Measurement of the Form Factors in the Decay $K_L{}^0 \rightarrow \pi\mu\nu$

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A Dalitz plot of 1.6×10^6 $K_L{}^0 \rightarrow \pi\mu\nu$ decays has been studied to measure the t dependence of the vector and scalar form factors. The observed slopes, $\lambda_+ = 0.030\pm0.003$ and $\lambda_0 = 0.020\pm0.003$, are compatible with current-algebra and soft-pion predictions, μ-e universality, and K^* (890) dominance of the vector form factor.

We present herewith the analysis of an experiment performed at the Stanford Linear Accelerator Center, in which the Dalitz plot of 1.6×10^6 $K_{\mu3}{}^0$ decays was studied to extract the strong-interaction form factors of the decay.

With the assumption of $V-A$ Cabibbo theory and time-reversal invariance, the matrix element for the $K_L{}^0 \rightarrow \pi\mu\nu$ decay may be written in terms of two real form factors f_+ and f_-, which depend only on t, the square of the four-momentum transfer to the lepton pair:

$$M \propto [\bar{u}_l\gamma_\mu(1+\gamma_5)u_\nu] \times [f_+(t)(p_K+p_\pi) + f_-(t)(p_K-p_\pi)],$$

where $t=(p_K-p_\pi)^2$. The Dalitz-plot distribution is usually written in terms of $f_+(t)$ and $\xi(t)\equiv f_-(t)/f_+(t)$. We have chosen, however, to analyze the Dalitz-plot distribution using the form factors $f_+(t)$ and $f_0(t) = f_+(t) + [t/(m_K{}^2 - m_\pi{}^2)]f_-(t)$, which represent the vector and scalar exchange amplitudes, respectively. These form factors are more directly related to theoretical predictions, and are less strongly correlated. The Dalitz-plot distribution is then given by

$$\frac{d^2N(E_\pi, E_\mu)}{dE_\pi\, dE_\mu} = \alpha f_+(t)^2 + \beta f_+(t) f_0(t) + \gamma f_0(t)^2,$$

where α, β, and γ are kinematic factors. Note that since the form factors are functions only of t, their t dependence may be extracted without prior parametrization by analyzing the Dalitz plot in bands of constant E_π.

The t dependence of f_+ and f_0 is expected to be understood in terms of poles of definite mass which saturate the dispersion relations for these form factors. Further, with the assumption that the eight vector and axial-vector currents of the weak interactions form a chiral SU(3) $\otimes$ SU(3) algebra, and using the hypothesis of partial conservation of axial-vector current, the value *and* slope of f_0 at unphysical points are predicted to be[1,2]

$$f_0(m_K{}^2) = f_K/f_\pi = (1.27\pm0.03)f_+(0),$$

and

$$m_\pi{}^2 \frac{df_0}{dt}\bigg|_{t=m_K{}^2+m_\pi{}^2} = \frac{m_\pi{}^2}{2(m_K{}^2-m_\pi{}^2)}\left(\frac{f_K}{f_\pi} - \frac{f_\pi}{f_K}\right) = 0.021\pm0.003,$$

respectively, in the limit of vanishing pion mass. These results are expected to be valid to 10% for the physical pion. The history of the experimental determination of these form factors has been confusing: The recent review of Chounet, Gaillard, and Gaillard[3] attempted to bring some order to the situation, concluding that the current-algebra and partial conservation of axial-vector current ideas, which have been successful in other areas, were not in agreement with K_{l3} data.

This experiment was performed at the Stanford Linear Accelerator Center K^0 spectrometer facility,[4] concurrent with the measurement of the charge asymmetry in the decay $K_L{}^0 \rightarrow \pi^\mp\mu^\pm\nu$.[5] Briefly, some features of the experiment are (1) a high-energy $K_L{}^0$ beam with low neutron contamination, (2) determination of the $K_L{}^0$ momentum by time-of-flight measurement with a resolution of $\pm\frac{1}{3}$ nsec over a flight path of 75 m, corresponding to $\Delta p/p = 0.005p^2$, p in GeV/c, (3) momentum analysis of charged decay products with a 12.6 kG m magnet, (4) a muon filter consisting of 7.7 interaction lengths of lead, and (5) a resolution on the Dalitz plot compatible with 5×5-MeV binning.

