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Non-Abelian Gauge Theories of the Strong Interactions*

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A class of non-Abelian gauge theories of strong interactions is described, for which parity and strangeness are automatically conserved, and for which the nonconservations of parity and strangeness produced by weak interactions are automatically of order α/m_W^2 rather than of order α. When such theories are "asymptotically free," the order-α weak corrections to natural zeroth-order symmetries may be calculated ignoring all effects of strong interactions. Speculations are offered on a possible theory of quarks.

Recently Gross and Wilczek and Politzer have made the exciting observation that non-Abelian gauge theories can exhibit free-field asymptotic behavior at large Euclidean momenta.[1] However, the physical application of this discovery raises serious problems: (1) Why don't the weak interactions produce parity and stangeness nonconservations of order α? (This problem finds a natural solution when the strong interactions are described by *Abelian* gauge models,[2] but not, to the best of my knowledge, in non-Abelian models of the "Berkeley" type.[3]) (2) Even with asymptotic freedom, when can the strong interactions actually be neglected? (3) Even if asymptotic freedom explains the success of naive quark-model calculations, why don't we see physical quarks? This note will describe a class of non-Abelian gauge models which provide a complete answer to (1), a partial answer to (2), and a possible answer to (3).

Consider a renormalizable gauge theory of the strong, weak, and electromagnetic interactions with the following properties: (A) The gauge group is a direct product of a "strong" gauge group G_S and a "weak" gauge group G_W. The coupling constants associated with G_S and G_W are of order 1 and e, respectively. The spin-$\frac{1}{2}$ hadrons are non-neutral with respect to both G_S and G_W, but the generators of G_S all commute with the generators of G_W. That is, the quarks would have to form a matrix, with weak and strong interactions producing transitions along columns and rows, respectively, as in the colored-quark[4] or Pati-Salam[5] models. The leptons are neutral under G_S. (B) The strong gauge group G_S is *nonchiral.* (C) There are various weakly coupled spin-0 fields with very large vacuum expectation values (~ 300 GeV), which break G_W and give the associated vector bosons large masses (~ 30 GeV) and also produce or contribute to the zeroth-order fermion mass matrix m. However, these spin-0 fields are neutral under G_S, so their vacuum expectation values do not contribute to the mass of the strongly interacting neutral vector bosons.

To avoid massless strongly interacting gauge bosons, it is natural to suppose that G_S is also spontaneously broken. We might assume that this occurs through the vacuum expectation values of various strongly interacting scalar fields, but it appears[1] that the introduction of these fields would prevent a free-field asymptotic behavior. Alternatively, we might assume that there are no strongly interacting spin-0 fields, but that G_S is broken dynamically.[6] However, this still leaves the question of why observed hadron states should belong to simple G_S multiplets, like the color singlets of Ref. 4.

There is one other possible alternative: that G_S is *not* broken, so that the G_S gauge bosons do have zero mass, and the quarks are "color" or "column" degenerate. On the basis of a preliminary analysis, it appears that the infrared divergences in such a theory could make the rate for production of any number of G_S-non-neutral particles in a collision of G_S-neutral particles *vanish.* The world would than consist of composite G_S singlets, like ordinary hadrons, which are not affected by these infrared divergences; the G_S-non-neutral hadrons, such as quarks and massless vector bosons, although present in Feynman diagrams, Wilson expansions, etc., would never appear as physical particles. This conjecture is under further study.

Leaving aside scalar fields, any theory governed by assumptions (A), (B), and (C) will have an effective "zeroth-order" strong-interaction Lagrangian of the form

$$\mathcal{L}_{\text{strong}} = -\bar{\psi} Z_\psi \gamma^\mu D_\mu \psi - \bar{\psi} m \psi - \tfrac{1}{4}(Z_A)_{ab} F_{a\mu\nu} F_b^{\mu\nu}, \qquad (1)$$

