transitions to the ground and 4.63-MeV levels of ^{7}Li, which is probably due to mixed-L transfers.

To extract spectroscopic factors which can be quantitatively compared to theoretical predictions, one must account for kinematic effects which might affect the relative excitation of states. As a crude preliminary approximation one can neglect these effects and look at the ratio of yields at the first maximum. [A fairly extensive attempt was made to fit these data via the zero-range distorted-wave Born approximation (DWBA) assuming an α-cluster transfer. Unfortunately, only poor fits were obtained, although at 65 MeV the angular momenta in the entrance and exit channels are well matched. This failure may be due to the fact that the ^{8}Be optical potential is unknown; in addition neglect of finite-range effects may contribute.] At the first maximum, the experimental ratios of the differential cross sections of the first excited to ground states of both ^{12}C and ^{8}Be are closely equal to 2, while the corresponding ratios of spectroscopic factors[2,3] are 5.5 and 1.3, respectively.

While it may require detailed excitation function studies to conclusively determine the direct nature of the (α, ^{8}Be) reaction at 65 MeV, the strong population of only those states which are predicted to have significant α-structure amplitudes implies a dominant direct reaction mechanism. Hopefully, a description of this α-transfer process by an exact DWBA approach will enable quantitative tests to be made of spectroscopic predictions. Furthermore, using this relatively simple ^{8}Be identifier, extensive comparisons with other α-pickup reactions like (d, ^{6}Li) and (^{3}He, ^{7}Be) will be made possible.

We would like to thank Dr. F. D. Becchetti for useful discussions on direct reaction theory.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

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[8] A Si target was irradiated at $\theta_{lab} = 20.0°$ to understand the small background in the SiO_2 data arising from ^{24}Mg states populated by (α, ^{8}Be) on ^{28}Si.

[9] Our PSD was obtained from Edax International Inc.

[10] Events from the (α, $^8Be_{2.9}$*) reaction on light targets, if two-body, would have ~ 500 keV higher energy than those from (α, $^8Be_{g.s.}$) transitions to the same final states, as a result of a kinematic effect.

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Theory of the Pion-Nucleus Optical Potential with Crossing*

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(Received 16 July 1973)

A theory of the π-nucleus optical potential is developed from the exact propagator of a pion in the presence of a nucleus. The crossed π-nucleus diagrams, absent in previous work, are shown to have significant effect on the cross sections and the elastic-scattering wave functions of low-energy pions.

There have recently appeared in the literature a number of papers discussing ways to improve the π-nucleus optical potential.[1] Although we find that most of these efforts have been primarily concerned with the description of the basic πN interaction, there is an additional feature of π-nucleus scattering which is physically significant and should be included in the construction of an improved optical potential. This is the role of π-nucleus crossing. Though the importance of crossing in the description of the πN interaction has generally been appreciated, the distinct physical processes described by crossed π-nucleus diagrams have not previously been considered. In this report we examine the role of crossing in the construction of the optical potential and the effects of crossing on the π-nucleus total cross section and the elastic-scattering wave functions of low-energy pions.

To derive an optical potential one must convert the many-particle problem into the one-particle problem of a pion moving in an effective potential. A convenient method to accomplish this is to consider the one-pion Green's function

$$\langle\beta\vec{k}'|G(\omega)|\alpha\vec{k}\rangle = -i\int dt\, e^{i\omega t}\langle\Psi_0|T[a_{\beta\vec{k}'}(t)a_{\alpha\vec{k}}^\dagger(0)]|\Psi_0\rangle/\langle\Psi_0|\Psi_0\rangle \tag{1}$$

which describes the propagation of a pion from the state with momentum $\vec{k}$ and isospin $\alpha = 1, 2, 3$ to the state $(\beta, \vec{k}')$ is the presence of the nucleus in its ground state, described by the exact wave function Ψ_0. The symbol T denotes chronological time ordering of the second-quantized pion creation and annihilation operators, $a_{\alpha\vec{k}}^\dagger(0)$ and $a_{\beta\vec{k}'}(t)$, respectively. We neglect nuclear recoil and work in the nuclear rest frame. Then, if the propagator can be written in the form