A sample of 0.8×10^6 accepted Monte Carlo events was used to determine the detection efficiency as a function of position on the Dalitz plot, and to search for biases in the data. The Monte Carlo program generated tapes in the same format as raw data tapes; these were then processed through the same analysis programs as the data. Although the results are insensitive to the form factors used in the Monte Carlo computation, the latter were chosen to correspond to the experimentally observed values. The momentum spectrum of the K_L^0 beam was initially derived from a sample of $K_L^0 \to \pi^+\pi^-\pi^0$ decays obtained with a looser trigger, and later refined by comparison with the actual $K_L^0 \to \pi\mu\nu$ decays.

Several sources of contamination were investigated. The $\pi \to \mu\nu$ decays following $K_{\pi 3}^0$, K_{e3}^0, $K_{\mu 3}^0$, and $K_{\pi 2}^0$ decays were studied by Monte Carlo methods. The subtraction of such events amounted to about 5% of the data remaining after the $P'_0{}^2$ cut, which was used to reject a majority of the $K_{\pi 3}^0$ decays. Pion penetration was less than 0.1%, as demonstrated by the excellent agreement between the number of 2μ events (mostly $K_{\mu 3}^0$ followed by π decay) in the data and in the Monte Carlo calculation. Decays of K_L^0's which had diffractively scattered before the decay volume were also subtracted. This 1% correction had no noticeable effect on the results, but did significantly improve the agreement of some of the experimental distributions with the Monte Carlo results. K_L^0 interactions in helium were investigated by taking separate data with a 1-in.-thick carbon slab at several positions within the decay volume. Less than 0.1% of the accepted data were due to interactions and such events were evenly distributed over the Dalitz plot.

A detailed study was made to look for systematic differences between the data and the Monte Carlo computation, particularly with respect to geometrical biases in the wire chambers which could directly affect the Dalitz plot. No such discrepancies were found. A comparison of several experimental and Monte Carlo distributions is shown in Fig. 1.

The Dalitz-plot analysis is complicated by a quadratic ambiguity associated with the transformation of the charged decay products to the K_L^0 rest frame. In principle, the K_L^0 time-of-flight measurement can resolve this ambiguity, but because of the high momentum of our beam, this information alone was insufficient for part of the data. Hence the beam momentum-spectrum information was also used. Monte Carlo studies indicated that this procedure chose the correct

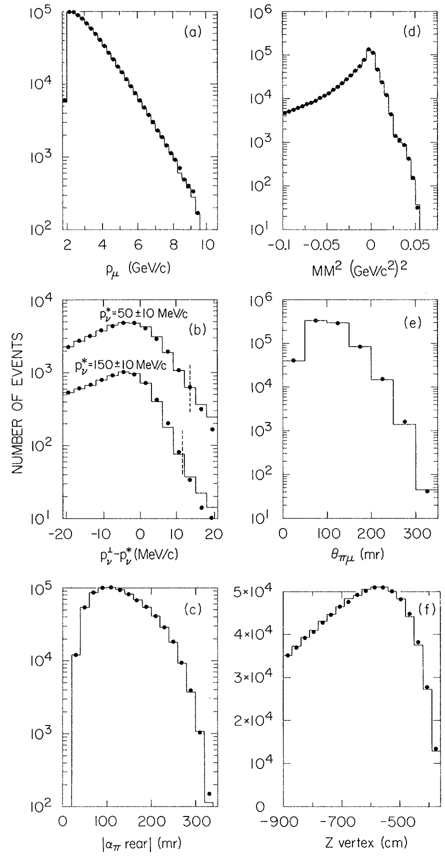

FIG. 1. Comparison of several experimental (solid lines) and Monte Carlo distributions (dots). (a) Laboratory muon momentum. (b) Difference between the transverse momentum of the neutrino and the total momentum of the neutrino in the K_L^0 rest frame; the accepted data were required to lie to the left of the dashed lines. (c) Absolute value of the direction cosine of the pion in the horizontal plane in the rear chambers. (d) Missing mass squared. (e) π-μ opening angle in the laboratory system. (f) Decay-vertex distribution along the beam direction.

solution for 80% of the data. The same procedure has been applied to the Monte Carlo events, and a detailed study indicates that no bias is introduced.