where ψ and $D_\mu\psi$ are the spin-$\frac{1}{2}$ field multiplet and its G_S-gauge-covariant derivative; $F_{a\mu\nu}$ is the G_S-covariant gauge-field-strength term; and Z_ψ, m, and Z_A are G_S-invariant matrices, the first two of which may involve both the γ_5 and 1 Dirac matrices. Because Z_ψ and m commute with the generators of G_S, we can use Schur's lemma to redefine the fermion fields so that Z_ψ becomes unity and m becomes real, diagonal, and γ_5 free, while the generators of G_S remain γ_5 free and commute with m. Once we define the fermion fields in this way, the theory automatically conserves parity, if we assign positive parity to all spin-$\frac{1}{2}$ fields and identify all G_S gauge fields as polar vectors. In addition, any quantum number such as strangeness, charm, etc., which can be expressed in terms of the number of elementary fermions, summing over "color," is also automatically conserved. (The same is true if we introduce strongly interacting spin-0 fields, provided they belong to representations of G_S which prohibit Yukawa couplings to the fermions. All spin-0 fields can then be assigned positive parity and zero strangeness, charm, etc.) Also, it will often happen[7] that, for a wide range of parameters in the original Lagrangian, the spontaneous breakdown of G_W leaves some of the elements of the diagonalized fermion mass matrix equal or zero. This equality will then extend to all colors, and the strong interactions will have a "natural" zeroth-order unitary or chiral symmetry,[7] in addition to the conservation of parity, strangeness, etc.

Now let us consider the effect of second-order weak and electromagnetic interactions which break these symmetries. Just as in the neutral vector gluon model,[2] it is found after canceling gauge-dependent terms that the corrections to a general hadronic S-matrix element fall into three classes: tadpolelike terms, photon terms, and heavy vector boson exchange terms. The tadpolelike terms just amount to a G_S-invariant correction to the matrix m in (1), and therefore any parity or strangeness nonconservation they introduce can be transformed away as before. The photon terms clearly cannot change parity or strangeness. The remaining terms are of the form

$$\delta S_{FI} = (2\pi)^4\delta^4(P_F - P_I) \times \int d^4k\, \mathfrak{F}_{\alpha\beta}{}^{FI}(k)(k^2+\mu'^2)_{\alpha\beta}{}^{-1}, \qquad (2)$$

where

$$\mathfrak{F}_{\alpha\beta}{}^{FI}(k) \equiv \tfrac{1}{2}i(2\pi)^{-4} \times \int d^4x \langle F|T(J_{\alpha\mu}(x)J_\beta{}^\mu(y))|I\rangle e^{ik(x-y)}. \qquad (3)$$

Here the $J_{\alpha\mu}$ are the currents of G_W, formed from the spin-$\frac{1}{2}$ hadron fields, and μ' is the G_W-vector boson mass matrix, except that the photon is here given an arbitrary large mass Λ, to compensate for a regulator mass Λ used in evaluating the true photonic term.[2] The only terms in (2) that are of order α rather than α/m_W^2 are those arising from terms in $\mathfrak{F}(k)$ which decrease no faster than k^{-2} as $k\to\infty$. Such terms may be picked out from the Wilson operator-product expansion[8] of the direction-averaged matrix element:

$$\int d\Omega_k \mathfrak{F}_{\alpha\beta}{}^{FI}(k) \underset{k\to\infty}{\longrightarrow} \textstyle\sum_N \langle F|O_N|I\rangle U_{\alpha\beta}{}^{(N)}(\sqrt{k^2}). \qquad (4)$$

The asymptotic behavior of $U_{\alpha\beta}{}^{(N)}(\kappa)$ in perturbation theory is

$$U_{\alpha\beta}{}^{(N)}(\kappa) = O(\kappa^{2-d_N} \times \text{powers of } \ln\kappa), \qquad (5)$$

where d_N is the "naive" dimensionality of the operator O_N. Hence the only terms that contribute to δS_{FI} in order α are those with exponent $2-d_N$ not less than -2. The change in the S matrix is therefore equivalent to a change in the strong-interaction Lagrangian:

$$\delta\mathfrak{L} = \textstyle\sum_N O_N \int_0^\infty U_{\alpha\beta}{}^{(N)}(\kappa)(\kappa^2+\mu'^2)_{\alpha\beta}^{-1}\kappa^3 d\kappa, \qquad (6)$$

the sum extending only over "renormalizable" operators O_N, with $d_N \leq 4$. But these operators are all Lorentz invariant, because we have averaged over momentum-space directions in (4), and they are all G_S-gauge invariant, because the currents $J_{\alpha\mu}$ are neutral under G_S. Hence the order-α correction term (6) must be of precisely the same form as the original zeroth-order Lagrangian (1), and so any order-α violation of parity and strangeness conservation introduced by the weak interactions can be eliminated by redefining the fermion fields as before. The only symmetry breaking remaining after this redefinition will take the form of shifts in the fermion masses, which can break the other natural zeroth-order symmetries[7] mentioned above. On the other hand, the truly weak corrections of order α/m_W^2 cannot be expressed in terms of renormalizable corrections to the Lagrangian, so we expect both parity and strangeness conservation to be violated by such nonleptonic truly weak interactions.

