of identifying high-spin states, a domain of investigations which has been generally claimed for the heavy-ion-induced reactions. We are presently extending similar measurements to other calcium and titanium isotopes.

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Imaginary Optical Potential for the Compound Nucleus ^{209}Pb

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A nonlocal energy-dependent imaginary optical potential for *s*-wave neutrons incident on ^{208}Pb is calculated in the intermediate structure model with particle-vibration coupling. The energy dependence is studied in the range of 0–12 MeV. The absorption cross section is calculated and compared to experiment below 2.6 MeV, the inelastic threshold. The agreement is quite good. The radial dependence is also investigated.

The optical potential is a first step towards understanding the scattering of a nucleon by a nucleus. However, despite its fundamental importance, the major efforts (with a few exceptions) have been towards a phenomenological fitting of the scattering data. In the present paper we present a derivation from a basic model of the imaginary part of the optical potential. The imaginary part is extremely interesting because it involves the dynamics of the system. The model we employ is that of intermediate structure (doorway states) with particle-vibration coupling. The results give information about nuclear structure as well as the scattering process.

O'Dwyer, Kawai, and Brown[1] and Rao, Reeves, and Satahler[2] have also performed imaginary-optical-potential calculations for ^{58}Ni and ^{208}Pb targets, respectively. Our calculations differ from Refs. 1 and 2 in that we include both bound and continuum intermediate states and determine a nonlocal energy-dependent potential that we use to compare with observed neutron cross sections over an energy range.

In the intermediate structure formalism,[3] the imaginary optical potential for elastic scattering may be written with notation similar to that of Auerbach *et al.*[4] as

$$W_{ljE}(r',r) = \mathrm{Im}\sum_q \frac{\langle r',lj|V|q\rangle\langle q|V|r,lj\rangle}{E-E_q+iI/2}. \tag{1}$$

The channel vector $|r,lj\rangle$ describes a particle in the continuum incident with angular momentum lj and energy E at the radial coordinate r. *The radial wave function is not included in* $|r,lj\rangle$. Similarly $\langle r',lj|$ describes the outgoing particle. The sum is over states q more complex than single particle. The energy of these states is E_q. The interaction is V and I is an energy interval over which the sharp energy dependence of the potential is averaged.

In the weak-coupling particle-vibration model the interaction is that written by Mottelson.[5] This interaction limits the sum over q in Eq. (1) to doorway states, i.e., states one step more complicated than single particle; specifically, these are particle-vibration levels.

The matrix elements in Eq. (1) are of the form

$$\langle q|V|r,lj\rangle \equiv \langle[\lambda(\alpha l'j')]j|V|r,lj\rangle = (2j+1)^{-1/2}(\hbar\omega_\lambda/2C_\lambda)^{1/2}\langle l'j'\|Y_\lambda\|lj\rangle\varphi_{\alpha l'j'}(r)k(r). \tag{2}$$

The symbol λ represents a vibration in the target nucleus, $(\hbar\omega_\lambda/2C_\lambda)^{1/2}$ is the one-phonon vibrational amplitude, $\varphi_{\alpha l'j'}(r)$ is the radial wave function of the coupled single particle of quantum numbers $\alpha l'j'$, and

$$k(r) = -dV(r)/dr, \tag{3}$$

where $V(r)$ is the real central optical potential.

We emphasize that the doorways consist of two types: a vibration coupled (i) to a bound particle $(nl'j')$ or (ii) to an unbound particle $(E'l'j')$. The only restrictions on the choice of the single-particle states is that total angular momentum and parity be conserved. When doorways of type (ii) contribute, then the sum in Eq. (1) is replaced by an integral over energy. Equation (1) thus becomes

$$W_{ljE}(r',r) = (2j+1)^{-1}k(r')k(r)\sum_\lambda \frac{\hbar\omega_\lambda}{2C_\lambda}\sum_{l'j'}|\langle l'j'\|Y_\lambda\|lj\rangle|^2\,\mathrm{Im}\left\{\sum_n \frac{\varphi_{nl'j'}(r')\varphi_{nl'j'}(r)}{E-E_\lambda-E_{nl'j'}+iI/2} + \int_0^\infty \frac{\varphi_{E'l'j'}{}^*(r')\varphi_{E'l'j'}(r)\rho(E')\,dE'}{E-E_\lambda-E'+iI/2}\right\}, \tag{4}$$

where E_λ, $E_{nl'j'}$, and E' are, respectively, the energies of the vibration, the coupled bound single particle, and the coupled continuum particle. The density of continuum states is represented by $\rho(E')$.