Radiative corrections which take into account the resolution and efficiency of the apparatus were applied to the data. We used the virtual corrections of Ginsberg,[6] but calculated the inner bremsstrahlung contribution by generating a large sample of Monte Carlo $K_L^0 \to \pi\mu\nu\gamma$ events according to the matrix element of Fearing, Fischbach, and Smith,[7] and subsequently analyzing them as $K_{\mu 3}^0$ decays. This treatment of radiative corrections significantly reduced the anomalous behavior of f_0 at low t as presented in a preliminary report of these data.[8]

We have made unparametrized fits to the data, as well as a two-parameter fit assuming linear t dependence of f_+ and f_0 with $f_0(0) \equiv f_+(0)$. Excellent fits to the data have been obtained by both methods. The results are shown in Fig. 2 and the following table:

Fit	χ^2	Degrees of freedom	λ_+	λ_0
Unparametrized	332	352	0.032 ± 0.004	0.017 ± 0.005
Two-parameter fit	396	398	0.030 ± 0.003	0.020 ± 0.003

The errors quoted in λ_+ and λ_0 derived from the two-parameter fit are twice the statistical error, to allow for possible systematic effects.

We have investigated the variation of λ_+ and λ_0 among many subsets of the data, such as low, medium, and high K_L^0 momenta; pion charge; inbending and outbending topology; and low, medium, and high muon center-of-mass energy. In addition, two subsets involving the quadratic ambiguity were also examined, containing either events in which one solution was at least 20 times more probable than the other, or events in which both solutions fell in the same 5×5-MeV bin on the Dalitz plot. None of these subsets yielded results which differed significantly from those obtained with the entire sample.

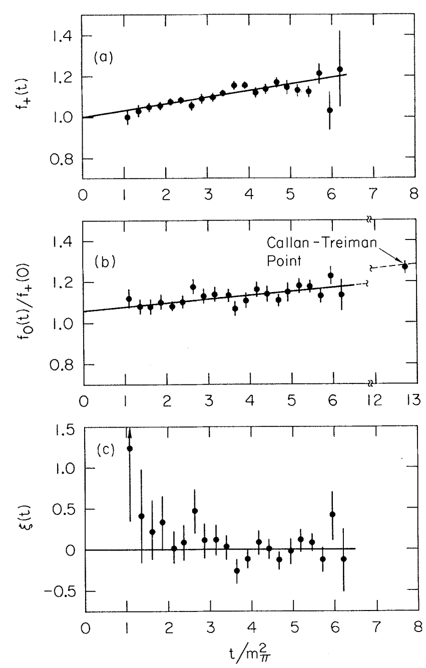

FIG. 2. Values of (a) vector form factor normalized to unity at $t = 0$, (b) scalar form factor, and (c) ξ parameter as obtained from the unparametrized fits. The Callan-Treiman point was not used in the fit. The solid line indicates the best linear fit in (a) and (b), and the best fit to a constant in (c). The unparametrized fit yields $f_0(0)/f_+(0) = 1.06 \pm 0.03$. We do not take this difference from unity to be significant, in view of the quality of the two-parameter fit with the constraint $f_0(0)/f_+(0) \equiv 1$.

