Dispersion of First Sound in Liquid Helium near the λ Transition*

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The velocity of first sound in liquid helium has been measured for frequencies between 5.4 and 208 kHz and at temperatures 1 μK$< |T_\lambda - T| <$5 mK. The dispersion $u(\omega)-u(0)$ for each frequency shows a single asymmetric peak at T_λ. The Landau-Khalatnikov order-parameter relaxation process alone is not adequate to describe the magnitude or the shape of the dispersion. The maximum of the dispersion remains frequency dependent even at the highest frequencies.

The acoustic modes of liquid helium have been extensively studied to investigate the superfluid phase transition. Rudnick and his co-workers have recently measured the velocity and attenuation of first sound near T_λ.[1-3] Their results have been compared with theoretical predictions.[4-8] In this Letter we report on measurements of the velocity $u(\omega)$ of first sound in liquid helium at frequencies between 5.4 and 208 kHz and in the temperature range 1 μK$< |T_\lambda - T| <$5 mK. The dispersion $u(\omega)-u(0)$ calculated from the data shows a single asymmetric peak at T_λ. We have compared our data with the theory of Landau and Khalatnikov (LK) for order-parameter relaxation,[4] as modified by Pokrovskii and Khalatnikov (PK).[5] The measured dispersion is larger than this prediction, and seems to contain contributions from other processes. The maximum of the dispersion remains frequency dependent even at our highest frequencies, and shows a trend consistent with high-frequency data at 1[8,9] and 500 MHz.[10]

The velocity of first sound in liquid helium was measured in a cylindrical copper resonator (2.0 cm long, 0.55 cm i.d.) with a vertical axis. The ends of the resonator were terminated by identical condenser microphones with aluminized Mylar diaphragms used to excite and detect plane-wave resonances of the cavity.[2] The resonator was isolated from the He II bath by placing it in an evacuated can. After filling the resonator at 1.3 K with liquid helium, a low-temperature valve in the vacuum can was closed and isolated the sample from the bath. A liquid-vapor interface in a small ballast volume between the resonator and the cold valve assured that all data were taken under saturated vapor pressure. The temperature was measured with a carbon resistance thermometer mounted axially in the resonator in direct contact with the liquid. The power dissipated in it was 2×10^{-10} W. The temperature resolution was better than 2 μK. We did not detect a plateau or a jump in the resistance of our thermometer at T_λ. We consider this an indication that the temperature gradients in our system near T_λ were not appreciably larger than our temperature resolution. The temperature of the resonator was regulated with a heater wound on its outer wall. The data were taken while the temperature of the liquid in the resonator drifted up at a controlled rate of 3 to 10 μK/min for $|T_\lambda - T| <$ 500 μK. A wave analyzer was locked to one of the resonant first-sound harmonics of the helium-filled cavity, and automatically tracked the resonant frequency as the temperature drifted.[2] The phase of the electronic loop of this feedback system was checked frequently and adjusted when necessary.

Because of the gravitational pressure gradient in the sample and the slope of the λ line, the transition temperature at the bottom of our sample was 2.5 μK lower than at the top.[11] Since we did not observe any anomaly in the thermometer at T_λ, we used the temperature of the inflection point of the velocity as the temperature $T_\lambda{}^s$ at which the λ temperature reached the upper end of our sample. This choice is correct for $\omega = 0$.[8] We also measured the temperature at which the maximum attenuation occurred. Comparing these temperatures with the data of Ref. 3, it was found that the inflection point stays at $T_\lambda{}^s$ to within 2 μK at all our frequencies. The behavior of the minimum of the velocity is quite different. It occurs when the λ temperature is at the bottom of a helium sample of finite height only for $\omega=0$, and is displaced as a result of dispersion to lower temperatures with increasing frequency. All temperatures given in this Letter are referenced to the temperature $T_\lambda{}^s$.

Figure 1 shows the velocity difference $u(\omega) - u_\lambda(0)$ for the range 80 μK$> T_\lambda{}^s - T > -$50 μK for six frequencies between 5.4 and 208 kHz, as well as the values calculated for $\omega = 0$.[8] The data at

