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Null Search for Bursts of Gravitational Radiation

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A calibrated detector of kilohertz-band gravitational radiation has been built which has sufficiently improved sensitivity over Weber's apparatus to allow comparison with his two-detector coincidence results. No events were observed by us at 710 Hz during a recent three-month period of observation. Weber's data at 1661 and 1030 Hz would imply that we should have seen more than 400 events. During these observations, our sensitivity to gravitational bursts was many times that of Weber during 1969–1970. Absolute limiting flux values are given.

Large bursts of kilohertz-band gravitational radiation have been reported by Weber.[1-3] The most significant features of Weber's observations and analysis are his claims of (1) excess coincidences (above chance due to noise) in a two-detector system and (2) a sidereal correlation of these coincidence events. Estimates[4,5] of Weber's detection sensitivity to gravitational radiation imply mass loss rates of $(10^4-10^5)m_{\odot}$/yr for our galactic nucleus, if the source is there. Questions regarding Weber's sensitivity claims have been raised by others.[5-7] We examine here only the implications of claim (1). Sufficiently large bursts should produce unmistakable output on a single, large detector of adequate sensitivity. A two-detector experiment is not necessary if local interference is small. Although Weber's detectors are not calibrated directly, existing data on his system provide some yardstick for comparison of our independent results. In this Letter we examine Weber's limiting noise using published data,[3,8] report our observations, and compare our flux limit with our estimate of Weber's minimum detectable flux.

It is necessary to perform the classic Weber two-detector coincidence experiment in order to verify the claimed flux of gravitational radiation (GR); occasional signals in a one-detector system might come from some local source of interference. However, given this claimed observation of GR above a certain flux level, a more sensitive single-detector null result is sufficient to disprove the claim.

For the mechanically resonant aluminum bar (one-dimensional oscillator) detectors discussed here, the energy of oscillation varies slowly, except during shock excitation. Per mode, the resonant bar has an rms thermal potential or kinetic energy (Brownian motion) of $\frac{1}{2}kT$. A sudden increase in this energy due to a burst of gravitational radiation can be distinguished from Brownian motion. For two detectors in coincidence, this ultimate energy resolution depends on the type of signal processing and extra noise present. In nuclear counting, the signal-to-noise ratio (S/N) is greatly improved by using two detectors in coincidence because the "noise" pulses are brief compared to the time between pulses. However the detection systems discussed here are limited (see below) by wide-band white noise, and a two-detector coincidence system gains no more than a factor of 2 in S/N over one detector. More exactly, a two-detector coincidence system is not more than twice as sensitive than one detector if the signal is less than the limiting Brownian-motion and preamplifier noises. However, a two-detector system does offer immunity from occasional local interference. Finally, in a null-check experiment of this kind, we must be careful to address the questions of detector frequency, threshold sensitivity, and signal signature assumed in the data analysis.

In maximizing the sensitivity of a detector system to gravitational radiation, we must optimize several parameters.[9] The transducer has capacitance C_2. We define β as that fraction of the mechanical energy in one longitudinal mode and one degree of freedom (i.e., potential energy) of the bar which is present instantaneously as electrical energy $\frac{1}{2}C_2V^2$ at the transducer output.[10] The integrated absorption cross section $\int\sigma(\nu)\,d\nu$ over the antenna resonance at frequency ω is proportional to $m\omega^2l^2$, where m is the detector mass and l its length. The fraction of a gravitational wave burst of energy spectral density ϵ (erg cm^{-2} Hz^{-1}) appearing as electrical energy is then proportional to $m\omega^2l^2\epsilon\beta$. Similarly, the Brownian motion appears as an electrical energy $\frac{1}{2}kT\beta$ if the electromechanical transducers instrument either the bar potential or kinetic energy. However, as in the classic Brownian motion problem[11] where the particle and observer have different response times, the effective electrical energy due to Brownian motion is given by $\frac{1}{2}kT\beta$ multiplied by t/τ, where t is the observation time and τ is the decay time of the amplitude of vibration of the bar ($t \ll \tau$). Of course, the preamplifier contributes noise, proportional to the electronics bandwidth ($\sim t^{-1}$). The general expression for the power signal-to-noise ratio for one degree of freedom is given by

$$S/N \approx \alpha m\omega^2l^2\epsilon\beta(kT\beta t/2\tau + N/t)^{-1}, \qquad (1)$$

where $\alpha \simeq G/c^3$ and $N(\omega)$ is an electrical noise factor[9] given (for high input impedance preamplifiers) by

$$N = (4/\pi)kT(R_sC_2 + \omega^{-1}\tan\delta) + (\text{preamplifier parallel noise}).$$

