the vortex cores and flow in an inhomogeneous pattern which reflects the structure of the vortex lattice. Clearly, further investigation both experimentally and theoretically is warranted.

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Sideband Instability: Observations and Comparison with Theory*

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In the trapped-electron sideband instability of a nonlinear plasma wave of potential φ_0, initial enhanced damping occurs for a distance $\propto \varphi_0^{-1/2}$, beyond which signals grow at a rate scaling nearly linearly with φ_0. The growth remains exponential even in cases where the main wave is significantly damped. This behavior is more consistent with a quasilinear spectral development than with the commonly cited parametric bounce-frequency instability of a stationary Bernstein-Greene-Kruskal equilibrium.

The original observation by Wharton, Malmberg, and O'Neil[1] that satellite frequencies of a large-amplitude collisionless electron-plasma wave are unstable and grow with distance has stimulated a number of recent experimental and theoretical works. The frequency separation between the sideband peak and the "main" (large-amplitude) wave was found to be proportional to the bounce frequency

$$\Omega_b = k_0(e\varphi_0/m)^{1/2}, \tag{1}$$

where φ_0 and k_0 are the main-wave electric potential and wave number, respectively. Since Ω_b is the frequency of oscillation of an electron trapped in the wave potential well, the instability was associated with the resonant electrons. No measurements of growth-rate dependence on external parameters were made in connection with this first observation.

Subsequent experiments have verified that the instability also occurs for ion-acoustic waves[2] and that it is possible to have a series of sidebands, with observed frequency separations given by the Doppler-shifted bounce frequency, growing from the background plasma noise.[3]

In this paper we describe new observations of the instability, designed specifically to test the various theories which have been advanced. To facilitate this approach we note that the theories may be classified generally according to which of two principal physical mechanisms is assumed responsible for sideband generation.

The first mechanism, originally proposed by Kruer, Dawson, and Sudan,[4] involves a parametric interaction between the main wave and the trapped electrons acting as Doppler-shifted harmonic oscillators. While these parametric sideband theories[4-7] differ in calculational detail, they share common features. In particular, (1) they test the stability of a Bernstein, Greene, and Kruskal[8] (BGK) state, which is a nonlinear stationary-wave equilibrium; and (2) they derive an instability growth rate $\gamma \propto \varphi_0^{1/2}$ (above a threshold,[7] in some cases).

The other class of theories[9,10] assumes that the main wave deforms the electron velocity distribution in the manner calculated (in one limit) by O'Neil.[11] Quasilinear theory may then, in principle, be used to deduce the growth rate from the slope of the distribution function at nearby phase velocities.

In our experiments, which measure the development of both spontaneous and test-wave excitations, an initial enhanced damping is observed for a distance x_t proportional to $\varphi_0^{-1/2}$. The instability growth then commences and is thereafter approximately exponential with distance, the rate scaling nearly linearly with φ_0, even in cases where the (initially) large wave has decayed to the same level as the test wave. We conclude from these experimental results that the instability cannot be described adequately by parametric constant-potential sideband theories, whereas it exhibits several features of quasilinear behavior involving the development of the distribution function.

The experiments were performed in a thermally ionized potassium plasma column[12] of diameter 2.5 cm, density $\sim 2\times 10^7$ electrons/cm^3, and temperature $\sim \frac{1}{4}$ eV. The plasma column is coaxial with a 5-cm-diam wave guide and a 2-kG longitudinal magnetic field over a length of 98 cm, giving a configuration in which the electron-plasma-wave linear dispersion and Landau damping are consistent with the parameters given above, and in which the electromagnetic mode is strongly cut off.[12] Axial density gradients along the 50-cm experimental region were less than 2%, typically 1%. The geometry is single ended; an ionizer plate and a cold plate (which is usually negatively biased) terminate the plasma column. Signals are launched and detected on transverse wire probes 0.025 cm in diameter. The two transmitters are fixed and the receiver is axially movable.