We expect each integral to be a smooth function of energy E; hence, we let I become vanishingly small. In this case the integrals may be evaluated by the principal-value theorem.[6] We assume that the continuum radial wave functions in Eq. (4) are real, and that the contribution of each integral in the case $E<E_\lambda$ is negligible. Finally we get

$$W_{ljE}(r',r) = -(2j+1)^{-1}k(r')k(r)\sum_\lambda \frac{\hbar\omega_\lambda}{2C_\lambda}\sum_{l'j'}|\langle l'j'\|Y_\lambda\|lj\rangle|^2 \times\left\{\frac{1}{2}\mathrm{Im}\sum_n \frac{\varphi_{nl'j'}(r')\varphi_{nl'j'}(r)}{(\mathcal{E}_\lambda-E_{nl'j'})^2+I^2/4} + \pi\rho(\mathcal{E}_\lambda)\varphi_{\mathcal{E}_\lambda l'j'}(r')\varphi_{\mathcal{E}_\lambda l'j'}(r)\theta(\mathcal{E}_\lambda)\right\}, \tag{5}$$

where $\mathcal{E}_\lambda = E - E_\lambda$ and $\theta(\mathcal{E}_\lambda)$ is a unit step function defined by

$$\theta(\mathcal{E}_\lambda) = \begin{cases} 0, & \mathcal{E}_\lambda < 0 \\ 1, & \mathcal{E}_\lambda \geq 0 \end{cases}. \tag{6}$$

We interpret the finite sum in Eq. (5) as the contribution from *compound-nucleus formation* and the second term as intermediate *inelastic target excitations* (pictorially, the target may be considered to be in a vibrational excited state while the single particle is in the continuum at energy $\mathcal{E}_\lambda$). We note that the potential is a sum of separate symmetric terms in r and r'.

We have calculated W [Eq. (5)] for s-wave neutrons incident on ^{208}Pb in the range $E=0$–12 MeV.

The bound single-particle wave functions ($2g_{9/2}$ to $3d_{3/2}$) are derived from a real optical potential with the parameters of Blomqvist and Wahlborn.[7] Essentially the same real potential is used to generate the intermediate continuum single-particle wave functions of all possible $l'j'$ values. The following are the vibrations used in ^{208}Pb: 3^- [2.6 MeV, 39.5 Weisskopf units (W.u.)], 5^- (3.2 MeV, 14.0 W.u.), 5^- (3.7 MeV, 1.85 W.u.), 2^+ (4.1 MeV, 8.00 W.u.), 4^+ (4.3 MeV, 15.00 W.u.), 6^+ (4.4 MeV, 5.5 W.u.), 8^+ (4.6 MeV, 4.00 W.u.). The energies and transition strengths of these are taken from experiment and are given by Divadeenam and Beres.[8] In addition, we include the giant quadrupole resonance at 11.5 MeV in ^{208}Pb.[9] The strength in this case is calculated from the $T=0$ sum rule[10] and is found to be 96 W.u.

Since the imaginary optical potential is peaked in our model at $r=r'=R$ (the nuclear radius parameter of the real optical potential), we give in Fig. 1 as a first step in our study a plot of $W_0 = -W_{0\,1/2\,E}(R,R)$ as a function of energy. In order to display the broad features of the potential, the averaging interval I is taken to be 0.75 MeV for all the doorways except the giant quadrupole where a value of 2 MeV is used. The solid line gives the total contribution of both the compound-nucleus formation and inelastic excitations. The various peaks, except the one at about 6 MeV, occur at the energies of the doorways. The broad peak at about 10 MeV is due to the doorways formed by the giant quadrupole vibrational resonance and the $3d_{3/2}$ and $3d_{5/2}$ bound single-particle states. The tall peak at about 6 MeV is due to a resonance in the $2h_{11/2}$ single-particle continuum wave function which is coupled to a 5^- vibration. The dashed line represents the inelastic contribution only. We note that this is zero below about 4 MeV and is relatively smooth except for the resonance just described. If there were no giant quadrupole resonance, then the inelastic excitations would be essentially the only contributors to the potential above about 5 MeV. When the resonance at 6 MeV is treated as a bound state by using a harmonic-oscillator wave function and $I=0.75$ MeV, we get the dash-dotted line.

Next we use our calculated potential to determine the absorption cross section for s-wave neutrons, $\sigma_{abs}{}^0$, in the energy range 0 to 2.6 MeV (the neutron inelastic threshold). In order to better analyze the fine structure in this small energy range, we use a smaller value of I than in Fig. 1. A change in I simply alters the widths of the peaks in the potential and does not change the general features given in Fig. 1. The absorption (i.e., resonance elastic) cross section is related

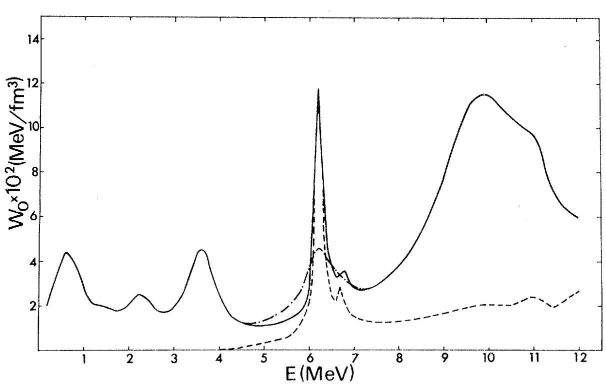

FIG. 1. Curves of $W_0\times 10^2$ versus E (see text for definition of W_0). Solid line, sum of compound nucleus and inelastic excitations; dashed line, only the inelastic contribution; dash-dotted line relates to treating the $2h_{11/2}$ resonance as a bound state (see text for details).

