion relation is given approximately by

$$1/\epsilon_{\rm I}+1/\epsilon_{\rm II}+i\pi 2ak_x/(\epsilon_{\rm II}-\epsilon_{\rm I})=0. \tag{7}$$

Consequently, the damping rate Γ becomes

$$\Gamma\sim\tfrac{1}{4}\pi(2ak_x)\omega_0, \tag{8}$$

where $2a\simeq[(d\ln\rho_m/dy)_{y=0}]^{-1}$ is the scale length of the density variation at the plasmapause. The sense of the wave polarizations depends on the sign of k_x and on which side of the discontinuity the observations are being made. Near the discontinuity the wave is linearly polarized. As one can expect from a surface wave, the amplitude of the surface eigenmode varies as $\exp(-|k_xy|)$. Therefore, the value of k_x can be estimated from the latitudinal distribution of the amplitude.

From the observations (cf. Fig. 1) of the damped sinusoidal pulsations, we obtain $\omega_0\sim 0.07$ rad/sec, $\Gamma/\omega_0\sim 0.08$, and $k_x\sim 3.2\times10^{-7}$ m^{-1}. If the pulsation is assumed to be the fundamental oscillation of the surface eigenmode at the plasmapause, the half-wavelength π/k_z is the length of the field line l, that is, $k_z\simeq\pi/l\sim 7.7\times10^{-8}$ m^{-1}. For the observed parameters and using Eqs. (5) and (8), $v_{\rm A\,II}\sim 650$ km/sec and $2a\sim 0.05R_E$. The estimated values are interpreted as the mean Alfvén speed along the field line just inside the plasmapause and the e-folding distance of the density variation at the plasmapause. Both derived quantities are in good agreement with observational values obtained with satellite measurements.[16,17] Thus, the wave frequency and damping time of the damped sinusoidal pulsations observed on the ground can be used to infer both the plasma density inside the plasmasphere and the plasma density gradient at the plasmapause. Such ground-based diagnostic methods may well prove extremely useful in future diagnostic studies of the dynamics of the plasmapause.

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Tricritical Behavior in a Liquid-Crystal System*

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The isotropic-cholesteric and cholesteric–smectic-*A* phase transitions in cholesteryl oleyl carbonate have been studied for pressures up to 7 kbar. The latter transition possesses a tricritical point at 2.66 kbar and 60.3°C, where the behavior changes from first to second order.

Recently, there has been considerable interest in the study of the tricritical point, where a line of phase transitions changes from first to second order. This interest is largely motivated by the fact that the conventional scaling approach, which has been so highly successful in describing second-order transitions, breaks down near a tricritical point.[1]

Examples of systems having tricritical points include the metamagnetic-antiferromagnetic transitions in dysprosium aluminum garnet,[2] $FeCl_2$,[3] and $Ni(No_3)_2 \cdot 2H_2O$,[4] the λ transition in He^3-He^4 mixtures,[5] and the orientational order-disorder transitions in NH_4Cl [6] and ND_4Cl [7] crystals. It is the primary purpose of this Letter to report the discovery of a new type of tricritical point which occurs at higher pressures for the transition between the smectic-*A* and cholesteric liquid-crystalline phases of cholesteryl oleyl carbonate (COC).

Until recently, all liquid-crystal transitions were believed to be first order. It was originally suggested by McMillan,[8] based upon mean-field calculations for a specific interaction model, that the transition between the smectic-*A* and the nematic or cholesteric phases might be second order under certain conditions. In particular, he predicted that for a homologous series of liquid crystals, the discontinuity in the order parameter at the transition should decrease with chain length and eventually vanish for sufficiently short molecules, thus rendering the transition second order. This trend was subsequently verified experimentally by Doane *et al.*,[9] who found one instance of a nematic–smectic-*A* transition which was second order or at least very nearly second order.

It seemed quite plausible that the behavior observed by varying chain length could also be induced for molecules of a given size through changes in the density. Therefore, we began an investigation of the phase transitions of the liquid crystal COC at elevated pressures. This substance was chosen primarily because of its ready availability and convenient temperature ranges for the smectic-*A* and cholesteric phases. To the best of our knowledge, the only other liquid crystal which has been investigated at high pressures is *p*-azoxyanisole,[10] which exhibits only the nematic mesophase.

The COC used in this study was obtained from Eastman Organic Chemicals and was not further purified. A sample of approximately $\frac{1}{2}$ mm thickness was sandwiched between two cylindrical glass windows and contained at the perimeter by a tightly stretched piece of fluran tubing. This arrangement was placed in a high-pressure optical cell filled with Octoil *S*, which was used as the pressurizing fluid. High pressures were generated by means of a reciprocating hand pump and measured with a Bourdon gauge having an accuracy of 100 lb/in.^{2} The temperature of the cell