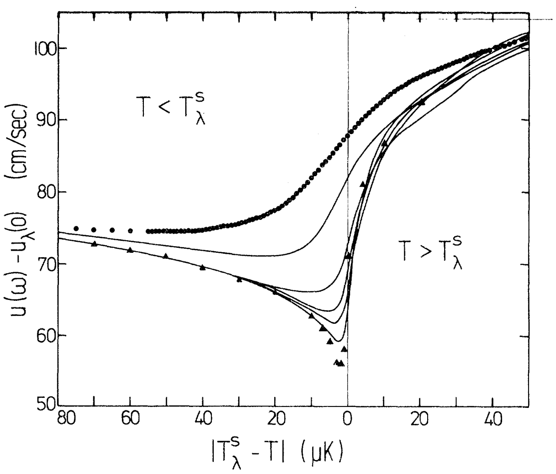

FIG. 1. Measured first-sound velocity $u(\omega)$ minus $u_\lambda(0)$ as a function of $|T_\lambda{}^s - T|$. $u_\lambda(0)$ is the thermodynamic velocity in a zero-height sample at T_λ; $T_\lambda{}^s$ is the λ temperature at the top of our sample. Circles, data at $\omega/2\pi = 208$ kHz; triangles, calculated thermodynamic velocity for $\omega/2\pi = 0$ in a sample of 2 cm height (Ref. 8). Full lines, smooth curves drawn through data points at $\omega/2\pi = 104$, 54, 27, 16, and 5.4 kHz, in decreasing order.

each frequency were taken in individual runs. The experimentally determined velocity is $u(\omega)$, and $u_\lambda(0)$ is the thermodynamic velocity at T_λ in a sample of zero height, taken as 217.3 m/sec from Ref. 2. The velocity of sound was determined from the measured frequencies by normalizing to the data of Ref. 2 at $T_\lambda{}^s - T = 500$ μK. The normalization was done so that an accurate comparison could be made to the data of Ref. 2 and the calculations of Ref. 8. Our data are independent of any choice of the normalization temperature as long as it was chosen from $T_\lambda{}^s - T > 300$ μK, where there was no dispersion. The total change of the velocity in the temperature range of Fig. 1 is only 0.2%. The velocity data are precise to $\Delta u/u = 10^{-5}$. It is apparent from Fig. 1 that there were temperature inhomogeneities in the liquid for $T > T_\lambda$ which are of the order of a few microkelvins. These small temperature inhomogeneities become noticeable in this experiment because of its precision and the strong temperature dependence of the first-sound velocity at $T \gtrsim T_\lambda$.

From the velocity data $u(\omega) - u_\lambda(0)$ we determined the dispersion $u(\omega) - u(0)$, where $u(0)$ is the thermodynamic zero-frequency velocity for our 2-cm-high sample.[8] The combined errors for the dispersion for $T < T_\lambda$ are 0.4 cm/sec, and for $T > T_\lambda$ the errors may be as large as 3 cm/sec because of the temperature inhomogeneity. The data show a rapid decrease of the dispersion for $T > T_\lambda$, and demonstrate a strong asymmetry of the dispersion peak. The dispersion measured by Barmatz and Rudnick at $\omega/2\pi = 22$ and 44 kHz is consistent with our data if the gravitational effects are considered for their 4.4-cm-high sample.[2,12]

The sound attenuation for 0.6 MHz $< \omega/2\pi < 3.17$ MHz was analyzed as resulting from two contributions[3]: one below T_λ due to an order-parameter (LK) relaxation process and one contributing both above and below T_λ arising from critical order-parameter fluctuations. The attenuation arising from order-parameter fluctuations is not understood in detail.[7] The mechanisms giving rise to attentuation should also determine the dispersion of sound. There are also no detailed theoretical predictions for the dispersion resulting from critical fluctuations with which we could compare the data. Qualitatively, this part of the dispersion is expected to occur both above and below T_λ with its maximum at T_λ.[6,7] We have compared our data with the dispersion due to the order-parameter relaxation process.[4,5] This dispersion is given by

$$u(\omega) - u(0) = \Delta u\, \omega^2\tau^2/(1 + \omega^2\tau^2), \tag{1}$$

where τ is the order-parameter relaxation time, and $\Delta u = u(\infty) - u(0) = A\epsilon^{0.022}/C_p{}^2$ [$\epsilon = (T_\lambda - T)/T_\lambda$; C_p = specific heat].[5,7,13] We have obtained $A = (1.3 \pm 0.2)\times 10^5$ from the data for the attenuation maximum of Williams and Rudnick,[3] and the data of Ahlers[14] for C_p (J/mole K). The temperature dependence of Δu has usually been neglected in the discussions of dispersion or attenuation data,[2,3,8,9,12,15] although Δu changes by about 50% per decade of $T_\lambda - T$.