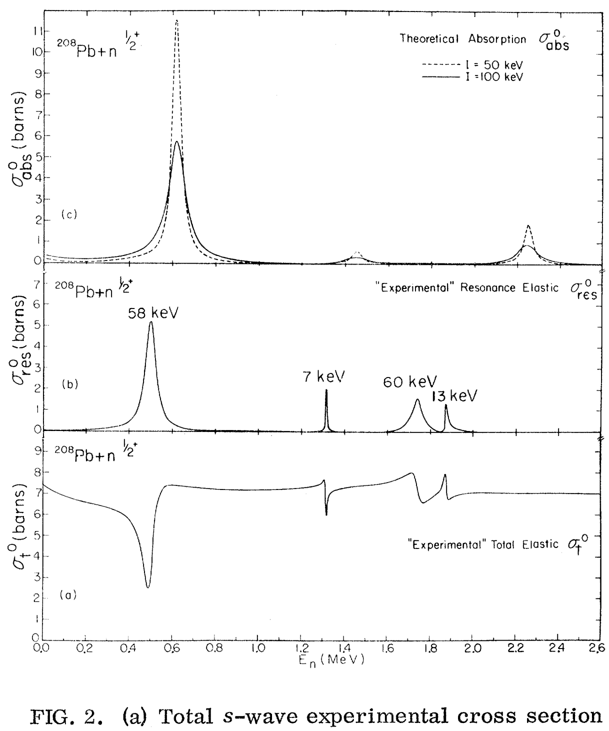

FIG. 2. (a) Total s-wave experimental cross section as generated from an R-matrix code using parameters that fit the data of Farrell *et al.* (Ref. 11) and Fowler (Ref. 12). The word experimental is in quotes to indicate that the background term corresponding to p waves is not included. (b) R-matrix-generated resonance-elastic cross section using the same parameters as in (a). (c) Calculated absorption cross section $\sigma_{abs}{}^0$ [Eq. (7)].

to $W_{0\,1/2\,E}(r', r)$ [Eq. (5)] via

$$\sigma_{\mathrm{abs}}{}^0 = (-2/\hbar v)\iint \psi_{0\,1/2\,E}{}^*(r') W_{0\,1/2\,E}(r', r)\psi_{0\,1/2\,E}(r) r^2 r'^2\,dr\,dr', \tag{7}$$

where v and $\psi_{0\,1/2\,E}(r)$ are, respectively, the incident neutron speed and radial wave function. We emphasize that the complete nonlocal potential is used in Eq. (7) and not just $W_{0\,1/2\,E}(R, R)$ as in Fig. 1.

The results from Eq. (7) using $I = 50$ and 100 keV are given in Fig. 2(c). The three predicted resonances in order of increasing energy are due to the doorways $(4^+ \otimes 2g_{9/2})$, $(6^+ \otimes 1i_{11/2})$, and $(2^+ \otimes 3d_{5/2})$. We note that other doorways including those based on the giant quadrupole resonance contribute negligibly to the cross section. Using an R-matrix code with parameters obtained from experiment the observed s-wave total cross section $\sigma_t{}^0$ is given in Fig. 2(a) and the resonance elastic cross section determined from experiment is given in Fig. 2(b). The cross section in Fig. 2(b) does not include potential scattering and is to be directly compared to the calculated cross section in Fig. 2(c). It is clear that the theory fits experiment quite well. Even better agreement as to the positions of the resonances could be obtained if we changed our single neutron energies, which we did not attempt to do. We cannot, however, predict the extra resonance seen experimentally at about 1.9 MeV.

We have also made a preliminary calculation of the cross section up to about 12 MeV, and except for a bump near 6 MeV the cross section is quite small. Even the giant quadrupole vibration does not contribute significantly to the cross section. The indication is that inelastic intermediate states, excluding single-particle resonances, are not important up to about 12 MeV. We intend to extend our cross-section calculation to higher energies.

Finally, we have studied the nonlocal character of our potential $W_{0\,1/2\,E}(r',r)$ for different values of E. We have attempted to approximate the radial dependence by a Gaussian of the form

$$W_{0\,1/2E}(r', r) = -W_0 \exp\left[-\left(\frac{r'-R}{\beta}\right)^2 - \left(\frac{r-R}{\beta}\right)^2\right], \tag{8}$$

and we found that the range parameter $\beta = 1.04$ fm gives a good description over the energy range 0–12 MeV. It is interesting that Eq. (8) is a separable potential, rather than a sum of separable terms as implied from Eq. (5). More details of this study will be given in a longer paper.

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