result of a more complex mechanism.

It seems surprising that there is not a more substantial polarization at neutron energies near $E_n = 8$ MeV, which is the energy of neutrons produced in the n-p final-state interaction (FSI) region.[12,13] A large fraction of the n-p pairs produced with low relative energy ($E_{np} \approx 0$) are produced in the $^3S_1$ state.[13] These pairs might be expected to have a substantial vector polarization because the inelastic reaction $p + d \to d^*(^3S_1, E_{np} \approx 0) + p$ resembles the elastic $d + p \to d(^3S_1, E_{np} = -2.2\ \text{MeV}) + p$ scattering. At this angle $[\theta_d(\text{c.m.}) = 142°]$, the deuteron vector polarization is large $(iT_{11} \approx 0.15)$,[2] which should result in large vector polarizations for the neutrons which come from the decay of the $d^*(^3S_1, E_{np} \approx 0)$. We plan to investigate the possible influence of the n-p FSI on the neutron polarization by performing a more thorough measurement of the polarization in the n-p FSI region.

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Compactness Criterion for the Formation of Averaged Trapped Surfaces in Gravitational Collapse*

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An average trapped surface is defined as a closed two-surface with the property that the area of the wave fronts formed by light rays emitted in the orthogonal inward and outward directions are both decreasing. A sufficient criterion is given for the formation of average trapped surfaces in a general gravitational collapse. The criterion has the qualitative form that if a given amount of matter is compacted into a sufficiently small region, then an average trapped surface must be formed. In the case of nearly spherically symmetric gravitational collapse, it is shown that average trapped surfaces always form to second order in the perturbations of spherical symmetry.

An outstanding issue today in the theory of gravitational collapse is whether an event horizon always forms in a general gravitational collapse and, if it is formed, whether it shields all singularities from an exterior observer.[1] A closely related question is whether trapped surfaces and average trapped surfaces always form in a general gravitational collapse. By an average trapped surface we mean a compact, spacelike, two-surface, such that light rays orthogonal to it generate wave fronts of decreasing area for both outward- and inward-directed rays.

This definition is to be distinguished from Penrose's[2] definition of a trapped surface. In a trapped surface the orthogonal inward and outward light rays must be converging *at every point* so that every small bundle of light rays will generate areas which are decreasing in both directions. Clearly every trapped surface is an average trapped surface, but not necessarily vice versa.

Trapped surfaces and average trapped surfaces are connected to the issue of the formation of event horizons in the following way. In the conventionally conjectured picture of nonspherical collapse,[1] a domain of trapped surfaces (and therefore also average trapped surfaces) is formed when the matter has collapsed into a sufficiently small region. The boundary of this domain (the apparent horizon) expands and eventually coincides with the event horizon. All singularities are formed inside the domain of trapped surfaces and therefore are shielded from distant observers. In testing the pieces of this conjectured picture it is easier to obtain conditions for the formation of trapped surfaces than for the event horizon, because trapped surfaces are defined locally while the event horizon is a globally determined surface. Pajerski and Newman[3] and Demmie and Janis[4] have given conditions sufficient for the formation of trapped surfaces in general space-times. These conditions take the form of certain restrictions on the characteristic data specified on the apparent horizon. It would be desirable, however, to have a sufficient condition which would guarantee the formation of a trapped surface if collapse should compact matter into a sufficiently small dimension. In this note we find a compactness criterion which, if satisfied, guarantees the formation of an average trapped surface in a general gravitational collapse. While this is a weaker result than a compactness criterion for a trapped surface, it is important for two reasons. First, the formation of average trapped surfaces is clearly a necessary condition for the formation of trapped surfaces. Second, the formation of averaged trapped surfaces is one of the few interesting features of spherically symmetric collapse which appears to be simply treatable in the nonspherical case. The investigation of the situations in which they are formed, and of their connections, if any, to the formation of singularities and event horizons, therefore become potentially useful avenues to a better understanding of nonspherical collapse.