In Fig. 2 we show the measured dispersion $u(\omega) - u(0)$ for $\omega/2\pi = 208$ kHz. In addition, we show Eq. (1) evaluated with a constant $\Delta u = 21.6$ cm/sec,[3] as well as evaluated with $\Delta u = (1.3\times 10^5) \times \epsilon^{0.022}/C_p{}^2$. We used the relaxation time τ given in Ref. 3.[15] Figure 2 also shows the expected influence of the gravitational field on the dispersion at 208 kHz for our 2-cm-high sample. Taking the temperature dependence of Δu into account substantially decreases the magnitude of the LK dispersion near T_λ. Both this temperature dependence and the gravitational average shift the position of the peak of the LK dispersion and of the measured dispersion to $T < T_\lambda{}^s$. The gravitational average affects the dispersion at lower frequencies more because the peak is nar-

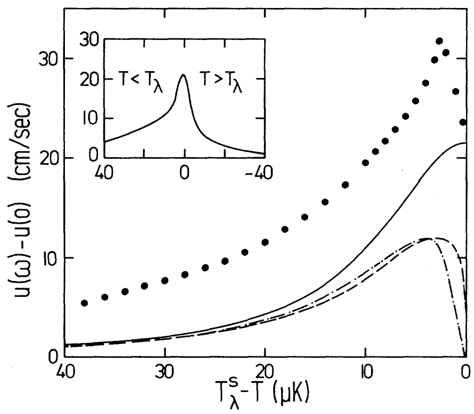

FIG. 2. First-sound dispersion at $\omega/2\pi = 208$ kHz as a function of $T_\lambda{}^s - T$. Circles, present data. The theoretical dispersion is calculated from Eq. (1) for $\Delta u = 21.6$ cm/sec (solid line) (Ref. 3), for $\Delta u = (1.3\times 10^5)\times \epsilon^{0.022}/C_p{}^2$ (dashed line) (Ref. 13), and including the effects of gravity (dash-dotted line). The inset shows the dispersion which remains when the gravity-average calculation (dash-dotted line) is subtracted from the measured data.

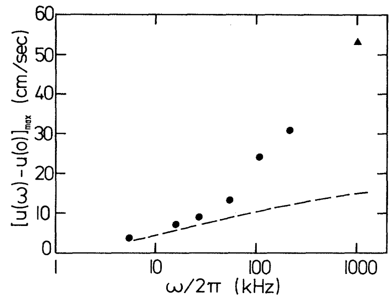

FIG. 3. Maximum dispersion of first sound as a function of frequency. Circles, data of the present work; triangle, value at 1 MHz (Refs. 7 and 8); dashed line, maximum measurable dispersion calculated from Eq. (1) including the temperature dependence of Δu and the gravity average for a 2-cm-high sample.

rower. The inset in Fig. 2 shows the dispersion remaining after subtracting the gravity-averaged PK dispersion (dash-dotted line) from the experimental data. The additional dispersion shows a peak at T_λ which may be attributed to order parameter fluctuations, as has been suggested for the attenuation.[3] The apparent asymmetry of this part of the dispersion may be due to the temperature inhomogeneity for $T > T_\lambda$ and the uncertainty in the theoretical calculation of the PK dispersion for $T < T_\lambda$. At all frequencies we see contributions to the dispersion in addition to the LK relaxation process, if Eq. (1) correctly describes this process. Only for temperatures such that $\omega\tau < 0.1$ do our data allow an $\omega^2\tau^2$ behavior of the dispersion.

Figure 3 shows the measured maximum of the first-sound dispersion as a function of frequency. In addition, Fig. 3 shows the maximum dispersion expected from order-parameter relaxation calculated with Eq. (1) including the temperature dependence of Δu and the influence of gravity for our sample. Especially at the higher frequencies the maximum of the dispersion is larger than expected from the PK theory and is a strongly increasing function of frequency. The continuous increase of the dispersion maximum with frequency is supported by the value at 1 MHz,[8,9] and the recent value for 500 MHz, 2.3 ± 0.2 m/sec, occuring very close to T_λ.[10] Our calculation using the PK theory[5,7] predicts a maximum dispersion of 0.5 m/sec at $T_\lambda - T = 6$ mK for 500 MHz. In the megahertz range the attenuation could be separated into two contributions of about equal magnitudes.[3] Our data are consistent with the assertion that the dispersion, too, arises from the LK relaxation phenomenon and from order-parameter fluctuations. Clearly, there is a need for a theory explaining the observed attenuation and dispersion of first sound in liquid helium near T_λ.

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[1]I. Rudnick and K. A. Shapiro, Phys. Rev. Lett. 15, 386 (1965).

[2]M. Barmatz and I. Rudnick, Phys. Rev. 170, 224 (1968).

[3]R. D. Williams and I. Rudnick, Phys. Rev. Lett. 25, 276 (1970).

[4]L. D. Landau and I. M. Khalatnikov, Dokl. Adad. Nauk SSSR 96, 469 (1954).

[5]V. L. Pokrovskii and I. M. Khalatnikov, Pis'ma Zh. Eksp. Teor. Fiz. 9, 225 (1969) [JETP Lett. 9, 149 (1969)]; I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. 57, 489 (1969) [Sov. Phys. JETP 30, 268 (1970)].

[6]R. A. Ferrell, N. Menyhárd, H. Schmidt, F. Schwabl, and P. Szépfalusy, Ann. Phys. (New York) 47, 565 (1968).