This expression for N gives the electrical noise arising from the three remaining irreducible sources (δ = transducer loss). Generally, the term containing the preamplifier series-noise resistance R_s is the largest, for state-of-the-art field-effect transistors.

Equation (1) indicates that in order to maximize sensitivity to GR at some fixed frequency ω, one should maximize $m\omega^2l^2\beta\tau \sim m\beta Q$, and choose some integration time t such that the two terms in the noise denominator are equal. If amplifier parallel noise is much greater than transducer parallel noise, one must choose a transducer such that $R_sC_2 \approx \omega^{-1}\tan\delta$. We have done this with an 8000-lb aluminum bar as the detector mass. We shall now describe the system and our observational results, and compare this with the Weber system and observations. Although we have previously used other detection schemes employing various types of transducers and demodulation methods, we confine this description to the system which produced the three months' results quoted herein.

The 3.6×10^6-g, 357-cm-long aluminum bar is instrumented with four symmetrically placed PZT-8 ferroelectric transducers around its middle. These are wired in parallel and go to four parallel preamplifiers (each $R_s \approx 40\ \Omega$). These outputs are added and narrow-banded at the bar's first longitudinal mode frequency (710 Hz) with a bandwidth of 3 Hz. The antenna has a loaded Q of 220 000. Linear detectors in phase quadrature detect the 710-Hz signal (reference oscillator within 5 ppm). These quadrature components (averaged 100 msec) are recorded on a digital tape system (10 Hz sampling rate) and are also squared, summed, and square rooted to obtain the distance from the origin in the complex plane (amplitude). This averaged amplitude (1 sec) is recorded on a chart recorder (4 sec/mm) alongside outputs of a narrow-band microphone at 710 Hz and other seismic and magnetic detectors. The antenna is in a vacuum of $<10^{-2}$ Torr and the vacuum tank is floating on triaxial air mounts of $\frac{1}{2}$ Hz resonance. This system is inside an acoustic enclosure and the electronics and data acquisition are isolated from the power line and electromagnetically shielded.

The antenna is calibrated[9] by introducing known mechanical forces through a capacitively coupled end plate. The Brownian motion (Fig. 1) is easily observed and is thermal (290°K) in spectrum, energy probability distribution, and absolute value (from calibration). Other relevant parameters are the transducer capacitance $C_2 = 50$ nF, and the total coupling $\beta = 2\times10^{-4}$. This represents a 10^2-fold increase in βQ over Weber's 1969–1970 system. Our Brownian motion observed corresponds to an rms strain $\Delta l/l = 3\times10^{-16}$. We have increased our S/N over Weber's detectors by raising the differential absorption cross section ($\times 6$) through increased mass and length (this lowered the resonant frequency from 1660 to 710 Hz), larger electromechanical coupling, higher Q, lower noise preamplifiers, and optimization of transducer capacitance. As a result, total wide-band noise is $\frac{1}{9}$ the total Brownian motion, corresponding to a limit strain of $\Delta l/l = 3.5\times10^{-17}$ and, for the computer-processed data, an instrumentation noise temperature of 10°K. (In this program, the computer looks at the differences between the expected and actual positions of the antenna vector in the complex plane.) We now report observations with this detector.