We identify the instability by its conformity to previously reported behavior, principally the dependence of the frequency separation on φ_0 described above. Spectra show a dominant lower sideband for which the following data are reported, as well as a smaller upper satellite in many cases.[13]

The sideband processes may be studied in detail over a range of main-wave amplitudes above $e\varphi_0/\kappa T \sim 0.05$ by applying to a separate antenna a second low-level test wave at a frequency of interest, and recording the detected power (in a 100-kHz bandwidth) as a function of receiver-antenna position. An illustrative record is shown in Fig. 1 for a superimposed test-wave signal at the lower sideband-peak frequency, with the measured amplitude development in the absence of the instability (i.e., with the main wave off) also

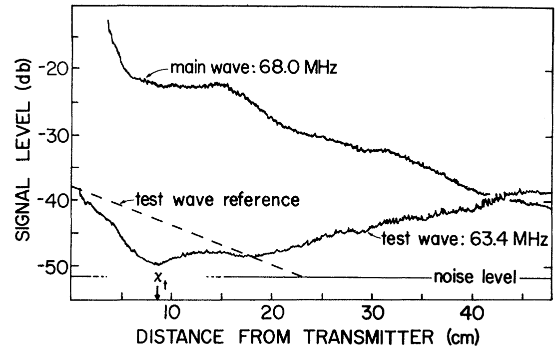

FIG. 1. Amplitude of a main wave and a test wave versus distance from main-wave launching antenna in the extreme case of $v_0 = 2.6v_{\text{th}}$. Dashed line is from measurements of test wave with main wave off. The test wave is applied at $x = -18$ cm; the main-wave amplitude is $\varphi_0 = 35$ mV.

indicated for comparison by the dashed line. It is seen that the test signal initially damps more strongly in the presence of the main wave until a "turn-on" point x_t is reached, whereafter the signal grows with slight oscillations about an exponential rate. These observed features will now be examined more closely in the context of theoretical predictions.

From recordings such as in Fig. 1, the dependence of the growth rate on φ_0 can be measured for either test waves or spontaneous sidebands. As an example, Fig. 2 shows the data obtained for a spontaneous lower satellite. The frequency at which the growth is measured was adjusted to be at the sideband peak for consistency in the interpretation of $\gamma(\varphi_0)$. The wave potential is obtained from the input level and the (extrapolated) received power at $x = 0$, using the symmetry of the transmitter-plasma and plasma-receiver coupling

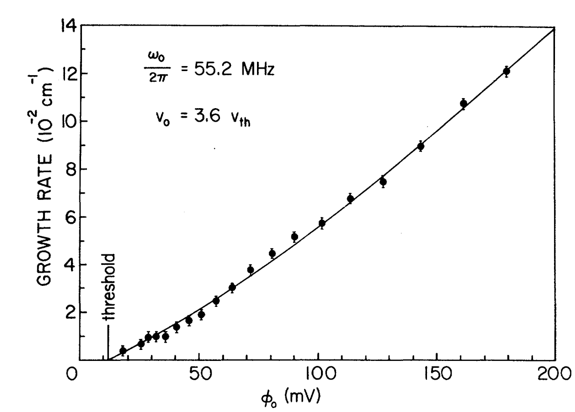

FIG. 2. Spontaneous lower-sideband growth rate versus φ_0. Solid curve is $\gamma - \gamma_0 \propto \varphi_0^{1.2}$, where γ_0 is inferred from the threshold effect shown.

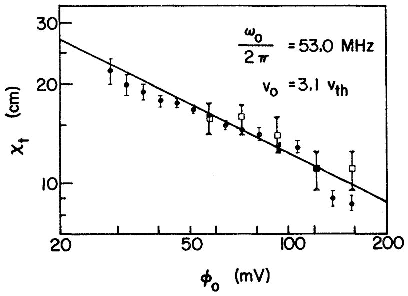

FIG. 3. Turn-on distance x_t versus φ_0. Solid line, $\lambda_b \equiv 2\pi v_0/\Omega_b$. Squares, measurements of main-wave bounce length.

losses. These particular data are fitted by $\gamma - \gamma_0 \propto \varphi_0^{1.2}$, where γ vanishes at a threshold potential of 12 mV. The fitted zero-potential damping rate γ_0 (< 0) is a consistent feature which is not entirely explained by the Landau result.

Determinations of the exponent of φ_0 in the γ dependence range from 0.8 to 1.4, which disagree with the $\varphi_0^{1/2}$ prediction of parametric sideband theories. In quasilinear treatments, the growth is computationally determined for different initial amplitudes and no direct comparison is presently available.

Another competing process which has bearing on theoretical predictions of the growth-rate dependence is the occurrence of passive four-wave mode coupling between upper and lower sidebands in the unstable-frequency range.[13] In theory this additional parametric effect,[14] since it scales as φ_0^2, eventually dominates the bounce-frequency resonance and changes the dependence on φ_0. Our test-wave measurements, which do not show this dominance,[13] will be detailed elsewhere in the context of a specific discussion of mode-coupling mechanisms.