[7]P. C. Hohenberg, in "Critical Phenomena, Proceedings of the International School of Physics 'Enrico Fermi,' Course 51," edited by M. S. Green (Academic, to be published).

[8]G. Ahlers, Phys. Rev. 182, 352 (1969).

[9]C. E. Chase, Phys. Fluids 1, 193 (1958).

[10]G. Winterling, F. S. Holmes, and T. J. Greytak, in Proceedings of the Thirteenth International Conference

on Low Temperature Physics, Boulder, Colorado, 1972 (to be published), and to be published.

[11] G. Ahlers, Phys. Rev. 171, 275 (1968).

[12] G. Ahlers, J. Low Temp. Phys. 1, 609 (1969).

[13] The temperature dependence of Δu is mainly determined by the behavior of $1/C_p^2$. The exponent of the weaker term, $\epsilon^{0.022}$, was evaluated from the data of G. Ahlers [Phys. Rev. A 3, 696 (1971)] invoking scaling laws.

[14] Ahlers, Ref. 13.

[15] The attenuation data were analyzed neglecting the temperature dependence of Δu (Ref. 3). An analysis with the PK theory (Ref. 5) leads to a maximum at $\omega\tau < 1$. Taking the temperature dependence of Δu into account strongly influences the value of the relaxation time τ. Such an analysis does not affect the calculated PK dispersion by more than the errors quoted above for A. The conclusions in this Letter about the dispersion are not influenced by the details of the analysis of the attenuation data.

Instability of an Unneutralized Relativistic Electron Beam

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Experimental observations of the propagation of an intense, unneutralized, annular, relativistic electron beam along a homogeneous magnetic guide field show that it is unstable. The instability is accompanied by a loss of beam particles and intense microwave emission.

High-current relativistic electron beams are now being used in both plasma heating and confinement experiments.[1-4] While these experiments use neutralized electron beams, the diode and injection regions can be unneutralized. Techniques now exist for the production and transport of low-impedence, electrostatically unneutralized beams.[5,6] We describe a series of experiments investigating the stability of an unneutralized electron beam similar to that described by Friedman and Ury[7] and also used in microwave generation experiments.[8,9] In the experiments reported here the operation is at relatively high impedance with beam energies of 400 keV, diode currents of up to 30 kA, and pulse durations of 60 nsec. The beam is an annulus with a 3.8-cm diameter and a thickness of 0.2 cm. It is propagated along a magnetic guide field through a 3-m-long metallic drift tube of 5.0-cm diameter. The base pressure in the drift tube is below 5×10^{-5} Torr.

Figure 1 shows Lucite witness plates mounted at different positions along the drift tube for a guide-field strength of 8 kG. The beam thickness has grown from its initial value of about 1.5 mm at injection to 3.0 mm at the first witness-plate location. Within the next 0.75 m the beam expands radially inward until it fills the central part of the tube. As seen in Fig. 1, the filling is not completely uniform but exhibits an azimuthal structure with seven or eightfold azimuthal variation. This feature is always present but not extremely well defined or completely repeatable as regards the azimuthal mode number. A more quantitative estimate of the expansion rate is shown in Fig. 2 which plots the thickness of the annular beam as a function of the ax-

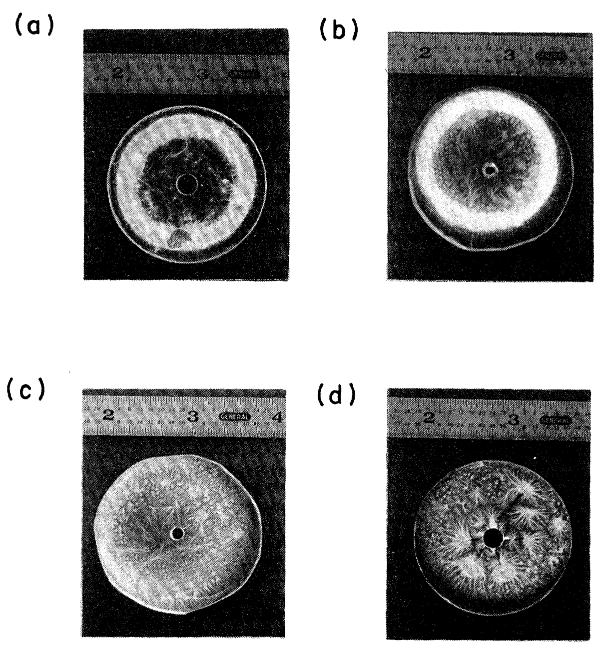

FIG. 1. Witness plates mounted at (a) 0.75 m, (b) 1.2 m, (c) 1.5 m, and (d) 3.0 m along the drift tube. The beam remained centered in the tube in all cases.