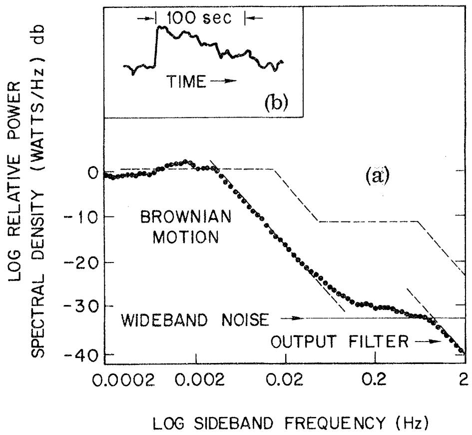

FIG. 1. (a) Line shape of our detector resonance obtained through computer analysis of our data tapes. The power spectral density at the line center (Brownian motion) is 34 dB above the wide-band preamplifier and transducer noise spectral density. Integrating our spectrum, this wide-band noise limit corresponds to a limiting strain sensitivity of $\Delta l/l = 3.5\times10^{-17}$. The noise spectrum of the bar drops off at 20 dB/decade above $(4\tau)^{-1}$ Hz exactly as expected for linear detection of a resonance with Lorentzian (Breit-Wigner) shape. Included is our estimate of Weber's 1970 noise spectrum (dashes). In (b) is shown our detector amplitude signature (typical) for an artificial $\frac{1}{4}kT$ shock excitation. In several hundred seconds the baseline wanders over a range $\sim kT/2$ due to Brownian motion.

Because of the increased S/N, we do not search far below the noise. We have closely examined our chart records for 10 December 1972 through 14 February 1973, and 5 March through 4 April 1973 looking for an increase in potential energy (risetime $\lesssim 1$ sec) of $\frac{1}{4}kT$ or larger, followed by the expected long decay [see Fig. 1(b) for signature]. We have found no such events. We should have seen <1 due to chance. Our results are consistent with purely thermal (Brownian) noise down to a threshold of $\Delta l/l = 2\times10^{-16}$. We have routinely applied precise low-level mechanical shocks, 710-Hz mechanical bursts, and slowly swept signals from our calibrator of intensity $\frac{1}{4}kT$, reproducing the expected output [Fig. 1(b)]. [There is some evidence for signals far below the noise. Continuous convolution[12] of the tape data for the detector complex-plane output with the expected (measured) response to shock excitation has shown that our detector is occasionally excited (in nonthermal fashion) at a level as high as $kT/8$. At this time we cannot rule out local interference at this level ($<kT/8$).] Our null observations imply $\epsilon\int\sigma(\nu)\,d\nu < kT/4$, with burst duration <1 sec. This gives an upper limit to the GR energy spectral density per burst of $\epsilon < 3\times10^6$ erg cm^{-2} Hz^{-1} at 710 Hz. Corresponding burst energy densities range from 3×10^6 erg cm^{-2} (1 sec duration) to 3×10^9 erg cm^{-2} (10^{-3} sec duration).

We now show that, if claim (1) is correct, and if similar GR bursts were incident during our observing period, we should have clearly seen more than 400 events during these reported observations. To do this, we must compare our S/N to that of Weber. We use his system of 1969–1970 (hereafter called W1) as a means of comparison for several reasons: The system parameters for his 1969–1970 detector are published[8]; in addition, this was the period of time during which he obtained most of his data[3] (published) which yields a large coincidence excess and sidereal effect; improvements in his system ($\sim 10\times$ in S/N) have apparently not produced a notable increase in event rate.[13] We first show that it was impossible for the W1 system to resolve the Brownian motion of the bar in 0.5 sec or less. The W1 system[8] had the following parameters: $\omega = 2\pi(1661)$ sec^{-1}, $C_2 = 100$ nF, $\beta = 5\times10^{-6}$, $Q = 7.7\times10^4$. Although not reported, the series-noise resistance R_s of Weber's Nuvistor pentode must have been at least 700 Ω.[9] Nevertheless, much of the resulting wide-band noise was rejected by Weber's inductive input feedback (R_f) preamplifier. Using Weber's[8] equivalent circuit, we compute Brownian-power-to-wide-band-noise ratio $(S/N)_B$ referred to the output of the preamplifier. The mean signal voltage squared in this case is $\langle V_{\rm Brown}^2\rangle \approx 4kT\Delta\nu R_f^2/R_1$, where $\Delta\nu = \pi\nu_0/2Q$ is the noise bandwidth of the bar resonance. Summing the squares of all the wide-band noises, we obtain