Now, what of the actual calculation of $\delta\mathfrak{L}$? After symmetric integration, the $U^{(N)}$ functions that contribute to (6) all have the naive asymptotic behavior κ^{-2}, so the integral (6) appears logarithmically divergent. However, the trace $U_{\alpha\alpha}$ is G_W invariant, so it cannot contribute to the corrections to natural zeroth-order symmetries, and thus all logarithmic divergences cancel.[2] If we assume that the "free-field" asymptotic behavior sets in at $\kappa < m_W$, then we can evaluate (6) using the asymptotic formula[9]

$$U_{\alpha\beta}{}^{(N)}(\kappa) \to \kappa^{-2}B_{\alpha\beta}{}^{(N)} \exp\{\textstyle\int_a^k \gamma_N(a')da'/a'\} \qquad (7)$$

where $B_{\alpha\beta}{}^{(N)}$ is a constant, independent of the strong-interaction coupling constant; $\gamma_N(a)$ is an anomalous dimension; and the lower limit a depends on the renormalization prescription used to define the operator O_N. In general, γ_N is not zero, so "asymptotic freedom" does *not* say the strong interactions can be ignored in calculating $U^{(N)}$. However, γ_N is in fact zero for the functions that interest us here. By the same arguments as those used in Ref. 2, we can conclude that the only O_N in (6) that matters is the bilinear product $\bar\psi_n\psi_m$. This does have a nonzero anomalous dimension $\gamma_{\bar\psi\psi}$, but we are interested here in the *nonleading* terms in the corresponding U function of order κ^{-2}, and the presence of a fermion mass factor in these nonleading terms introduces a term $-\gamma_{\bar\psi\psi}$ which cancels the previous term. (Details will be published elsewhere.) Using (7) in (6), our result then is that the terms of order α in δS_{FI} are the same as would be produced by a change in the effective strong-interaction Lagrangian

$$\delta\mathfrak{L} = \tfrac{1}{2}\bar\psi B_{\alpha\beta}\psi(\ln\mu'^2)_{\alpha\beta},$$

where $(B_{\alpha\beta})_{nm}$ is the B matrix for the operator $\bar\psi_n\psi_m$. This is independent of the strong-interaction coupling constants, and therefore must be the same as would be given by a one-loop perturbation calculation. The same is trivially true of the tadpole terms, so, apart from the purely photon terms, *all corrections to the effective Lagrangian are correctly given to order α by neglecting all effects of strong interactions.* Of course, we must not use perturbation theory to calculate any physical hadron mass or matrix element, but only to calculate $\delta\mathcal{L}$; the result must then be used as an input to current-algebra[10] or parton-model[11] calculations of physical quantities.

I am grateful for valuable discussions with T. Appelquist, C. Callan, S. Coleman, S. Glashow, D. Gross, R. Jackiw, K. Johnson, F. Low, D. Politzer, and H. Schnitzer.

*Work supported in part by the National Science Foundation under Grant No. GP-30819X.

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Why Do Total Cross Sections Grow with Energy?*

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By using exact inclusive sum rules we infer that the growth with energy of the total pp cross section is connected with the mechanism which is responsible for the appearance of a sharp peak near the kinematical boundary in the process $p+p\to p+$ anything. We discuss this mechanism in the context of Regge models and suggest tests of this idea which involve K^+p scattering experiments.

The apparent rise of the total cross section for pp scattering at CERN intersecting-storage-ring (ISR) energies[1,2] has renewed the interest in the asymptotic behavior of cross sections. In this paper, we connect, in a model-independent way, the increase of $\sigma_{pp}(s)$ with the appearance of a peak in the inclusive proton spectrum near $|x|=1$. This provides a mechanism which explains *why and at what energy* the increase in $\sigma_{pp}(s)$ occurs. It is, of course, possible to construct models[3] of various degrees of plausibility which can fit the observed behavior of the total cross section. According to our observation, viable models should explain simultaneously both the rise in the total cross section and the peak near $|x|=1$.

Let us denote the inclusive cross section for the process $a+b\to c+X$ by

$$E_c d\sigma_{ab}{}^c/d^3p_c = f_{ab}{}^c. \quad (1)$$

Then we can write the sum rule which expresses the conservation of energy,[4] in the c.m. system, as

$$\sum_c \int (d^3p_c/E_c)E_c f_{ab}{}^c = \sqrt{s}\,\sigma_{ab}(s). \quad (2)$$

In the above the dominant contributions arise from the fragmentation regions of a and b since the weighting factor of E_c effectively suppresses the pionization region.

We remark that if the $f_{ab}{}^c$ show limiting behav-