$$\langle\beta\vec{k}'|G(\omega)|\alpha\vec{k}\rangle = \langle\beta\vec{k}'|[\omega - h - U(\omega)]^{-1}|\alpha\vec{k}\rangle, \tag{2}$$

with h the single-pion free Hamiltonian, $h|\vec{k}\rangle = \omega_k|\vec{k}\rangle$, $\omega_k^2 = \vec{k}^2 + m_\pi^2$, $U(\omega)$ can be identified as the pion optical potential to be used in the Schrödinger equation. The representation (2) can be obtained by a perturbation expansion of the right-hand side of (1) in a Goldstone-type linked-cluster expansion. $U(\omega)$ is then identified as the sum of all proper diagrams, defined such that any diagram can be made up by connecting these proper diagrams with single pion lines propagating forward in time, for which the propagator is $(\omega - h)^{-1}$.

For the perturbation expansion we write the Hamiltonian of the π-nucleus system as $H = H_0 + H_1$. H_0 includes the free-field Hamiltonians for the pion and nucleon fields, counter terms to shift the bare masses to physical masses, and a nucleon-nucleus potential of the Brandow[2] type relevant for the nucleus under consideration. $H_1 = H_{\pi N} + H_{NN}$ − counter terms, where $H_{\pi N}$ represents the pion-nucleon interaction and H_{NN} describes the nucleon-nucleon interaction. Here we need not specify H_1 in more detail as the formal developments in this paper will be independent of the specific forms of both $H_{\pi N}$ and H_{NN}.

The eigenfunction of H_0 with the same baryon number as the target nucleus and the smallest eigenvalue will be called the vacuum. Clearly, this is the model wave function of the ground state of the nucleus in the Brueckner theory. If we now carry out the linked-cluster expansion of $G(\omega)$, we find the following: The subdiagrams which represent conventional mass renormalization are canceled by the counter terms, and the subdiagrams which represent the renormalization of the nucleon propagator due to the interaction between two or more nucleons produced by H_{NN} or by $H_{\pi N}$ (through second- or high-order effects) are canceled by the nucleon-nucleus potential (by definition). The remaining series of proper diagrams are shown schematically in Fig. 1, where the ground-state nucleus is taken as the vacuum.

Figure 1(a) represents the most important class of proper diagrams, i.e., those where the pion of momentum $\vec{k}$ comes in, interacts with a nucleon and excites the nucleus which remains in an excited state until the pion finally comes out with momentum $\vec{k}'$. Its value will be represented by $\langle\vec{k}'|D(\omega)|\vec{k}\rangle$. (Here and in the following we will ignore isospin symbols.) The diagrams in D can be broadly grouped into two sets—one containing the diagrams where the incoming and the outgoing pions interact with the same nucleon, and the other containing all remaining diagrams. The first set represents the elementary πN scattering in the nuclear medium. The resulting amplitude differs from the free πN scattering amplitude through three important effects, viz. the exclu-

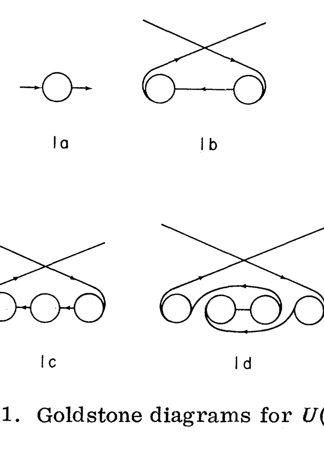

FIG. 1. Goldstone diagrams for $U(\omega)$.

sion principle, the nuclear excitation energy, and the rescattering of the internal pions. In the nucleon-nucleus optical potential the analog of D is approximated by summing a relevant subset of diagrams, e.g., the ladder diagrams. The history of π-nucleon scattering has shown that a similar approximation scheme does not work here, and the only viable approach is something like the Chew-Low theory. The other set of diagrams in D represents multiple scattering and may be evaluated in terms of the elementary πN amplitude.

