Does Pickup to Analog States Exhibit a Charge Dependence of the Nuclear Force?

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Discrepancies between observed ratios of $d,{}^3\mathrm{He}$ and d,t yields populating analog states in $^{18}\mathrm{O}$ and $^{18}\mathrm{F}$ and ratios calculated by standard distorted-wave theory, which were thought to provide evidence for a charge-dependent nuclear interaction, are resolved by extending the conventional theory to include both an explicit charge-exchange channel in the continuum and form factors generated from a microscopic analysis.

Bussoletti and Garvey have reported[1] inconsistencies between theoretical calculations and experiments involving single-nucleon pickup reactions from $^{19}\mathrm{F}$. They observed the two processes $d,{}^3\mathrm{He}$ and d,t populating various excited analog states in $^{18}\mathrm{O}$ and $^{18}\mathrm{F}$, respectively. Of particular interest were the ratios of $^3\mathrm{He}$ to t yields which, on the basis of strict charge independence, would be predicted to be exactly 2. Electromagnetic effects introduce changes in these ratios which are not reproduced by conventional distorted-wave analyses. Since the calculated ratios are rather insensitive to changes in the model parameters, one might be tempted to view these discrepancies as possible evidence for a violation of charge independence of the nuclear force.

Because of the serious implications of this interpretation, we have thought it worthwhile to carry out a more detailed theoretical analysis of these reactions in the hope that the discrepancies may be understood without invoking explicit charge dependence. We have extended the distorted-wave calculations to include coupling[2] via an isospin-conserving $\vec{t}\cdot\vec{T}$ interaction[3] to the charge-exchange channel, and have also made use of improved form factors[4] obtained from structure calculations which take excplicitly into account the mixing of isospin in the initial and final nuclear states. The calculations are essentially parameter free and in large measure remove the apparent inconsistencies.

There are several reasons for using an optical potential of the form

$$U = U_0 + (U_1/A)\vec{t}\cdot\vec{T}. \tag{1}$$

The potential has a simple form, is charge independent, and has had considerable success[5] in explaining both charge-exchange data and nuclear symmetry effects. An immediate consequence of such a force is that the distorted waves are solutions of coupled differential equations,[6] and the transition amplitude includes a term[6] which describes the mirror channel. Thus, for example, the $d,{}^3\mathrm{He}$ reaction amplitude may be written as

$$T_{d,{}^3\mathrm{He}} = T_{\mathrm{drct}}{}^{d,{}^3\mathrm{He}} + T_{\mathrm{chrg\ exchn}}{}^{d,t}, \tag{2}$$

and similarly for the d,t amplitude. The coherent nature of Eq. (2) implies that even a weak charge-exchange contribution may have a significant effect on the cross-section ratios.

Some calculations were performed in which both the bound and continuum wave functions were obtained as solutions of the Lane equations. We computed the ratio

$$R = \sigma_{d,{}^3\mathrm{He}}/\sigma_{d,t},$$

where $\sigma_{d,x}$ is the integrated cross section for deuteron-induced reactions on $^{19}\mathrm{F}$ and x is the detected cluster (either $^3\mathrm{He}$ or t). In order to facilitate the comparison with experiment, the integration for both theoretical and experimental cross sections was confined to the range of angles for which data were available. The individual contributions were not weighted by a possible $\sin\theta$ factor. The optical-model and bound-state parameters which were employed in these calculations are listed in Table I.

Our calculations were for the first two analog states in the mass-18 system, the 0^+ and 2^+ levels. The resulting values for R are presented in the third row of Table II. Also tabulated are the experimental ratios and ratios predicted by a conventional distorted-wave analysis. While the Lane model provides excellent agreement for the 0^+ state, the ratio given for the 2^+ level is actually worse than the conventional one.