In another significant reduction of data, the position of the turn-on point x_t is followed as φ_0 varies. An example of the results appears in Fig. 3, compared with calculated and measured values of the main-wave bounce length $\lambda_b \equiv 2\pi v_0/\Omega_b$, where $v_0 \equiv \omega_0/k_0$ is the main-wave phase velocity and Ω_b is calculated from φ_0. From this plot, x_t is seen to scale as $\lambda_b \propto \varphi_0^{-1/2}$. The bounce resonance required in parametric theories would not (as a practical matter) manifest itself until at least λ_b, so this result is not inconsistent with them. However, a quantitative prediction of the turn-on point or the existence of the initial enhanced damping is beyond the scope of present steady-state parametric theories.

On the other hand, the quasilinear calculation of Brinca[10] exhibits both of these features explicitly. In that treatment the temporal growth rate of the mode at the peak sideband frequency initially goes negative, reaching a minimum at $t = 2\pi/\Omega_b$, then rapidly increases, crossing zero at $t \approx 3\pi/\Omega_b$. Converting time to distance via v_0, this prediction compares within a factor of 1.5 with our observations.

The sideband instability occurs for phase velocities ranging from 2.5 to 4.5 times thermal velocity (v_{th}). At the lower velocities, small-amplitude signals experience fairly strong linear collisionless damping. Large slow signals, for which γ_L/Ω_b is finite,[11] exhibit amplitude oscillations superimposed on a net decay. The example shown in Fig. 1 is for $v_0 = 2.6 v_{th}$ and thus is an extreme case with relatively large main-wave decay, weak bounces, and small instability growth.[15] The important feature to note is that the sideband growth remains exponential even though the main wave decays by more than 40 dB and eventually becomes smaller than the satellite.

The assumption of a uniform BGK state, made by parametric theories, is a poor approximation to this behavior since the magnitude of the main-wave decay shown here implies a large decrease in the local bounce frequency. A steady sideband growth such as that observed would not then be expected because γ would eventually decrease to one-tenth its original value (obtained from φ_0 at $x = 0$).

In this regard, it should be noted that the measured sideband-peak frequency separation remains constant with distance from the transmitter and agrees within 10% with the value calculated from φ_0. Therefore, the instability is characterized by the initial main-wave level, and not by the local level which can be orders of magnitude smaller.

The behavior in this case does not preclude a quasilinear description since the growth would arise from a convecting distortion of the local electron-velocity distribution, and not from the local electric-field–determined bounce frequency.

We conclude that the experimental results cannot be explained by the parametric sideband theories. Such formulations fail to predict the correct φ_0 dependence of the growth rate, and cannot deal with the initial behavior. Moreover, the instability occurs over a range of main-wave phase velocities including those for which the assumption of a stationary nonlinear state is violated, and the persistent exponential growth in these cases is inexplicable in terms of a varying bounce-frequency resonance.

On the other hand, at least one quasilinear theory predicts the initial enhanced damping and instability turn-on point with reasonable accuracy. Further theoretical work seems needed at this time to see if the quasilinear mechanism can successfully predict the correct scaling of the growth rate with φ_0. Any such work should also self-consistently include the change in amplitude of the main wave.

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Light-Scattering Measurements of Turbulence in a Normal Shock*

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Measurements on laser-light scattering have been made of the turbulence in a normal shock. Wave-number spectrum and phase velocity were determined. The turbulence has not been identified with any instability. The predominant direction of propagation is shown not to coincide with the current vector.

A number of papers[1-5] recently have been discussing which instabilities lead to the turbulence responsible for the enhanced resistivity observed in collisionless shocks. At the moment, two types of instabilities appear to be likely candidates, namely, (1) ion-acoustic waves, and (2) electron Bernstein waves resonating either with ion-acoustic waves or directly with the thermal ions. Only a limited number of experiments have investigated this problem,[6,7] and they each seem to favor their own explanation, originating from different conditions of the experiments. We are reporting data on laser-light scattering obtained in an experiment having conditions very similar to those of Paul *et al.* The results are in some respects similar, but we are presenting more extensive data with respect to some parameters, and we do differ in our interpretation of the type of instability observed.

Our apparatus was built mainly to study oblique shock waves, but does at the same time produce shocks propagating radially inward perpendicular to the magnetic field. All data presented in this Letter are on the perpendicular shock. We will