In conclusion, we find that both the vector and scalar form factors in $K_{\mu 3}{}^0$ decay exhibit a linear t dependence in the physical region, with slopes $\lambda_+ = 0.030 \pm 0.003$ and $\lambda_0 = 0.020 \pm 0.003$, respectively. This value of λ_+ is consistent with $K^*(890)$ dominance of the vector form factor,[9] and with the current world averages for λ_+ as determined in studies of K_{e3} decay, in accord with μ-e universality.[10] An extrapolation of f_0 using $\lambda_0 = 0.020$ yields a value of $(1.25 \pm 0.04) f_+(0)$ at $t = m_K^2$, in excellent agreement with the Callan-Treiman-Mathur-Okubo-Pandit current-algebra prediction. Further, the extrapolated slope of f_0 confirms the prediction of Dashen and Weinstein. The traditional parameter ξ is found by a separate unparametrized fit to be 0.01 ± 0.04, and to be independent of t. Our experimental results thus support the hypotheses that chiral SU(3) ⊗ SU(3) and SU(2) ⊗ SU(2) are good symmetries of the strong interactions, and that symmetry-breaking terms are small. A detailed discussion of this experiment is in preparation.

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[1]C. G. Callan and S. B. Treiman, Phys. Rev. Lett. 16, 153 (1969); V. S. Mathur, S. Okubo, and L. K. Pandit, Phys. Rev. Lett. 16, 371 (1966).

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[3]L.-M. Chounet, J.-M. Gaillard, and M. K. Gaillard, Phys. Rep. 4C, 199 (1972).

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[6]E. S. Ginsberg, Phys. Rev. D 1, 229 (1970).

[7]H. W. Fearing, E. Fischbach, and J. Smith, Phys. Rev. D 2, 542 (1970).

[8]G. Donaldson *et al.*, in *Proceedings of the Sixteenth International Conference on High Energy Physics, The University of Chicago and National Accelerator Laboratory, September 1972,* edited by J. D. Jackson and A. Roberts (National Accelerator Laboratory, Batavia, Ill., 1973).

[9]If the vector form factor is given by K^* dominance, $f^+ = m_{K^*}^2/(m_{K^*}^2 - t)$, then our data should yield $\lambda_+ = 0.029$ if the linear parametrization is used. We are grateful to Professor J. J. Sakurai for a stimulating comment on this point.

[10]S. Wojcicki, in *Proceedings of the Sixteenth International Conference on High Energy Physics, The University of Chicago and National Accelerator Laboratory, September 1972,* edited by J. D. Jackson and A. Roberts (National Accelerator Laboratory, Batavia, Ill., 1973), Vol. 2, p. 209.

ERRATA

TUNABLE FAR-INFRARED RADIATIONS FROM HOT ELECTRONS IN n-TYPE InSb. K. L. I. Kobayashi, K. F. Komatsubara, and E. Otsuka [Phys. Rev. Lett. 30, 702 (1973)].

In the third line of the first column on page 702, the phase "for the first time" should be omitted. The sentence beginning on the last line of that column should be replaced by the sentence, "The experimental results give the first evidence of the existence of the energy-loss process due to photon emission."

The following sentence should be inserted between the fourteenth and fifteenth lines of the first column on page 703: "Recently, Gornik[12] also observed tunable radiation from n-type InSb using a tunable narrow-band detector."

[12]E. Gornik, Phys. Rev. Lett. 29, 595 (1972).

INVERSE PROBLEM WITH CONSTRAINTS. D. J. Ernst, J. E. Monahan, C. M. Shakin, and R. M. Thaler [Phys. Rev. Lett. 30, 929 (1973)].

A prior solution to many of the mathematical questions involved in this study had been given by Girardeau and Wannier.[1] The authors deeply regret that through an oversight on their part no reference whatsoever to this prior work appeared in their Letter.

OPTICAL DIELECTRIC FUNCTION OF THE LITHIUM-FLUORIDE CRYSTAL. W. Paul Menzel, Chun C. Lin, Doyle F. Fouquet, Earl E. Lafon, and Roy C. Chanel [Phys. Rev. Lett. 30, 1313 (1973)].

The graphs of Figs. 1 and 2 should be interchanged.

OBSERVATION OF EXCITONIC POLARONS AT CYCLOTRON RESONANCE IN GERMANIUM. E. Otsuka, T. Ohyama, and T. Sanada [Phys. Rev. Lett. 31, 157 (1973)].

On page 4, line 9, instead of $V_q = ie\gamma/qV$, read $V_q = ie\gamma/qV^{1/2}$.