Figure 1(b) represents the simplest crossed (proper) diagram; and Fig. 1(c) represents the second term of a series of cross diagrams. These diagrams describe the process whereby the nucleus emits two pions and returns to its ground state. One of the pions comes out in the final state, while the other suffers a series of D interactions and is eventually absorbed, as is the incoming pion. The π-nucleus crossing described here has not been incorporated into previous calculations of the π-nucleus interaction based on the various multiple scattering theories,[3] nor is it present in the many-body theory of Dover and Lemmer.[4] We note that diagrams such as the one shown in Fig. 1(d), where we have crossing within crossing, are not proper diagrams, but are iterations [in the context of Eq. (4)] of the series of crossed diagrams.

Summing the proper diagrams gives for the optical potential

$$U(\omega)=D(\omega)-D(\omega)[\omega+h+D(\omega)]^{-1}D(\omega), \qquad (3)$$

and the fully off-shell scattering amplitude satisfies the Lippmann-Schwinger equation

$$T(\omega)=U(\omega)+U(\omega)(\omega-h)^{-1}T(\omega). \qquad (4)$$

Introducing $\hat{T}(\omega)=(2h)^{1/2}T(\omega)(2h)^{1/2}$ and the invariant potential

$$V(\omega)=(2h)^{1/2}D(\omega)(2h)^{1/2}, \qquad (5)$$

and using (3) in (4) gives

$$\hat{T}(\omega)=V(\omega)+V(\omega)(\omega^2-h^2)^{-1}\hat{T}(\omega). \qquad (6)$$

With the normalization $\langle\vec{k}|\vec{l}\rangle=(2\pi)^3\,2\omega_k\,\delta(\vec{k}-\vec{l})$, the matrix elements of $\hat{T}$ and V are invariant quantities, and the matrix elements $\langle\vec{k}'|\hat{T}(\omega_k)|\vec{k}\rangle$, with $|\vec{k}|=|\vec{k}'|$, are invariant scattering amplitudes; that is,

$$\mathrm{Im}\langle\vec{k}|\hat{T}(\omega_k)|\vec{k}\rangle=-k\sigma_{\mathrm{tot}}(\omega_k)/4\pi. \qquad (7)$$

Thus while $U(\omega)$ is the potential to be used in the Schrödinger equation, $V(\omega)$ is the invariant potential for the Klein-Gordon equation.

If we now consider the optical potential constructed without the addition of the crossed π-nucleus diagrams, i.e., $U(\omega)=D(\omega)$, and introduce $\hat{T}_1=(\omega+h)^{1/2}T_1(\omega+h)^{1/2}$, where T_1 is the Lippmann-Schwinger amplitude, we find

$$\hat{T}_1(\omega)=V_1(\omega)+V_1(\omega)(\omega^2-h^2)^{-1}\hat{T}_1(\omega), \qquad (8)$$

$$V_1(\omega)\equiv\left(\frac{\omega+h}{2h}\right)^{1/2}V(\omega)\left(\frac{\omega+h}{2h}\right)^{1/2}. \qquad (9)$$

The potential V_1 is formally what has been used in previous calculations. Again, $\langle\vec{k}|\hat{T}_1(\omega_k)|\vec{k}\rangle$ is the scattering amplitude in the sence of Eq. (7).

To demonstrate the role of crossing we present in Table I the π-C^{12} total cross sections for several pion energies, with and without crossing, which are obtained from Eqs. (6)–(8). The calculations were done with a V obtained by summing off-shell πN amplitudes $h_\mu(\omega)$, which are projected onto the four spin-isospin channels ($\mu=1, 2, 3, 4$), and are given by the Chew-Low theory.[5,6] Since the target has zero spin and zero angular momentum, only the scalar-isoscalar part of the πN amplitude is relevant. Furthermore, for this illustrative calculation the exclusion principle is taken into account only partially. Thus we use

$$\langle\vec{k}'|V(\omega)|\vec{k}\rangle=-16\pi^2 v(k)v(k')\vec{k}'\cdot\vec{k}\tilde{\rho}\times(|\vec{k}'-\vec{k}|)[\beta(k)\beta(k')]^{1/2}H(\tilde{\omega}), \qquad (10)$$

where $H=h_1+2h_2+2h_3+4h_4$. $\tilde{\rho}(\vec{q})$ is the Fourier transform of the nuclear density $\rho(\vec{r})$, which is normalized to the number of nucleons; $v(k)$ is a form factor for the πN interaction.[6] The energy variable $\tilde{\omega}$ which appears as the argument of the Chew-Low functions is related to ω, the pion energy in the nuclear rest frame, according to[7]

$$\tilde{\omega}=-\tfrac{1}{2}M_N+\tfrac{1}{2}[M_N{}^2+4(M_N\omega+1)]^{1/2}-\Delta, \qquad (11)$$

where Δ represents the average energy needed

TABLE I. π-C^{12} total cross sections σ_T.