Inasmuch as the Lane model is a single-particle model, it is not able to accomodate a number of many-body effects which are known to play important roles in the nuclear-structure problem. Pauli blocking, configuration mixing, and the consequent fragmentation of spectroscopic strength are examples of such effects. Where these effects are relatively unimportant, as in the pickup

TABLE I. Optical and bound-state parameters for d, ^{3}He and d,t reactions. A Woods-Saxon form was used for all interactions. The surface interaction (sf) was taken to be a Woods-Saxon derivative. For all calculations appearing in this paper, the geometries for both spin-orbit and Lane potential are the same as the central term. The quantity $\lambda_{s.o.}$ is the spin-orbit parameter defined by $V_{s.o.}(r) = -(\lambda_{s.o.}/45.2r)[dV_0(R)/dr]\vec{1}\cdot\vec{s}$.

	Real potential							Imaginary potential				
	V_0 (MeV)	V_1(sf) (MeV)	$\lambda_{s.o.}$ (MeV fm^2)	r_0 (fm)	r_c (fm)	a (fm)		W_0 (MeV)	W_0(sf) (MeV)	W_1(sf) (MeV)	r_0 (fm)	a_0 (fm)
^{3}He-t	−152.0	120.0	0.0	1.14	1.40	0.71	^{3}He-t	−12.1	0.0	36.0	1.67	0.78
d	−104.4	···	0.0	1.05	1.30	0.80	d	0.0	−19.0	···	1.37	0.77
Nucleons	− 60.0	105.54	25.0	1.25	1.25	0.60	···	···	···	···	···	···

of an $s_{1/2}$ nucleon leading to the predominantly $d_{5/2}{}^2$ 0^+ state in ^{18}O or ^{18}F, the Lane model may provide useful results. Where structure effects are important, the Lane model becomes unreliable. The existence of 2^+ $T=0$ levels lying in the immediate vicinity of the lowest 2^+ $T=1$ level in ^{18}F suggests that contamination of the isospin purity of the latter state may be responsible for the discrepancy between experiment and the Lane-model prediction for that transition.

In order to overcome the deficiencies of the Lane model with regard to the structure problem, we have also obtained form factors from a microscopic model which takes into account the detailed structure of both the target and residual nuclei. We have taken the ^{16}O core to be inert, so that the mass-19 and mass-18 Hamiltonians are given by

$$H(A=19) = H(A=16) + \sum_{i=1}^{3}(T_i + U_i) + V_{12} + V_{13} + V_{23} \quad (4)$$

$$= H(A=18) + T_3 + U_3 + V_{13} + V_{23}, \quad (5)$$

where U_i represents the interaction of nucleon i with the ^{16}O core, and V_{ij} is the interaction between nucleons i and j. If Ψ_{18} and Ψ_{19} are eigenstates for the mass 18 and mass 19 systems, respectively, the desired form factor satisfies the differential equation[4]

$$(E_3 - T_3 - U_3)F = \langle \Psi_{18} | V_{13} + V_{23} | \Psi_{19} \rangle, \quad (6)$$

where E_3 is the observed separation energy. Equation (6) is equivalent to a set of coupled equations for the various mass-18 form factors. A useful approximation is obtained if the eigenfunctions on the right are replaced by shell-model eigenfunctions. In this approximation, many channels are included, but the form factors in channels other than the channel of interest are approximated by shell-model functions and, hence, treated in lowest order only. This may be contrasted with the Lane model, which couples only two channels together, but includes the coupling to all orders.

Shell-model calculations were carried out to obtain two-nucleon and three-nucleon eigenstates in the $2s$-$1d$ shell. Two types of two-nucleon interactions were employed: a phenomenological central Gaussian potential given by True,[7] and a set of properly renormalized[8] Sussex matrix elements.[9] Single-particle energies were taken directly from the observed ^{17}O and ^{17}F spectra. Oscillator basis functions were employed with $\nu = m\omega/\hbar = 0.375$ fm^{-2}. The potential U_3 was constructed by folding a short-range Yukawa interaction with exchange with the orbits representing an ^{16}O core to produce an effective nonlocal single-particle interaction[10] which correctly reproduces bound-state and continuum data in the ^{17}O and ^{17}F systems.

We note that the above calculations contain no adjustable parameters. Because the form factor is obtained from an inhomogenous equation, it possesses a definite normalization and one is able to extract new spectroscopic amplitudes. These were found to be consistent with the amplitudes obtained from the shell model, suggesting that our calculations are reasonably self-con-

TABLE II. Cross-section ratio of the reaction ^{19}F(d, ^{3}He)^{18}O to ^{19}F(d,t)^{18}F for the first two analog states.