Consider a general gravitational collapse such

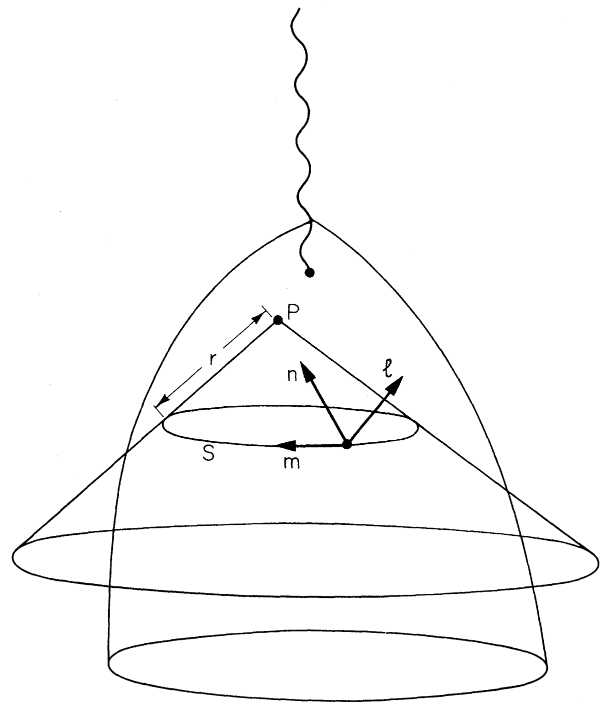

FIG. 1. Definition of average trapped surface. A schematic space-time diagram showing the collapse of a star (its outline represented by curved cone) and eventual formation of a singularity (wiggly line). P is some event inside the collapsing matter. One chooses a spacelike two-surface S in the past null cone of P having a constant affine distance r from P. For every event in S, null tetrad vectors n^μ, l^μ, m^μ are defined such that n^μ points along the generators of the null cone, m^μ lies in S, and l^μ is orthogonal to S. If the set of light rays emitted in the outward direction (along l^μ) at each point of S have a wave front of decreasing area, then S is trapped on average.

as is illustrated in Fig. 1. Choose any event P whose past null cone does not intersect a singularity. We will assume that this past null cone can be chosen so that it extends smoothly out to past null infinity with none of its generators having end points. This is tantamount to assuming that the surface is inner trapped. Parametrize this past null cone by an affine parameter r which takes the value 0 at P and is positive towards the past. Consider the two-surfaces of constant r which lie in this null cone. The area of the wave front generated by the ingoing light rays orthogonal to this surface is decreasing by the assumed construction of the null cone. The rate of change of the area generated by the outgoing orthogonal light rays we will denote by $\dot{A}$. Implicit in Hawking's 1972 paper on black holes[5] is a powerful formula for the rate of change of $\dot{A}$ with respect to r:

$$d\dot{A}/dr = 4\pi - 2\int da\,[|\tau|^2 + \Phi_{11} + 3\Lambda]. \quad (1)$$

The integral is taken over the area of the two-surface of constant r. The quantities in brackets are defined, as in Newman and Penrose,[6] in terms of the Ricci tensor and a tetrad $l^\mu, n^\mu, m^\mu, \bar{m}^\mu$ of null vectors (where the bar denotes complex conjugation) whose only nonvanishing scalar products are[7] $l_\mu n^\mu = 1$ and $m^\mu \bar{m}_\mu = -1$. The tetrad is oriented so that (1) $n^\mu = dx^\mu/dr$, where $x^\mu(r)$ are the geodesic generators of the past null cone, and (2) the spacelike vectors m^μ and $\bar{m}^\mu$ lie in the two-surface. The outgoing null vector l^μ is then necessarily orthogonal to the two-surface. If we write the *outgoing* null vector as $l^\mu = dx^\mu/dv$ thus defining a parameter v, $\dot{A}$ is dA/dv. In terms of this tetrad,

$$\tau = \tfrac{1}{2} l_{\mu;\nu}(m^\mu n^\nu + m^\nu n^\mu), \quad (2)$$

$$\Phi_{11} = -\tfrac{1}{4} R_{\mu\nu}(l^\mu n^\nu + m^\mu \bar{m}^\nu), \quad (3)$$

$$\Lambda = R/24, \quad (4)$$

and $\dot{A}$ is determined by $\dot{A} = -2\int \rho\, da$, where the convergence ρ is $\rho = l_{\mu;\nu} m^\mu \bar{m}^\nu$. To derive Eq. (1) one needs only the Newman-Penrose equations and the Gauss-Bonnet theorem.[8] The details of this derivation will be given elsewhere.

The useful fact about the integrand in Eq. (1) is that it is positive. The quantity $|\tau|^2$ is manifestly positive while $\Phi_{11} + 3\Lambda$ is positive from the dominant energy condition.[9] The integral therefore acts in the opposite direction to the factor of 4π in changing $\dot{A}$. For example, suppose one starts at large values of r where Φ_{11} and Λ vanish, where τ can be chosen to vanish, and where $\dot{A}$ is positive. As one moves to smaller values of r the factor of 4π acts to decrease $\dot{A}$, while the integral acts to increase it. If the integral does not become too large, $\dot{A}$ may cross zero at some positive value of r, and the two-surface at that value of r will be an average trapped surface.

A compactness criterion for the formation of average trapped surfaces can be obtained by examining Eq. (1) for small values of r. At P, space is locally flat. The rate of increase $\dot{A}$ and spin coefficient τ vanish at P, and the area integral varies like r^2. Initially, therefore, $\dot{A} = 4\pi r$ and is positive. This merely reflects the fact that a pulse of light started at a point must at least begin to increase in area before gravitational attraction of matter inside can cause it to decrease. If we write

$$\tilde{M}(r) = (4\pi)^{-1}\int_0^r dr \int da\,[|\tau|^2 + \Phi_{11} + 3\Lambda], \quad (5)$$

then the condition for the two-surface at affine parameter r is that

$$2\tilde{M}(r)/r > 1. \quad (6)$$

An important lower bound on $\tilde{M}$ can be obtained by neglecting $|\tau|^2$ in Eq. (5) since it is positive. Expressing the result in terms of the stress energy tensor one has

$$\tilde{M}(r) \geq \int_0^r dr \int da\,[T_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta} T] m^\alpha \bar{m}^\beta. \quad (7)$$

This is useful because it bounds $\tilde{M}$ below by a quantity which only depends on the distribution of the matter and on the geometry of the null cone.