$$R_1^{-1}(S/N)_B^{-1} = (\Delta f/\Delta\nu)[R_s(R_f+R_2)^2/R_f^2R_2^2 + R_2^{-1} + R_f^{-1}] + R_sR_1^{-2},$$

where $\Delta f = 1$ Hz is the electronics noise bandwidth. With his values of $R_f = 3\times10^5$ Ω, $R_1 = 2600$ Ω, and $R_2 = 2\times10^5$ Ω, we obtain $(S/N)_B = 1$.

Thus, we estimate that the W1 system was wide-band-noise limited at kT, i.e., at the rms Brownian noise level ($T_{\rm eff} \sim 600$°K). We can now make a direct comparison of our $(S/N)_{\rm BTL}$ with the W1 system: $(S/N)_{\rm BTL}/(S/N)_{\rm W1} \equiv R$. This relative S/N for detection of GR will be the product of the relative absorption cross section and the relative S/N for observation of the bar's Brownian motion.

Using all the above data, we obtain $R=[(m\omega^2l^2)_{BTL}/(m\omega^2l^2)_{W1}][kT/kT(t/\tau)_{BTL}]\approx 3(t/\tau)_{BTL}^{-1}$ for the relative S/N, where $(t/\tau)_{BTL}\simeq 10^{-3}$ (see Fig. 1), corresponding to our values $t=0.1$ sec, $\tau=10^2$ sec. This gives a relative improvement in S/N over the W1 system of $>10^3$, if the two systems were limited by thermal-like noise alone. However, this is calculated for both systems exactly at the noise level (wide-band noise limit). We must consider the thresholds in the two systems, taking into account the spectrum and statistics of the noise. The W1 system was thresholded so that the threshold was crossed 77 times per day ($N_AN_B=6000$).[3] Since the W1 system had[8] $t\lesssim 0.5$ sec, the threshold would have been crossed (on the average) about once per second if it had been set at the average Brownian plus wide-band noise level ($\frac{1}{2}kT+\frac{1}{2}kT$). Therefore, since Weber had a mean of $(N_AN_B)^{1/2}\simeq 77$ crossings per day (86 400 sec), the threshold had to be at a value which gave a decrease of roughly $86\,400/77\simeq 10^3\simeq e^7$ in crossing probability. Thus, we estimate that Weber was thresholding above $7kT$. Similarly, we did not threshold at our wide-band noise limit, but rather at $\frac{1}{4}kT$. Therefore we have an estimated increase in S/N for GR of $3\times 7\times 4=84$ over the W1 detectors.

Also we consider the question of detection efficiency. Experimentally, we detected test pulses $\geq\frac{1}{4}kT$ with ~60% efficiency (we look for sudden decreases as well as increases). The W1 system counted only positively going, fixed-threshold crossings and its efficiency has variously been estimated at $\lesssim 10\%$, in rough agreement with his quoted[13] triples-versus-doubles ratio. Accounting for the additional factor of 2 due to two detectors in the W1 system, this relative power S/N of 80/2 and relative efficiency of six gives a total sensitivity increase to GR bursts in excess of that required to perform a single-detector null search.

Recently, Weber has been detecting ~5 events per day,[13] about the same as the W1 system. Also, he reports[13] that there are as many events at 1030 Hz as there are at 1660 Hz. Thus, we should have seen at least 450 events (most of them off scale) during our period of observation, even assuming that the events detected by Weber are the only existing kilohertz events. A more reasonable pulse height distribution would predict many thousand. From Weber's experiments,[13] if he had been detecting short bursts of GR, the flux at 710 Hz would not be expected to be significantly less than at 1030 Hz. We do not speculate here on what Weber is observing, but it seems unlikely that these events are gravitational radiation (splash, slowly swept,[14] or otherwise) particularly since our detector is a Weber-type energy detector ($t\sim 1$ sec) and therefore is relatively insensitive to details of the wave form of the metric curvature.[15]

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