	Ratio			
	Experimental	Conventional	Land model	Microscopic
0^+	1.94 ±0.14	2.17	1.95	2.03
2^+	2.77 ±0.22	2.49	2.39	2.87

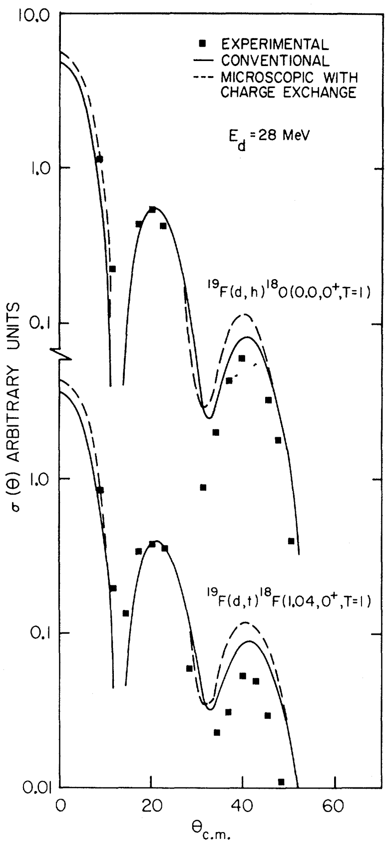

FIG. 1. Differential cross sections for the reactions $^{19}F(d, {}^3He)^{18}O$ and $^{19}F(d, t)^{18}F$. Data provided by Ref. 1.

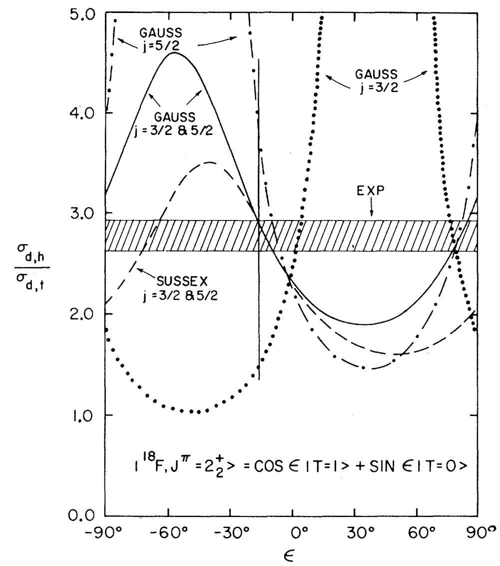

FIG. 2. Cross-section ratio versus isospin mixing angle ϵ.

sistent overall.

Cross sections for the 0^+ states are shown in Fig. 1. The new value of R agrees quite well with the experimental value. This ratio was calculated by replacing the form factors and spectroscopic amplitudes used in Eq. (2) by the corresponding quantities obtained from the microscopic model. In general, the ratios were found to be more sensitive to changes in the spectroscopic amplitudes than to changes in the radial shapes of the form factors. Note that we still allow for the additional charge-exchange transition amplitude which stems from the presence of the Lane operator in the optical potential.

Pickup to the 2^+ states is complicated by the incoherent mixing of two possible angular-momentum transfers and by the coherent mixing in ^{18}F of the two possible values of isospin. Both these effects are accounted for by the microscopic model, which provides a cross-section ratio in agreement with the observed ratio, as shown in Table II. Figure 2 illustrates the manner in which some of these factors affect the ratio. The isospin mixing angle ϵ describes the isospin structure of the lowest calculated predominantly $T=1$, 2^+ eigenstate in ^{18}F. To very good approximation, the two components are eigenstates of the Hamiltonian formed with averaged single-particle energies. It follows that changes in the treatment of the Coulomb interaction, or introduction of some small symmetry-breaking interaction would have little effect on the structure problem other than changing the mixing angle. The particular mixing angle predicted by True's force is about $-12°$ and is indicated on the figure by a vertical line.