Neither the quantity $\tilde{M}$ nor its bound given in (7) is the mass as might be defined at infinity. Nevertheless, both are positive-definite measures of the amount of matter contained on the past null cone inside an affine parameter distance r.

Equations (6) and (7) give a simple compactness criterion sufficient for the formation of average trapped surfaces in a general gravitational collapse. It should be noted that there is a great deal of freedom in applying this criterion. Not only can one choose the point P and thereby the null cone in an arbitrary way, but there is also freedom in choosing among the various affine parameters r. We expect that in a wide variety of nonspherical collapses, one will be able to choose a succession of points P coming closer to the singularity and an affine parameter r on each of past null cones, such that on successive null cones all the matter will be contained within a smaller and smaller affine parameter R. Further, we expect to be able to make these choices in such a way that $\tilde{M}(R)$ remains bounded below on the succession of null cones. Eventually, therefore, we expect the criterion (6) will be satisfied and an average trapped surface formed.

Some insight into the formation of average trapped surfaces may be obtained by studying the small perturbations of spherically symmetric collapse. Price[10] has studied this problem in some detail to first order in deviations from spherical symmetry. He finds, in particular, that trapped surfaces are always formed in this order. Using his results expressed in the formalism of Bardeen and Press[11] together with Eq. (1), we have shown that average trapped surfaces must be formed to *second* order in small deviations from spherical symmetry. We will sketch the derivation of this result here; the details will be published elsewhere.

Choose the apex, P, of the unperturbed null cone so that it lies at the center of spherical symmetry and is sufficiently close to the singularity so that the past null cone intersects no matter outside the Schwarzschild event horizon. Outside the horizon, then, Φ_{11} and Λ can be assumed always to vanish. The affine parameter r can be chosen so that the two-surfaces of constant r are natural spheres left invariant under the rotations which define the spherical symmetry. The quantity τ vanishes for the unperturbed spherical collapse and, therefore, the first-order perturbations in it determine $\dot{A}$ to *second order.* The perturbation in τ can be found by writing out the perturbed Newman-Penrose equations and separating the angular dependence by expanding in spin-weighted spherical harmonics as in Ref. 11. Letting $\tau^{(1)}$ denote the radial part of the perturbation in τ corresponding to a particular multipole l, one finds for the physically interesting $l \geqslant 2$ cases

$$\bar{\tau}^{(1)}(r) = Kr^{-2} + [(l-1)(l+2)/2]^{1/2} r^{-2} \times \int_\infty^r (dz/z^2) \int_\infty^z dx\, x^3 \Psi_4^{(1)}(x), \quad (8)$$

where the integrals are taken in the past null cone of P, $\Psi_4^{(1)}$ is the radial part of the perturbation of the Riemann tensor component $-R_{\alpha\beta\gamma\delta} \times n^\alpha \bar{m}^\beta n^\gamma \bar{m}^\delta$, and K is a constant depending on how r is chosen. The quantity $\Psi_4^{(1)}$ itself is the solution of a second-order homogeneous linear differential equation.[11] The solution corresponding to ingoing waves on the horizon is bounded there. Sufficiently far in the past, $\Psi_4^{(1)}$ will be calculable from the static precollapse geometry and, for the physically interesting $l \geqslant 2$ perturbations, will fall off sufficiently fast to make the integrals in Eq. (8) converge. The perturbation $\tau^{(1)}$ therefore will be finite in the neighborhood of the unperturbed horizon. Since the size of the perturbation is by definition small, one concludes by integrating Eq. (1) that an average trapped sphere must be formed slightly inside the unperturbed horizon to second order in the deviation from spherical symmetry. In other words, the property of a spherical collapse that average trapped surfaces are formed is stable under second-order perturbations from spherical symmetry.

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Proof of the Pomeranchuk Theorem for Unbounded Total Cross Sections

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If particle or antiparticle total cross sections σ_P, σ_A are unbounded as $E \to \infty$, using unitarity it is shown that on the average the ratio $\langle \sigma_P - \sigma_A \rangle / \langle \sigma_P + \sigma_A \rangle \to 0$.

Recent experimental data at the CERN insecting storage rings[1] indicate that the proton-proton total cross section increases with laboratory energy beyond the 300-GeV region. At the equivalent laboratory energy of 1500 GeV the proton-proton total cross section reaches 43 mb. The antiproton-proton total cross section decreases steadily as energy increases and reaches the