π energy (MeV)	σ_T (mb) Crossing	σ_T (mb) No crossing
20	9.98	8.62
30	43.9	31.9
40	369	214
50	493	541
100	565	608
150	642	656

to excite a nucleon from the Fermi sea, M_N is the nucleon mass, and the first term on the right-hand side of (11) represents the change in energy in going to the πN c.m. frame, where the nucleons are taken to be static. The factor $\beta(k)$ represents the effect of the Pauli principle on the interaction of the "external" (incoming and outgoing) pions with the bound nucleons and is related to the factor $F(k)$[8] by

$$\beta(k) = F(k) + [1 - F(k)](1 - n^2), \tag{12}$$

with n the average occupation probability of the states in the Fermi sea. The exclusion-principle factor $F(k)$ is appropriate for a 0°K Fermi gas, but it is felt that the modification (12) produces a factor which is more consistent with the use of exact nuclear states. In the present calculation we have taken $n = 0.8$.

There is also a need to include exclusion-principle effects in the interaction of the intermediate-state pions ("internal" pions) with the nucleons off which the external pions scatter. This may be accomplished by a modification of the πN intermediate-state contribution to the Chew-Low amplitudes. However, the nature of the Chew-Low equation is such that this correction, in conjunction with the β factor, may lead to an overstatement of the role of the exclusion principle. For this reason we have not included Pauli corrections for internal pions.

Using this model for V we have also examined the effects of crossing on the wave functions of low-energy pions. There are theories of pion production in which the effects of the pion's final-state interaction are approximately taken into account by using the π-nucleus elastic-scattering wave function.[9] A careful examination of such theories shows that the appropriate wave function to use is the Schrödinger wave function and not the corresponding Klein-Gordon wave function. (We have found that these wave functions in coordinate space differ by several percent at short distances, though become equal asymptotically, for low-energy elastically scattered pions.)

In Fig. 2 there are shown the p-wave radial Schrödinger wave functions $\psi(r)$ for 40-MeV pions elastically scattered from C^{12} calculated with and without π-nucleus crossing. The difference shown here is expected to be significant in pion-production calculations.

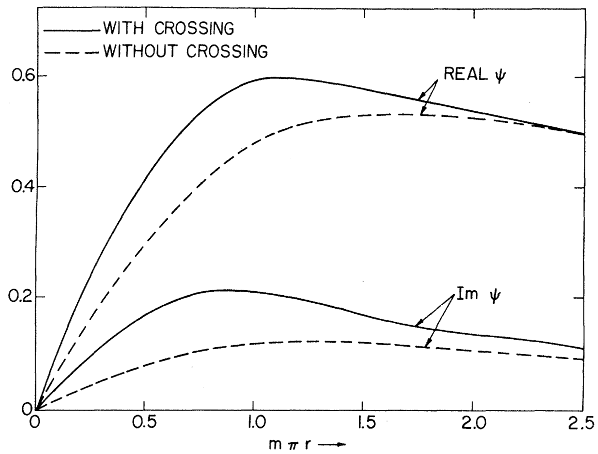

FIG. 2. 40-MeV p-wave Schrödinger wave functions.

Finally, we note that for both the cross sections and the wave functions the importance of crossing decreases with increasing pion energy. This can be understood from a comparison of (5) with (9), and from the fact that because of the finite size of the nucleus the matrix elements $\langle\vec{p}|V(\omega)|\vec{q}\rangle$ are largest when $1.6m_\pi \leq (m_\pi^2 + \vec{p}^2)^{1/2}$, $(m_\pi^2 + \vec{q}^2)^{1/2} \leq 3m_\pi$.

*Work supported by the U. S. Atomic Energy Commission. Computer time provided by the University of Maryland Computer Science Center.

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