Two important features are portrayed by Fig. 2. Firstly, both the Sussex and True models involve significant contributions from both possible angular-momentum transfers and, as the curves indicate, agreement with experiment is obtained only when both j values are included. It would be interesting to investigate the j mixing experimentally using, for example, a (p,d) reaction with a polarized proton beam. Secondly, while the two models give rather different predictions in general, they are in agreement within the shaded area which represents the experimental result.[1]

In summary, the conventional result is not in agreement with experiment. When the form factors are developed from the Lane model, there is substantial improvement for the 0^+ levels, but the method fails for the 2^+ states. As discussed above, this result can be attributed to the restrictions imposed by the single-particle nature of the Lane model. The good agreement achieved by the microscopic method indicates that structure effects are important. We conclude that the microscopic approach is required to provide a reliable account of the experimental ratios. Since we have assumed throughout that the nuclear force is charge independent, we further conclude that the observed ratios can be satisfactorily understood without requiring explicit charge dependence of the nuclear interaction.

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[1]J. E. Bussoletti and G. T. Garvey, Bull. Amer. Phys. Soc. 17, 443 (1972), and private communication.
[2]S. Cotanch and D. Robson, Phys. Rev. C 7, 1714 (1973).
[3]A. M. Lane, Nucl. Phys. 35, 676 (1962).
[4]R. J. Philpott, W. T. Pinkston, and G. R. Satchler, Nucl. Phys. A119, 241 (1968).
[5]G. R. Satchler, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969), p. 389.
[6]S. Cotanch and D. Robson, to be published.
[7]W. W. True, Phys. Rev. 130, 1530 (1963).
[8]T. T. S. Kuo and G. E. Brown, Nucl. Phys. 85, 40 (1966).
[9]J. P. Elliott, A. D. Jackson, H. A. Mavromatis, E. A. Sanderson, and B. Singh, Nucl. Phys. A121, 241 (1968).
[10]R. J. Philpott, Phys. Rev. C 5, 1457 (1972).

Observation of the Decay $f^0 \to \pi^+\pi^+\pi^-\pi^-$

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We have observed the decay mode $f^0 \to \pi^+\pi^+\pi^-\pi^-$ and determined the branching ratio $\Gamma(f^0 \to \pi^+\pi^+\pi^-\pi^-)/\Gamma(f^0 \to \pi^+\pi^-) = (5.5 \pm 1.0)\times 10^{-2}$.

Information about the decay $f^0 \to \pi^+\pi^+\pi^-\pi^-$ is sparse. The only observation of this decay mode has been reported by Ascoli *et al.*,[1] who found 78 events in the region of the f^0 mass. Other experiments[2] have obtained upper limits for this decay mode.

We report here on a search for the four-pion decay of the f^0 from a study of the reaction

$$\pi^+ + d \to \pi^+ + \pi^+ + \pi^- + \pi^- + p + p_s, \tag{1}$$

where p_s denotes the spectator proton. Data were obtained from a 16-event/μb exposure of the 30-in. Argonne National Laboratory bubble chamber to a 6-GeV/c π^+ beam. The sample consists of 7271 events which fit Reaction (1) by kinematics and are consistent with the observed ionization. We have included events with invisible spectator protons and have applied a momentum cutoff of 250 MeV/c for visible spectator protons.

The four-pion mass spectrum is shown in Fig. 1(a). Note that we observe less than 15 events between the four-pion threshold and 1 GeV. In Fig. 1(b), we show part of the mass spectrum for events up to mass 1.57 GeV with $|t| \leq 0.3$ (GeV/c)^2 where t is the four-momentum transfer between the incident π^+ and the outgoing four-pion system. An enhancement around 1.28 GeV for these peripherally produced events is clearly visible.

Since the energy dependence of the width of a resonance decaying into four particles is not known, it is not clear what shape is appropriate to describe this enhancement. Fortunately, the results presented here are insensitive to the specific choice for the assumed resonance shape. If we assume a two-body D-wave Breit-Wigner shape plus a polynomial background,[3] we obtain the following results for events with $|t| \leq 0.3$ (GeV/c)^2:

$$M = 1.283 \pm 0.010 \text{ GeV},$$

$$\Gamma = 0.185 \pm 0.025 \text{ GeV}, \quad N = 154 \pm 22,$$